Ivan Franko National University of Lviv

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of National Academy of Sciences of Ukraine

Taras Shevchenko National University of Kyiv

Institute of Mathematics of National Academy of Sciences of Ukraine

The 15th Ukraine Algebra Conference

Book of Abstracts

July 8–12, 2025

Ivan Franko National University of Lviv

Organizing committee

Taras Banakh
Yevhen Bondarenko
Viktoria Brydun
Nataliia Dzhaliuk
Andriy Gatalevych
Olena Hryniv
Marta Maloid-Glebova
Andriy Oliynyk
Vasyl' Petrychkovych
Andriy Plaksin
Olha Popadiuk
Iryna Raievska
Maryna Raievska
Oleh Romaniv
Andriy Sagan
Volodymyr Shchedryk
Iryna Tsyganivska
Mykhailo Zarichnyi

Scientific committee

Chairman: Yu. Drozd (USA & Ukraine)

Members:

O. Artemovych (Poland)
T. Banakh (Ukraine)
V. Bavula (UK)
L. Bedratyuk (Ukraine)
O. Bezushchak (Ukraine)
I. Burban (Germany)
M. Dokuchaev (Brazil)
V. Futorny (China)
R. Grigorchuk (USA)
I. Kashuba (China)
L. Kurdachenko (Ukraine & Spain)
V. Lyubashenko (Ukraine)
S. Maksymenko (Ukraine)
V. Mazorchuk (Sweden)
V. Nekrashevych (USA)
M. Nikitchenko (Ukraine)
A. Oliynyk (Ukraine)
B. Oliynyk (Poland & Ukraine)
A. Petravchuk (Ukraine)
V. Petrychkovych (Ukraine)
M. Pratsiovytyi (Ukraine)
I. Protasov (Ukraine)
A. Pulemotov (Australia)
O. Pypka (Ukraine)
V. Shchedryk (Ukraine)
I. Shestakov (Brazil)
Ya. Sysak (Ukraine)
V. Ustimenko (UK)
E. Zelmanov (China)
M. Zarichnyi (Ukraine)
A. Zhuchok (Germany & Ukraine)

These proceedings gather the papers presented at the 15th Ukraine Algebra Conference (XV UAC), held in a hybrid format from July 8–12, 2025 at Ivan Franko National University of Lviv. Traditionally co-organized by Taras Shevchenko National University of Kyiv and the Institute of Mathematics of the National Academy of Sciences of Ukraine, the UAC has met biennially since its inaugural meeting in Slovyansk (1997) to unite established and early-career researchers in algebra from Ukraine and around the world. Past venues include Vinnytsia (1999, 2019), Sumy (2001, 2023), Lviv (2003, 2013), Odesa (2005, 2015), Kamianets-Podilskyi (2007), Kharkiv (2009), Luhansk (2011), Kyiv (2017, 2021).

We thank all conference participants for their abstracts, the Editorial Committee for its dedicated efforts, and everyone for supporting Ukraine.

We trust these proceedings will both memorialize the conference's vibrant exchanges and inspire future developments in algebra. We look forward to continuing this tradition of collaboration at the 16th UAC in 2027.

— *The Scientific and Organizing Committees*

Contents

N. Andruskiewitsch[1], I. Heckenberger[2], L. Vendramin[3]

Nichols algebras over solvable groups

[1] Universidad Nacional de Córdoba, CIEM – CONICET, Córdoba, Argentina

[2] Philipps-Universität Marburg, Marburg, Germany

[3] Vrije Universiteit Brussel, Brussels, Belgium

Nichols algebras appear in various areas of mathematics, ranging from Hopf algebras and quantum groups to Schubert calculus and conformal field theory. In this talk, I will review the main challenges in classifying Nichols algebras over groups and discuss some recent classification theorems. In particular, I will highlight a recent classification result, achieved in collaboration with Andruskiewitsch and Heckenberger, concerning finite-dimensional Nichols algebras over solvable groups.

[1] N. Andruskiewitsch, I. Heckenberger, L. Vendramin. Pointed Hopf algebras of odd dimension and Nichols algebras over solvable groups. `arXiv:2411.02304 [math.QA] (2024)`

[2] I. Heckenberger, E. Meir, and L. Vendramin. Finite-dimensional Nichols algebras of simple Yetter–Drinfeld modules (over groups) of prime dimension. *Adv. Math.*, 444:Paper No. 109637, 2024.

E-mail: ✉[1] *nicolas.andruskiewitsch@unc.edu.ar,*
✉[2] *heckenberger@mathematik.uni-marburg.de,*
✉[3] *leandro.vendramin@vub.be.*

Nicolás Andruskiewitsch[1], Olivier Mathieu[2]

Noetherian enveloping algebras of simple graded Lie algebras

[1] Universidad Nacional de Córdoba and CONICET, Córdoba, Argentina

[2] CNRS and Université Claude Bernard, Lyon, France

We show that the universal enveloping algebra of an infinite-dimensional simple $\mathbb{Z}^n$-graded Lie algebra is not Noetherian, a partial answer to a well-known conjecture that is unavoidable for the classification of Noetherian Hopf algebras.

[1] Nicolás Andruskiewitsch and Olivier Mathieu, *Noetherian enveloping algebras of simple graded Lie algebras.* J. Math. Soc. Japan 1-15, (May, 2025) `DOI: 10.2969/jmsj/93619361`

E-mail: ✉[1] *nicolas.andruskiewitsch@unc.edu.ar*, ✉[2] *mathieu@math.univ-lyon1.fr.*

Nikita Arskyi

On semigroups whose divisibility relation is a partial order

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Let S be a semigroup. The *divisibility* relation $a|b :\Leftrightarrow b \in S^1aS^1$ on S is a partial order if and only if S is $\mathcal{J}$*-trivial*, i.e. the Green's relation $\mathcal{J}$ on S coincides with the equality relation. The class of all $\mathcal{J}$-trivial semigroups contains, in particular, all nilsemigroups.

Proposition 1. *The class of $\mathcal{J}$-trivial semigroups is closed under taking subsemigroups, arbitrary Cartesian products, Rees quotient semigroups and unions of ascending chains.*

By $\mathrm{Sub}(S)$, $\mathcal{L}\mathrm{Id}(S)$, $\mathcal{R}\mathrm{Id}(S)$ and $\mathrm{Id}(S)$ we denote the lattices of all subsemigroups, left ideals, right ideals and two-sided ideals of a semigroup S, respectively.

Proposition 2. *Let S be a $\mathcal{J}$-trivial semigroup. There is a one-to-one correspondence between the extensions of the divisibility relation $|$ to a linear order and maximal chains in the lattice $\mathrm{Id}(S)$.*

Proposition 3. *Let S be a $\mathcal{J}$-trivial semigroup and L be one of the lattices $\mathrm{Sub}(S)$, $\mathcal{L}\mathrm{Id}(S)$, $\mathcal{R}\mathrm{Id}(S)$ and $\mathrm{Id}(S)$. Between two elements $T_1, T_2 \in L$, $T_1 < T_2$, there are no other elements of L if and only if $|T_2 \setminus T_1| = 1$.*

Proposition 4. *All maximal left, right and two-sided ideals of a $\mathcal{J}$-trivial semigroup are its maximal subsemigroups. All maximal subsemigroups of a nilsemigroup are its maximal ideals.*

Theorem 5. *Let S be a nilpotent semigroup. Let L be one of the lattices $\mathrm{Sub}(S)$, $\mathcal{L}\mathrm{Id}(S)$, $\mathcal{R}\mathrm{Id}(S)$ or $\mathrm{Id}(S)$. Then for any $T_1, T_2 \in L$ such that $T_1 < T_2$, there exist an immediate successor T_1' of T_1 and an immediate predecessor T_2' of T_2, such that*

$$T_1 < T_1' \leq T_2, \qquad T_1 \leq T_2' < T_2.$$

[1] Ganyushkin O., Mazorchuk V. Classical Finite Transformation Semigroups: An Introduction. Springer-Verlag, 2009.

E-mail: ✉ *nikitaarskyi@knu.ua.*

Orest D. Artemovych

Minimal non-(finite dimensional) Lie algebras

Department of Mathematics, Silesian University of Technology, ul. Kaszubska 23, Gliwice 41-100, Poland

A. G. Gein (Dnestr Notebook, Question 3.28) asked: Does there exist

(a) an infinite dimensional Lie algebra all of whose proper subalgebras are finite dimensional,

(b) an infinitely generated Lie algebra all of whose proper subalgebras are finite dimensional,

(c) an infinitely generated Lie algebra all of whose proper subalgebras are finitely generated.

We obtain an affirmative answer to these question.

E-mail: ✉ *oartemovych@polsl.pl.*

Nikita Avramenko

Practical parallel LDPC-based δ-ensemble of threshold secret-sharing schemes

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Threshold secret sharing schemes are an important component of modern cryptography, allowing a secret to be split among multiple parties so that only authorized subsets can reconstruct it. In [1], a threshold scheme requiring only $O(N)$ additions was introduced by B. Applebaum, O. Nir, and B. Pinkas, offering a more effective approach for distributing secrets. Building on this foundation, we present a practical implementation of this construction with several technical enhancements.

One notable improvement over our previous implementations is the use of a more robust library of LDPC (Low-Density Parity-Check) codes [2]. This updated library offers improved code constructions and decoding routines that are better aligned with the algebraic structure of error-correcting codes.

The base scheme was further redesigned to support parallel processing. By dividing the codeword into independent chunks and encoding them separately, we enable efficient parallelism during both the sharing and reconstruction phases. For example, with a threshold setting of 4/5 and the Phif64 decoder, tests on a 6-core CPU demonstrated a 7-8x speedup compared to the sequential version.

Notably, certain modifications made to simplify and streamline the implementation have inadvertently resulted in the emergence of an ensemble of secret sharing schemes. When operating over sufficiently large finite groups, this construction exhibits the behavior of a δ-ensemble, where δ denotes the failure probability of the ensemble and is a negligible function of n.

Taken together, the revised implementation offers concrete technical enhancements that improve scalability. Parallelism boosts throughput, and framing the constructions as a δ-ensemble opens new avenues for analyzing scheme behavior.

[1] B. Applebaum, O. Nir, and B. Pinkas, "How to Recover a Secret with O(n) Additions," In *CRYPTO 2023*, Springer-Verlag, 2023, pp. 236–262.

[2] D. Estevez, *ldpc-toolbox*, Version v0.7.0, October 4th 2024. Available at: https://crates.io/crates/ldpc-toolbox

E-mail: ✉ *nikitaavramenko@knu.ua.*

Taras Banakh

The interplay between Fano and Desargues axioms

Ivan Franko National University of Lviv, Lviv, Ukraine

A *projective plane* is a pair $(\Pi, \mathcal{L})$ consisting of a nonempty set Π of points and a nonempty set $\mathcal{L}$ of lines (which are subsets of Π) such that the following two axioms are satisfied:

- any distinct points $x, y \in \Pi$ are contained in a unique line $\overline{x\,y} \in \mathcal{L}$,
- any two distinct lines contain a unique common point.

A *triangle* in a projective plane is any triple abc of non-collinear points. Two triangles abc and $a'b'c'$ are *perspective* if the lines $\overline{a\,a'}$, $\overline{b\,b'}$, $\overline{c\,c'}$ are distinct and contain a common point.

A projective plane $(\Pi, \mathcal{L})$ is defined to be

- *Fano* if for any quadrangle $abcd$ in Π the set $(\overline{a\,b} \cap \overline{c\,d}) \cup (\overline{a\,c} \cap \overline{b\,d}) \cup (\overline{a\,d} \cap \overline{b\,c})$ is contained in a line;
- *Desarguesian* if for any perspective triangles abc and $a'b'c'$ in Π, the set $(\overline{a\,b} \cap \overline{a'\,b'}) \cup (\overline{b\,c} \cap \overline{b'\,c'}) \cup (\overline{a\,c} \cap \overline{a'\,c'})$ is contained in a line;
- *uno-Desarguesian* (resp. *bi-Desarguesian*) if for any perspective triangles abc and $a'b'c'$ in Π with $a' \in \overline{b\,c}$ (and $b' \in \overline{a\,c}$), the set $(\overline{a\,b} \cap \overline{a'\,b'}) \cup (\overline{b\,c} \cap \overline{b'\,c'}) \cup (\overline{a\,c} \cap \overline{a'\,c'})$ is contained in a line.

Theorem 1. *For a projective plane* Π, *the following are equivalent:*

(1) Π *is bi-Desarguesian;*
(2) Π *is uno-Desarguesian or Fano;*
(3) *for every ternar* (R, T) *of* Π, *the binary operations* $+ : (x, y) \mapsto T(x, 1, y)$ *and* $\heartsuit : (x, y) \mapsto T(1, x, y)$ *are commutative;*
(4) *for every ternar* (R, T) *of* Π, *the binary operations* $+$ *and* $\heartsuit$ *are commutative and associative.*

If the projective plane Π *is finite, then* (1)–(4) *are equivalent to:*

(5) Π *is Desargusian;*
(6) *for every ternar* (R, T) *of* Π, *the operation* $+$ *is commutative;*
(7) *for every ternar* (R, T) *of* Π, *the operation* $+$ *is associative;*
(8) *for every ternar* (R, T) *of* Π, *the operation* $\heartsuit$ *is commutative;*
(9) *for every ternar* (R, T) *of* Π, *the operation* $\heartsuit$ *is associative.*

The equivalence (2) $\Leftrightarrow$ (5) is a combined result of Moufang and Gleason; (5) $\Leftrightarrow$ (6) $\Leftrightarrow$ (7) was proved by Kegel and Lüneburg in 1963.

[1] T. Banakh, *Linear Geometry and Algebra*, (`arxiv.org/abs/2506.14060`).

E-mail: ✉ *t.o.banakh@gmail.com.*

Taras Banakh[1], Vladyslav Pshyk[2]

A characterization of 3-dimensional affine spaces

Ivan Franko National University of Lviv, Lviv, Ukraine

In this talk we will present a simple characterization of an affine 3-dimensional space X by merely 4 axioms describing properties of lines. A *liner* is a pair $(X, \mathcal{L})$ consisting of a set X of points and a family $\mathcal{L}$ of subsets of X, called *lines*.

A set of point $A \subseteq X$ in a liner $(X, \mathcal{L})$ is called *flat* if a line $L \in \mathcal{L}$ is a subset of A whenever it has at least two common points with L. For a set of point $A \subseteq X$ we denote by $\overline{A}$ the smallest flat subset of X that contains the set A. The *rank* of a subset $A \subseteq X$ is defined as the smallest cardinality $|B|$ of a subset $B \subseteq X$ such that $A \subseteq \overline{B}$. Flat subsets of rank 3 in liners are called *planes*.

The principal result of this talk is the following theorem characterizing 3-dimensional affine spaces over corps.

Theorem 1. *Assume that a liner $(X, \mathcal{L})$ satisfies the following four axioms:*

- (Euclid) *Any two distinct points belong to a unique line;*
- (Playfair) *For every plane $P \subseteq X$, line $L \subseteq P$ and point $x \in P \setminus L$ there exists a unique line Λ in X such that $x \in \Lambda \subseteq P \setminus L$;*
- (Hilbert) *If two planes in X have a common point, then they have at least two common points;*
- (Rank) *There exist four points that do not belong to any plane.*

Then there exists a 3-dimensional vector space V over a skew-field R and a bijective function $f : X \to V$ such that

$$\{f[L] : L \in \mathcal{L}\} = \{x + R{\cdot}v : x \in V,\ v \in V \setminus \{0\}\}.$$

[1] T. Banakh, *Geometry and Algebra in Liners*, Lviv, 2025 `https://www.researchgate.net/publication/383409915`.

[2] V. Pshyk. *A characterization of 3-dimensional affine spaces*, Bachelor Thesis, Lviv, 2025.

E-mail: ✉[1] *t.o.banakh@gmail.com*, ✉[2] *vladyslav.pshyk@lnu.edu.ua.*

V. V. Bavula

Δ-locally nilpotent algebras, their ideal structure and simplicity criteria

SMPS, Division of Mathematics, University of Sheffield, Sheffield, UK

The class of Δ-locally nilpotent algebras introduced in [2] is a wide generalization of the algebras of differential operators on commutative algebras. Examples include all the rings $\mathcal{D}(A)$ of differential operators on commutative algebras in arbitrary characteristic, the universal enveloping algebras of nilpotent, solvable and semi-simple Lie algebras, the Poisson universal enveloping algebra of an arbitrary Poisson algebra, iterated Ore extensions $A[x_1, \ldots, x_n; \delta_1, \ldots, \delta_n]$, certain generalized Weyl algebras, and others.

In [1], simplicity criteria are given for the algebras differential operators on commutative algebras. To find the simplicity criterion was a long standing problem from 60'th. The aim of the talk is to describe the ideal structure of Δ-locally nilpotent algebras and as a corollary to give simplicity criteria for them. These results are generalizations of the results of [1]. Examples are considered.

[1] V. V. Bavula, Simplicity criteria for rings of differential operators, *Glasgow Math. J.*, **64** (2021), no. 2, 347–351; arXiv:1912.07379.

[2] V. V. Bavula, Δ-locally nilpotent algebras, their ideal structure and simplicity criteria, *J. Pure Appl. Algebra* **229** (2025), 107861, doi: https://doi.org/10.1016/j.jpaa.2024.107861.

E-mail: ✉ *v.bavula@sheffield.ac.uk.*

Leonid Bedratyuk

First order joint projective invariants

Khmelnytskyi National University, Khmelnytskyi, Ukraine

Let $G = PGL(3, \mathbb{R})$ act diagonally on n copies of the affine plane. Each point (x_i, y_i) is mapped via a projective transformation, and a smooth function $u^{(i)} = u(x_i, y_i)$ is attached to each point.

We consider the first jet space, that is, the space with coordinates

$$(x_i, y_i, u_{10}^{(i)}, u_{01}^{(i)}), \quad i = 1, \ldots, n,$$

where $u_{10}^{(i)} := \partial u^{(i)}/\partial x_i$ and $u_{01}^{(i)} := \partial u^{(i)}/\partial y_i$.

Let J_n denote the field of rational functions in these variables. The subfield $J_n^G \subset J_n$ consists of absolute first–order projective invariants, that is, rational functions invariant under the diagonal action of G.

We define the basic first–order expressions

$$\Phi_{i,j,k} := (x_i - x_j)\, u_{10}^{(k)} + (y_i - y_j)\, u_{01}^{(k)}, \quad 1 \le i < j \le n,\ k \in \{i, j\}.$$

Each $\Phi_{i,j,k}$ transforms covariantly under G, and the products

$$I_{i,j} := \Phi_{i,j,i} \cdot \Phi_{i,j,j}$$

are invariant under the projective group action.

We prove that the field J_n^G is generated by all such expressions:

$$J_n^G = \mathbb{R}(I_{i,j} \mid 1 \le i < j \le n).$$

Among them, exactly $4n - 8$ elements form a transcendence basis.

This provides a complete algebraic description of the absolute first–order differential invariants under the diagonal projective action.

E-mail: ✉ *leonidbedratyuk@khmnu.edu.ua.*

Jonah Berggren[1], Khrystyna Serhiyenko[2]

Consistent dimer models on surfaces with boundary

University of Kentucky, Lexington, USA

A dimer model is a quiver with faces embedded in a surface, which gives rise to certain noncommutative algebras called dimer algebras. Consistent dimer models on tori have been studied extensively in the physics literature, in relation to phase transitions in solid state physics, while those on the disk are related to Grassmannian cluster algebras. We define and investigate various notions of consistency for dimer models on general surfaces with boundary in [2]. We prove that the dimer algebra of a strongly consistent dimer model is bimodule internally 3-Calabi-Yau with respect to its boundary idempotents. As a consequence, its Gorenstein-projective module category categorifies the cluster algebra given by the underlying quiver.

Moreover, in [1] we give an explicit combinatorial description of boundary algebras of consistent dimer models on disks, which provide categorification of cluster structures on positroid varieties in the Grassmannian.

[1] Jonah Berggren and Khrystyna Serhiyenko, Boundary Algebras of Positroids, preprint arXiv:2404.02886, 2024.

[2] Jonah Berggren and Khrystyna Serhiyenko, Consistent Dimer Models on Surfaces with Boundary, preprint arXiv:2310.02454, 2023.

E-mail: ⌧[1] *jrberggren@uky.edu,* ⌧[2] *khrystyna.serhiyenko@uky.edu.*

Oksana Bezushchak

Jordan homomorphisms of algebras of triangular matrices

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Let A, B be associative algebras over a commutative associative ring Φ. A Φ-linear mapping $f\colon A \to B$ is called a *Jordan homomorphism* if $f(a^2) = f(a)^2$, $f(aba) = f(a)\, f(b)\, f(a)$ for all $a, b \in A$.

Reduction of Jordan homomorphisms to homomorphisms and anti-homomorphisms of the underlying associative algebras is known: (a) if the algebra A contains three pairwise orthogonal *full* idempotents (W. S. Martindale [4], N. Jacobson [3]); (b) if B is semiprime and f is injective (M. Brešar [2]).

Note that the algebra of $n \times n$ upper-triangular matrices is neither semiprime nor does it in general contain proper full idempotents.

D. Benkovič [1] described Jordan homomorphisms of algebras of upper-triangular matrices over an associative commutative ring $\Phi \ni \frac{1}{2}$ that are Φ-linear. We extend this result to noncommutative rings.

Let R be an associative algebra over an associative commutative ring $\Phi \ni \frac{1}{2}$, with unit 1. Denote by $T(n, R) = \left\{ A = (a_{ij}) \in M_n(R) \;\middle|\; a_{ij} = 0 \text{ for } i > j \right\}$. Let $T_0(n, R) = \left\{ A \in T(n, R) \;\middle|\; a_{ii} = 0 \text{ for } 1 \le i \le n \right\}$ be its ideal of strictly upper-triangular matrices, and let $Diag(\Phi)$ denote the subalgebra of all diagonal matrices with entries in Φ on the main diagonal.

Theorem 1. *Let $n \geq 2$ and let $\varphi\colon T_n(R) \longrightarrow S$ be a Jordan homomorphism. Then there exist a unique homomorphism $\varphi\colon T(n, R) \to S$, and a unique anti-homomorphism $\psi\colon T(n, R) \to S$ such that $\varphi = \psi$ on $Diag(\Phi)$, and $\varphi(a)\,\psi(b) = \psi(b)\,\varphi(a) = 0$ for all $a, b \in T_0(n, R)$, and $f = \varphi = \psi$ on $Diag(\Phi)$, $f = \varphi + \psi$ on $T_0(n, R)$.*

Theorem 2. *Under the assumptions $R = \Phi$, the same conclusion holds without any additional restriction on additive torsion.*

1. Benkovič D. Jordan homomorphisms on triangular matrices. Linear Multilinear Algebra 2005, **13**, 345–356.
2. Brešar M. Commuting traces of biadditive mappings, commutativity preserving mappings, and Lie mappings. Trans. Amer. Math. Soc. 1993, **335**, 525–546.
3. Jacobson N., Rickart C. Jordan homomorphisms of rings. Trans. Amer. Math. Soc. 1950, **69**, 479–502.
4. Martindale W.S. 3rd. Lie isomorphisms of prime rings. Trans. Amer. Math. Soc. 1969, **142**, 437–455.

E-mail: ✉ *bezushchak@knu.ua.*

Collin Bleak

Embedding certain automatic groups into the rational group R

School of Mathematics and Statistics University of St Andrews, Scotland, UK

We introduce left-continuous automatic groups as a subclass of the well-known automatic groups. The class of left-continuous automatic groups is a bit mysterious as a subclass of the automatic groups, but, we know at least that the class contains the CAT(0) Cubical Complex groups (CCC groups), which themselves represent a broad class of groups of topical interest. Our main theorem states that all left-continuous automatic groups embed as subgroups of the rational group R, a group introduced in 2000 by Grigorchuk, Nekrashevych, and Suschanskii. We will provide definitions and examples of these various groups along the way, and have some discussion as well of different forms of boundaries of groups. We also discuss how similar embedding theorems have been of use in larger programs of discovery. Joint work with Belk, Chatterji, Matucci and Perego.

E-mail: ✉ *cb211@st-andrews.ac.uk.*

Ievgen Bondarenko

The word problem and growth of groups

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Let G be a finitely generated group with a finite generating set S. The word problem of G with respect to S is the decision problem WP_G, which asks whether a given word over $S \cup S^{-1}$ represents the identity element in G.

In this talk, I will discuss the time complexity of the word problem with respect to deterministic Turing machines with a single tape. Let $\mathrm{DTIME}(t(n))$ be the complexity class of all languages solved in time $O(t(n))$ by such a machine. By a result of Anisimov (1971), $\mathrm{WP}_G \in \mathrm{DTIME}(n)$ if and only if G is finite. Kobayashi (1985) showed that $\mathrm{DTIME}(n) = \mathrm{DTIME}(o(n \log n))$.

Theorem 1. *Let G be a finitely generated group. Then* $\mathrm{WP}_G \in \mathrm{DTIME}(n \log n)$ *if and only if G is virtually nilpotent.*

I will present a connection between the word problem and the growth rate of a group. In particular, the word problem for groups of exponential growth requires at least quadratic time (on a deterministic single-tape Turing machine). A natural question arises:

Question. Does $\mathrm{WP}_G \in \mathrm{DTIME}(o(n^2))$ imply G is virtually nilpotent?

The question concerns specifically groups of intermediate growth between polynomial and exponential. A notable class of such groups arises as automaton groups. I will present results on the complexity of the word problem in automaton groups generated by bounded and polynomial automata.

[1] I. Bondarenko. The word problem and growth of groups. Journal of Algebra, Volume 677, P. 252–266, 2025.

E-mail: ✉ *ievgbond@gmail.com.*

Nataliia Bondarenko

Matrix representations of infinitely iterated wreath products of one-dimensional Lie algebras

Kyiv National University of Construction and Architecture, Kyiv, Ukraine.

Let p be a prime number and $n > 2$ a fixed integer. It is shown in [1] that the Lie algebra $L_{p,n}$ associated with the lower central series of the Sylow p-subgroup $S_{p,n}$ of the symmetric group $Sym(p^n)$ decomposes as the n-th iterated wreath product of one-dimensional Lie algebras over the field $\mathbb{F}_p$. We study matrix representations of the Lie algebras $L_n := L_{2,n}$ and of the infinite-dimensional Lie algebra L_∞, which is the inverse limit of L_n, that is, the infinitely iterated wreath product of one-dimensional Lie algebras over the field $\mathbb{F}_2$. It is worth noting that an embedding of the Lie algebra $UT_m(\mathbb{F}_p)$ into the Lie algebra $L_{p,n}$ was previously constructed in [2]. We construct an explicit embedding ϕ of L_n into the Lie algebra $UT_m(\mathbb{F}_2)$ of strictly upper triangular matrices for the minimal possible order $m = 2^{n-1} + 1$. As a consequence, we obtain an explicit embedding ϕ of the Lie algebra L_∞ into the Lie algebra $UT_\infty(\mathbb{F}_2)$ of infinite strictly upper triangular matrices. We show that this embedding can be constructed recursively using matrix schemes sh, introduced in [3].

Theorem. *The mapping $\phi : L_\infty \to UT_\infty$ satisfies the recursion:*

$$sh(\phi(u)) = \begin{pmatrix} \phi(u|_0) & u_1 E \\ \phi(u|_0) + \phi(u|_1) & \phi(u|_0) \end{pmatrix},$$

where for a tableau $u = [u_1, u_2(x_1), u_3(x_1, x_2), \ldots] \in L_\infty$, two tableaux $u|_0$ and $u|_1$ are defined as follows: $u|_c = [u'_1, u'_2(x_1), u'_3(x_1, x_2), \ldots] \in L_\infty$, $c = 0, 1$, and $u'_k(x_1, \ldots, x_{k-1})$ is obtained from $u_{k+1}(x_1, \ldots, x_k)$ by applying the substitution: $x_1 \mapsto c$, $x_2 \mapsto x_1$, $x_3 \mapsto x_2$, $\ldots$, $x_k \mapsto x_{k-1}$.

[1] Suschanskiy V.I., Netreba N.V. (2005). Wreath product of Lie algebras and Lie algebras associated with Sylow p-subgroups of finite symmetric groups *Algebra and Discrete Mathematics*, vol. 4, pp. 122–132.

[2] Bondarenko N.V. (2006). Lie algebras associated with Sylow p-subgroups of some classical linear groups *Bulletin of Kyiv University. Series: physical and mathematical sciences.*, vol. 2, pp. 21–27.

[3] Leonov Yu.G. (2004). Representation of finite-approximate 2-groups by infinite unitriangular matrices over a field of two elements *Mathematical studies*, vol. 22(2), pp. 134–140.

E-mail: ✉ *natvbond@gmail.com.*

Vitaliy Bondarenko

On representation type of the Hasse commutative quiver of nodal extensions of positive posets

Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

Through all posets are finite without the element 0 and subposets are full. A subposet A of a poset S is said to be upper if from $x < y$ with $x \in A$ and $y \in S$ it follows that $y \in A$. A poset S is called positive if so is its Tits quadratic form

$$q_S(z) := z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i.$$

All positive posets are first described in [2]). They can be serial if there is an infinite increasing sequence $S \subset S^{(1)} \subset S^{(2)} \subset \ldots$ with positive terms, and non-serial if otherwise. The number of non-serial positive posets is equal to 108, up to isomorphism and duality. The serial positive posets consist of two 2-parameter and one 3-parameter series.

For a poset S, denote by $\overrightarrow{H}(S)$ its (oriented) Hasse diagram as a commutative quiver. We call such quiver the Hasse commutative quiver of the poset S. A full subquiver of $\overrightarrow{H}(S)$ is said to be upper if so is the corresponding subposet of S.

We call an extension $T \supset S$ of a poset S upper if so is the subposet S of T, and nodal if any element of the subposet $T \setminus S$ of T is a node, i.e. is comparable to all elements of T; the number $m = |T \setminus S|$ is called the order of the extension T. In the case when T is an upper nodal extension of S we write $T = S^{(m)}$.

Theorem 1. *The commutative Hasse quiver $\overrightarrow{H}(S^{(m)})$ of an upper nodal extension of order m of a non-serial positive poset S is of finite representation type over a field k if and only if any its upper subquiver without cycles is a Dynkin graph.*

Theorem 2. *Let S and k be as in Theorem 1. If $\overrightarrow{H}(S^{(m)})$ is of finite and $\overrightarrow{H}(S^{(m+1)})$ of infinite representation types, then $\overrightarrow{H}(S^{(m+2)})$ is wild.*

These studies were carried out together with M. Styopochkina.

[1] V. M. Bondarenko, M. V. Styopochkina. (Min, max)-equivalence of partially ordered sets and the Tits quadratic form. *Collection of works of Inst. of Math. NAS Ukraine – Problems of Analysis and Algebra*. 2(3):18–58, 2005.

E-mail: ✉ *vitalij.bond@gmail.com.*

Alexandre Borovik[1], Şükrü Yalçınkaya[2]

Black Box Algebra

[1] Manchester University, Manchester, United Kingdom

[2] Istinye University, Istanbul, Turkey

We shall give a compact survey of principal ideas of probabilistic methods in computational algebra: what can be said about an algebraic structure (group, ring, Boolean or Heyting algebra, etc.) from observation of behaviour of a random sample of its elements? In the wider mathematics similar approaches are known under the umbrella term the *Monte Carlo method* and have a fantastic range of applications. The first, and still the most famous of them was the analysis, in 1946, of neutron diffusion paths for the hydrogen bomb.

In the case of finite algebraic structures, the Monte Carlo Method allows to analyse structures of astronomic size, not accessible to any deterministic approaches. We have interesting parallels with (logical) model theory and, which is even stranger, the Internal Set Theory of Edward Nelson, a conservative extension of the ZFC set theory which erases the boundary between finite and infinite.

In the first part of talk we will discuss these games on the boundary of infinity and explain the 'Black Box' ideology, in the second – give some concrete practically usable examples of Monte Carlo algorithms for finite groups.

In particular, we will look at a class of deceptively innocuous problems: You are given a few non-degenerate matrices $x_1, \ldots, x_m$ of size $n \times n$ over a large finite field F. What can you say about the group X generated by them in $\mathrm{GL}_n(F)$? For example, what is the order of X? The biggest group X of that kind where we managed to compute (without any supercomputers, on an old laptop) significant and important subgroups, and say something sensible about them, had about 10^{960} elements. The Observable Universe contains around 10^{80} electrons. We were computing in something which was 10^{880} times bigger than the Observable Universe.

E-mail: ✉[1] *alexandre@borovik.net*, ✉[2] *sukru.yalcinkaya@istinye.edu.tr*.

Matej Brešar

Jordan homomorphisms

University of Ljubljana and University of Maribor, Slovenia

In the first part of the talk, a brief historical overview of the theory of Jordan homomorphisms will be given. In the second part, new results will be presented. Some of them were obtained in collaboration with Efim Zelmanov.

E-mail: ✉ *matej.bresar@fmf.uni-lj.si.*

Igor Burban

Exceptional hereditary non-commutative curves and real curve orbifolds

Paderborn University, Germany

An exceptional hereditary non-commutative curve over an algebraically closed field is a weighted projective line of Geigle and Lenzing. However, over arbitrary fields, the theory of exceptional curves is significantly richer. In my talk I am going to explain the definition, examples and key properties of these classes of non-commutative curves, including their invariants and relation to squid algebras and canonical algebras.

E-mail: ✉ *burban@math.uni-paderborn.de.*

Andrii Chaikovs'kyi[1], Oleksandr Liubimov[2]

Boundedness of solutions of the first order linear multidimensional difference equations in critical case

Taras Shevchenko National University of Kyiv, Ukraine

In space $\mathbb{C}^d$ with Euclidean norm we consider the following difference equation

$$x(n+1) = Ax(n) + y(n), \ n \geq 1 \tag{1}$$

with respect to unknown sequence $\{x(n)\}_{n\geq 1} \subset \mathbb{C}^d$. The first element of this sequence $x(1)$, sequence $\{y(n)\}_{n\geq 1} \subset \mathbb{C}^d$ and square matrix $A \in \mathcal{M}_d(\mathbb{C})$ of order d are supposed to be known.

Let $J_\lambda^{(M)}$ denote the Jordan matrix of order M corresponding to an eigenvalue λ. The investigation of the boundedness of the solution of (1) can be fairly easy reduced to the investigation of the boundedness of the solutions of all the difference equations

$$x(n+1) = J_{\lambda_j}^{(M_j)} x(n) + y^{(\lambda_j)}(n), \ n \geq 1$$

where $\lambda_j \in \sigma(A)$ are eigenvalues of A.

The case of our interest is the so-called *critical case*: $\sigma(A) \cap \{z \in \mathbb{C} \mid |z| = 1\} \neq \varnothing$. In our paper [1] we present the following result

Theorem 1. *Let $M \geq 2$, $|\lambda| = 1$, $\{\widetilde{y}_m(n)\}_{n\geq 1} \subset \mathbb{C}$, $1 \leq m \leq M$ and the following conditions hold:*

- *For all $1 \leq m \leq M$: $y_m(n) = \frac{\widetilde{y}_m(n)}{n^{m-1}}$, $n \geq 1$*
- *For each $m \in \{1, \ldots, M\}$ sequence of sums $\left\{\sum_{n=1}^{N} \widetilde{y}_m(n)\lambda^{-n}\right\}_{N\geq 1}$ is bounded.*

Then solution of the equation $x(n+1) = J_\lambda^{(M)} x(n) + y(n)$, $n \geq 1$ is bounded if and only if for each $m \in \{2, \ldots, M\}$ we have:

$$x_m(1) = -\sum_{r=0}^{M-m} \left\{ (-1)^r \cdot \sum_{k=1}^{\infty} \left(\binom{k+r-1}{r} \cdot \lambda^{-k-r} \cdot y_{r+m}(k) \right) \right\}.$$

Here $x(n) = (x_1(n), \ldots, x_M(n)), y(n) = (y_1(n), \ldots, y_M(n))$.

[1] A. Chaikovs'kyi, O. Liubimov, Boundedness of solutions of the first order linear multidimensional difference equations, Bulletin of Taras Shevchenko National University of Kyiv, Physics and Mathematics. (To be published).

E-mail: ✉[1] *andriichaikovskyi@knu.ua,* ✉[2] *liubimov_oleksandr@knu.ua.*

Yevhenii Chapovskyi[1], Anatoliy Petravchuk[2]

Module structure of the Lie algebra $W_n(K)$ over $sl_n(K)$

[1] Institute of Mathematics of National Academy of Sciences of Ukraine, Kyiv, Ukraine

[2] Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Let K be an algebraically closed field of characteristic zero, and let $W_n := W_n(K)$ denote the Lie algebra of all K-derivations on the polynomial ring $K[x_1, \ldots, x_n]$. The Lie algebra W_n was studied by many authors from different viewpoints: maximal subalgebras of W_n were studied in [1; 2], solvable and nilpotent subalgebras in [4], automorphisms and derivations of $K[x_1, \ldots, x_n]$ and related structures in [1; 3]. The Lie algebra W_n admits a natural grading $W_n = \bigoplus_{i \geq -1} W_n^{[i]}$, where $W_n^{[i]}$ consists of all homogeneous derivations whose coefficients are homogeneous polynomials of degree $i+1$ or zero.

Theorem 1. *Let the Lie algebra $W_n(K) = W_n, n \geq 2$ be written as a direct sum of homogeneous components of the standard grading*

$$W_n = W_n^{[-1]} \oplus W_n^{[0]} \oplus \cdots \oplus W_n^{[m]} \oplus \ldots . \tag{1}$$

Then $L = W_n^{[0]}$ is a subalgebra of W_n, $L \simeq gl_n(K)$ and every summand of the sum (1) is a finite dimensional module over L and over the subalgebra $M_0 \subseteq W_n^{[0]}$ isomorphic to $sl_n(\mathbb{K})$. Every L-module $W_n^{[m]}, m \geq 0$ is a direct sum $W_n^{[m]} = M_m \oplus N_m$ of two irreducible submodules, where M_m consists of divergence-free derivations and N_m consists of all the derivations from $W_n^{[m]}$ that are polynomial multiple of the Euler derivation $E_n = x_1 \frac{\partial}{\partial x_1} + \cdots + x_n \frac{\partial}{\partial x_n}$.

[1] V. Bavula, The groups of automorphisms of the Lie algebras of polynomial vector fields with zero or constant divergence, Communications in Algebra, (2013), 45(3), 1114-1133.

[2] J. Bell, L. Buzaglo, Maximal dimensional subalgebras of general Cartan-type Lie algebras, Bulletin of the London Mathematical Society, (2024), v.57, issue 2, 605-624.

[3] O. Bezushchak, Derivations and automorphisms of locally matrix algebras. Journal of Algebra, (2021), v.576, 1-26.

[4] Ie. A. Makedonskyi, A.P. Petravchuk, On nilpotent and solvable Lie algebras of derivations. Journal of Algebra, (2014), 401, 245-257.

E-mail: ✉[1] *safemacc@gmail.com*, ✉[2] *apetrav@gmail.com.*

Corentin Correia

Isoperimetric profile and quantitative orbit equivalence for lamplighter-like groups

Université Paris Cité, Paris, France

It is a joint work with Vincent Dumoncel.

Two groups G and H are orbit equivalent if there exist two free probability measure-preserving G- and H-actions on a standard probability space, having the same orbits.

However Orstein and Weiss proved that two infinite amenable groups are orbit equivalent. To get an interesting theory, we add some quantitative restrictions on two maps called *cocycles*, which describe more precisely the orbit equalities of a given orbit equivalence between G and H. If G and H are amenable, quantitative orbit equivalence provides interesting information on their geometry, since the *isoperimetric profiles* of the groups give obstructions to the existence of quantitative versions of orbit equivalence (see [1, Theorems 1.1, Corollary 4.7]). In some sense, this is a more quantitative comparison between groups. The highest quantification we can get answers to the following question: if two groups are not quasi-isometric, how much do their geometry differ?

In a joint work with Vincent Dumoncel, we study quantitative orbit equivalence and isoperimetric profile for *lampshuffler groups*. Given a group H, the lampshuffler group over H is

$$\mathsf{Shuffler}(H) := \mathrm{FSym}(H) \rtimes H,$$

where $\mathrm{FSym}(H)$ is the set of permutations of H of finite support, and the action of H on it is given by $k \cdot \sigma \colon h \in H \to k\sigma(k^{-1}h)$.

Lampshufflers belong to a large class of groups which look like *lamplighter group*. They have been intensively studied in [2], where the authors found conditions for two lamplighters to be quasi-isometric, for two lampshufflers to be quasi-isometric, etc.

[1] T. Delabie, J. Koivisto, F. Le Maître, and R. Tessera. Quantitative measure equivalence between amenable groups. *Annales Henri Lebesgue*, 5:1417–1487, 2022.

[2] A. Genevois, R. Tessera. Lamplighter-like geometry of groups. ArXiv, 2024.

E-mail: ✉ *corentin.correia@imj-prg.fr.*

Oleksandra Desiateryk

Ideals of inverse symmetric semigroup connection to variants

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Ideals of the symmetric inverse semigroup IS_n, of all partial injective transformations of the set $N = \{1, 2, \ldots, n\}$ are considered.

In the paper [1] it was proved that a Brandt semigroup is not isomorphic to the variant of any semigroup.

It is well known that the set of all ideals of IS_n form the next chain with respect to inclusion $\{0\} = I_0 \subset I_1 \subset I_2 \subset \ldots \subset I_n = IS_n$.

In paper [2] it was shown that for every $1 \leq k \leq n$, the Rees quotient I_k/I_{k-1} is isomorphic to the finite Brandt semigroup.

Then it is obvious that I_1 is a Brandt semigroup. To be precise I_1 is a smallest Brandt semigroup which consists of five elements.

Earlier there was stated a hypothesis that such a statement holds for other ideals.

Now this hypothesis is proved. Hence we state it as following

Theorem 1. *Any ideal I_k, $1 \leq k \leq n-1$ of IS_n is not isomorphic to a variant of any semigroup.*

[1] Oleksandra O. Desiateryk, Olexandr G. Ganyushkin, Sandwich semigroups and Brandt semigroups. Algebra and Discrete mathematics Vol 38, No 1, 34-42 (2024)

[2] Olexandr Ganyushkin, Ivan Livinsky, Length of the inverse symmetric semigroup. Algebra and Discrete mathematics Vol 12, No 2, 64-71 (2011)

E-mail: ✉ *sasha.desyaterik@gmail.com.*

Martyn R. Dixon[1], Leonid A. Kurdachenko[2], Igor Ya. Subbotin[3]

On the structure of some nilpotent braces

[1] University of Alabama, Tuckaloosa, USA
[2] Oles Honchar Dnipro National University, Dnipro, Ukraine
[3] National University, Los Angeles, USA

A left brace is a set A with two binary operations $+$ and $\cdot$ satisfying the following conditions: A is an abelian group by addition $+$, A is a group by multiplication $\cdot$, and $a(b+c) = ab + ac - a$ for every $a, b, c \in A$.To facilitate the study of involutive set-theoretic solutions of the Yang–Baxter equation, W. Rump introduced the concept of braces in 2005 as a generalization of Jacobson radical rings. Let A be a left brace. Put $a \star b = ab - a - b$ for any elements a and b. If K, L are subbraces of A, then denote by $K \star L$ the subgroup of the additive group of A generated by the elements $x \star y$, where $x \in K, y \in L$. Put $A^{(1)} = A$ and, recursively, $A^{(\alpha+1)} = A^{(\alpha)} \star A$ for all of ordinal α and $A^{(\lambda)} = \bigcap_{\mu<\lambda} A^{(\mu)}$ for limit ordinals λ, and put $A^1 = A$ and, recursively, $A^{\alpha+1} = A \star A^{\alpha}$ for all of ordinals α and $A^{\lambda} = \bigcap_{\mu<\lambda} A^{\mu}$ for limit ordinals λ. Note that $A^{(\alpha)}$ is a ideal of A,while A^{α} is a left ideal for every α. We consider one-generator braces A such that $A^3 = \langle 0 \rangle$. We say that A is called Smoktunowicz-nilpotent ($\star -$ nilpotent) if there are positive integers n, k such that $A^{(n)} = \langle 0 \rangle = A^k$. Denote by $\mathcal{N}_{S(n,k)}$ the class of left braces satisfying $A^{(n)} = \langle 0 \rangle = A^k$ where n, k are the least integers that satisfy this property.

The one–generator braces A satisfying $A^3 = \langle 0 \rangle$ have been studied by several researchers. The next natural step is the investigation of left braces belonging to the class $\mathcal{N}_{S(4,4)}$. All considerations here become quite cumbersome. Note that the left braces in this class A are $\star -$ nilpotent nilpotent of class at most 16. This naturally leads to the idea of slightly changing the approach and focusing on a systematic study of the one-generator $\star -$ nilpotent braces. Our talk is dedicated to some results we have obtained in the study of such braces. We will also discuss new findings concerning the structure of two-generated $\star -$ nilpotent braces.

E-mail: ✉[1] *mdixon@ua.edu*, ✉[2] *lkurdachenko@gmail.com*, ✉[3] *isubboti@nu.edu.*

Mikhailo Dokuchaev

Twisted Steinberg algebras of not necessarily Hausdorff ample groupoids and regular inclusions

Instituto de Matemática e Estatística, Universidade São Paulo, Brasil

In a joint work with Ruy Exel and Héctor Pinedo, given a field K and an ample (not necessarily Hausdorff) groupoid G, we define the concept of a *line bundle* over G inspired by the well known notion from the theory of C*-algebras. If E is such a line bundle, we construct the associated *twisted Steinberg algebra* in terms of sections of E, extending the original construction introduced independently by Steinberg, and by Clark, Farthing, Sims and Tomforde. We also generalize the recent construction of (cocycle) twisted Steinberg algebras of Armstrong, Clark, Courtney, Lin, Mccormick and Ramagge. We then extend Steinberg's theory of induction of modules, not only to the twisted case, but to the much more general case of *regular inclusions* of algebras. Among our main results, we show that, under appropriate conditions, every irreducible module is induced by an irreducible module over a certain abstractly defined *isotropy algebra*. We also describe a process of *disintegration* of modules and use it to prove a version of the Effros-Hahn conjecture, showing that every primitive ideal coincides with the annihilator of a module induced from isotropy.

E-mail: ✉ *dokucha@gmail.com.*

Yuriy Drozd

Quasikrullian rings and their divisorial categories

Harvard University, Cambridge, MA, USA and
Institute of Mathematics of the NASU, Kyiv, Ukraine

Recall that a commutative ring R with the full ring of fractions Q is called *pseudonoetherian* [2] if the following conditions hold:

1. For every element $a \in R$ there is a finite set $V_{min}(a)$ of prime ideals containing a such that every prime ideal containing a also contains an ideal from $V_{min}(a)$.
2. For every $\mathfrak{p} \in V_{min}(a)$ the ring $R_{\mathfrak{p}}$ is noetherian.

   We call R *pseudokrullian* if, moreover, the following conditions hold:

3. R is *reduced*, i.e. has no nilpotent elements. Equivalently, Q is semisimple.
4. $R = R_{\mathcal{P}}$, where $\mathcal{P} = \{\mathfrak{p} \in \operatorname{spec} R \mid \operatorname{ht} \mathfrak{p} = 1\}$ and
   $R_{\mathcal{P}} = \{q \in Q \mid \forall(\mathfrak{p} \in \mathcal{P})\, \exists(r \in R \setminus \mathfrak{p})\ rq \in R\}$.

Let R be a pseudokrullian ring, $\mathcal{N}$ be a full subcategory of R-Mod consisting of such N that $N_{\mathfrak{p}} = 0$ for all $\mathfrak{p} \in \mathcal{P}$, $R\text{-}\widetilde{\text{Mod}} = R\text{-Mod}/\mathcal{N}$ (Serre quotient) and $T : R\text{-Mod} \to R\text{-}\widetilde{\text{Mod}}$ the natural projection. We also set $\mathcal{M} = \{M \in R\text{-Mod} \mid (N \subseteq M \,\&\, N \in \mathcal{N}) \Rightarrow N = 0\}$. We denote by $E(M)$ the *injective envelope* of the module M.

Theorem 1. *1. If $M \in \mathcal{M}$, also $E(M) \in \mathcal{M}$ and $E(TM) = TE(M)$.*

2. Indecomposable injective modules from $\mathcal{M}$ are $E(R/\mathfrak{p})$, where $\mathfrak{p} \in \mathcal{P}$, and Q_i, where Q_i are the simple components of the semisimple ring Q.

3. Injective modules from $\mathcal{M}$ are just coproducts of indecomposables.

Theorem 2. *1.* $\operatorname{inj.dim} TM = \sup_{\mathfrak{p} \in \mathcal{P}} \operatorname{inj.dim} M_{\mathfrak{p}}$.

2. $\operatorname{gl.dim} R\text{-}\widetilde{\text{Mod}} = \sup_{\mathfrak{p} \in \mathcal{P}} \operatorname{gl.dim} R_{\mathfrak{p}}$.

These results generalizes the results of I. Beck [1].

[1] I. Beck. *Injective modules over a Krull domain.* J. Algebra, 17, 116–131, 1971.
[2] Y. Drozd. *The semigroup of divisors of a commutative ring.* Trudy Mat. Inst. Steklov, 148, 156–167, 1978.

E-mail: ✉ *y.a.drozd@gmail.com.*

Yuriy Drozd[1], Andriana Plakosh[2]

Represenations and cohomologies of the alternating group of degree 4

[1] Harvard University, Cambridge, MA, USA, Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

[2] Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

Let A_4 be the alternating group of degree 4. We consider the *integral representations* of this group, that is $\mathbb{Z}\mathrm{A}_4$-modules M such that the abelian group of M is free of finite rank (A_4-*lattices*). Recall that a classification of 2-adic representations of A_4 was obtained by Nazarova [3]. Unfortunately, this classification gives no idea how to use it to calculate cohomologies of A_4-lattices. We propose another approach based on the technique of *Bäckström orders* [4]. Namely, since the group ring is always Gorenstein, all its 2-adic representations, except projective ones, are actually representations of an overring A [2]. In the case of $\mathbb{Z}_2\mathrm{A}_4$ this overring is a Bäckström order with the enveloping hereditary order $\mathbb{Z}_2 \times \mathbb{Z}_2[\theta] \times \mathrm{Mat}(3, \mathbb{Z}_2)$, where $\theta = \sqrt[3]{1}$, and the quotient $A/\mathrm{rad}A = \mathbb{F}_2 \times \mathbb{F}_4$. Using it, we relate 2-adic representations of A_4 with representations of the valued graph of type $\tilde{F}_4$ [1]:

$$\bullet \longleftarrow \bullet \longrightarrow \bullet \xleftarrow{\ 2,1\ } \circ \longrightarrow \circ,$$

where the fields associated with $\bullet$ are $\mathbb{F}_2$ and those associated with $\circ$ are $\mathbb{F}_4$. It allows to give a complete description of the Auslander–Reiten quiver of the category of A-lattices. We also describe all indecomposable integral representations of A_4 and explain non-uniqueness of decomposition of representations into indecomposables.

It is known that $\tau M \simeq \Omega M$ for every A-lattice M, where τ is the Auslander–Reiten transform and Ω is the syzygy of $\mathbb{Z}\mathrm{A}_4$-lattices [2]. Using it, we calculate Tate cohomologies of all A_2-lattices.

1. Dlab V., Ringel C. M. Indecomposable representations of graphs and algebras. Mem. Amer. Math. Soc., 1976, 73, 1–57.
2. Drozd Yu. A. Rejection lemma and almost split sequences. Ukr. Mat. Zh., 2021, 73, 908–929.
3. Nazarova L. A. Unimodular representations of the alternating group of degree four. Ukr. Mat. Zh., 1963, 15, 437–444.
4. Ringel C. M., Roggenkamp K. W. Diagrammatic methods in the representation theory of orders. J. Algebra, 1979, 60, 11–42.

E-mail: ✉[1] *y.a.drozd@gmail.com,* ✉[2] *andrianaplakosh@gmail.com.*

Vincent Dumoncel

Scaling groups and subgroups of wreath products

IMJ-PRG, University of Paris, France

The purpose of this talk will be to introduce scaling quasi-isometries and scaling groups, introduced recently by Genevois and Tessera in [2], in their quasi-isometric classification of some lamplighters (and, more generally, of some *halo products*).

The computation of the scaling group $\mathrm{Sc}(G)$ of an amenable group G can give access to algebraic informations on the group, that are not obvious to derive in a purely algebraic manner. This represents a good and elementary instance of the interaction that exists between geometry and group theory.

In a recent work [1], I proved the following result:

Theorem 1. *Let N be a polynomial growth group, and let G be a finitely presented group in the class $\mathcal{M}_{exp}$. Then $Sc(N \wr G) = \{1\}$.*

The class $\mathcal{M}_{\mathrm{exp}}$ has been introduced very recently by Bensaid, Genevois and Tessera [3], and encompasses many amenable groups, such as solvable Baumslag–Solitar groups $\mathrm{BS}(1, n)$, $n \geq 2$, or lamplighter groups.

The talk will focus on some nice consequences of Theorem 1, some algebraic and some geometric. In fact, results from [1] are more general and allow for instance to deduce a nice criterion to rule out the existence of quasi-isometries between some iterated wreath products.

References:

[1] Vincent Dumoncel, *Quasi-isometric rigidity for lamplighters with lamps of polynomial growth*, arXiv:2502.01849.

[2] Anthony Genevois, Romain Tessera, *Measure-scaling quasi-isometries*, Geom. Dedicata, 216, 34, 2022.

[3] Oussama Bensaid, Anthony Genevois, Romain Tessera, *Coarse separation and large-scale geometry of wreath products*, arXiv:2401.18025.

E-mail: ✉ *vincent.dumoncel@imj-prg.fr.*

Nataliia Dzhaliuk[1], Vasyl' Petrychkovych[2]

Methods for solving Sylvester-type matrix equations and the investigation of the structure of their solutions

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the NAS of Ukraine, Lviv, Ukraine

We consider the Sylvester-type matrix equation

$$AX + YB = C, \tag{1}$$

in two variables $X,\ Y$ over a ring of polynomials $F[\lambda]$, where F is a field, and over an adequate ring R. The solvability criterion for (1) is the well-known Roth's condition. In addition to the solvability conditions, solutions with certain properties are also required. In the case where, in (1) over $F[\lambda]$ at least one of matrices A or B is regular, it has been established in [1] the conditions for the existence and uniqueness of the solution $X_0,\ Y_0$ such that $\deg X_0 < \deg B$ and $\deg Y_0 < \deg A$.

In this report, for (1) over $F[\lambda]$ [2],[3], based on the standard form of polynomial matrices with respect to semiscalar equivalence, we:

- proved the existence of solutions with bounded degrees in the case where coefficients A and B are nonregular,
- described their structure in rows and columns,
- pointed out the necessary and sufficient conditions for the existence of solutions of given degrees,
- established the uniqueness criterion for these solutions.

For (1) over an adequate ring R [2], we used the standard form of a pair of matrices with respect to generalized equivalence in order to:

- derive the formulas for the general solutions,
- establish the uniqueness criterion for the particular solution.

We proposed methods for the construction of the mentioned solutions of matrix equation (1) over $F[\lambda]$ and over an adequate ring R.

[1] *Feinstein J., Bar-Ness Y.* On the uniqueness of the minimal solution to the matrix polynomial equation $A(\lambda)X(\lambda) + Y(\lambda)B(\lambda) = C(\lambda)$ // J. Franklin Inst. – 1980. – 310, No. 2. – P. 131–134.

[2] *Dzhaliuk N.S., Petrychkovych V.M.* Matrix linear bilateral equations over different domains, methods for the construction of solutions, and description of their structure // Journal of Mathematical Sciences. – 2024. – 282, No. 5. – P. 616–645. https://doi.org/10.1007/s10958-024-07206-w

[3] *Dzhaliuk N.S., Petrychkovych V.M.* Kronecker product of matrices and solutions of Sylvester-type matrix polynomial equations // Matematychni Studii. – 2024. – 61, No. 2. – P. 115–122. doi:10.30970/ms.61.2.115-122

E-mail: ✉[1]*nataliya.dzhalyuk@gmail.com,* ✉[2]*vas_petrych@yahoo.com.*

Gabriella D'Este

A theorem on support τ-tilting pairs

Department of Mathematics, University of Milano, Milano, Italy.

I will describe a bijection between the indecomposable summands of two modules of the form $P \oplus T$ and $P' \oplus T'$ such that (T, P) and (T', P') are two basic support τ-tilting pairs in the sense of [1]. The bijection obtained extends the bijections constructed in [2] and [3].

[1] Adaki T., Iyama O., Reiten I., τ-tilting theory, Composition Mathematica, 150(3), (2014), 415-452.

[2] D'Este G., Tekin Akcin H. M., A bijection between the indecomposable summands of two multiplicity free tilting modules, Bulletin of the Iranian Mathematical Society, 48 (2022), 2521-2538.

[3] D'Este G., Tekin Akcin H. M., Bijections between τ-rigid modules, to appear in Contemprorary Mathematics, Proceedings of the 14th Ukraine Algebra Conference.

[4] D'Este G., A theorem on support τ-tilting pairs, preprint.

E-mail: ✉ *gabriella.deste@unimi.it.*

Pavel Etingof

Twisted traces and positive forms on quantized Kleinian singularities of type A

Massachusetts Institute of Technology, USA

I will discuss twisted traces on quantizations of Kleinian singularities of type A_{n-1} and the corresponding orthogonal polynomials of semiclassical type. In particular, I'll give explicit integral formulas for these traces, which may be used to determine when a trace defines a positive Hermitian form on the corresponding algebra. This leads to a classification of unitary short star-products for such quantizations, a problem posed by Beem, Peelaers and Rastelli in connection with 3-dimensional superconformal field theory. In particular, this classification confirms their conjecture that for $n < 5$ a unitary short star-product is unique, and allows one to compute its parameter as a function of the quantization parameters, giving exact formulas for the functions computed numerically by Beem, Peelaers and Rastelli. If $n = 2$, this, in particular, recovers the theory of unitary spherical representations of $SL_2(\mathbb{C})$ (i.e., Harish-Chandra bimodules for $\mathfrak{sl}_2$). Thus these results may be viewed as a starting point for a generalization of the theory of unitary Harish-Chandra bimodules over enveloping algebras of simple Lie algebras to more general quantum algebras. Finally, I'll describe recurrences to compute the coefficients of short star-products corresponding to twisted traces, which are generalizations of discrete Painlevé systems, and a q-deformation of this story due to D. Klyuev. This is joint work with Daniil Klyuev, Eric Rains, Douglas Stryker.

E-mail: ✉ *etingof@math.mit.edu.*

Domink Francoeur[1], Rostislav Grigorchuk[2], Paul-Henry Leemann[3], Tatiana Nagnibeda[4]

On the structure of finitely generated subgroups of branch groups

[1] Universidad Autónoma de Madrid, Madrid, Spain
[2] Texas A& M University, College Station, USA
[3] Xi'an Jiaotong-Liverpool University, Suzhou, China
[4] Université de Genève, Genève, Switzerland

One way to study a group is to understand its subgroups lattice. Among subgroups, finitely generated subgroups are specifically important and having a characterisation of them is useful to study properties such as subgroup separability (LERF), the Ribs-Zalenski property and many others. In this work we investigate a class of branch groups containing the first Grigorchuk group as well as torsion GGS groups. Every group G in this class is defined by a nice faithful action on a regular rooted tree T. Technically, we say that G is *regularly branch* over some finite index subgroup K, meaning that for every vertex v of the tree, the subgroup $K@v \leq \mathrm{Aut}(T_v)$ (a copy of K acting below v) is contained in K.

In a regularly branch group, the $K@v$ are finitely generated subgroups. The same remains true for diagonal subgroups $\mathrm{diag}(K@v \times K@w)$ A product of such subgroups is called a *block subgroup*. This is a product of copies of K compatible with the action of G on the tree. Block subgroups are finitely generated subgroups of G. Our main result is that sometimes the converse holds:

Theorem 1. *Let G be either the first Grigorchuk groups or a torsion GGS group. Then every finitely generated subgroup of G is virtually a block subgroup.*

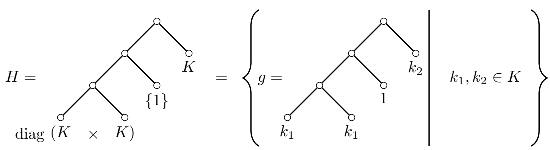

Figure 1: A block subgroup.

E-mail: ✉[1] *dominik.francoeur@uam.es*, ✉[2] *grigorch@tamu.edu*, ✉[3] *paulhenry.leemann@xjtlu.edu.cn*, ✉[4] *tatiana.nagnibeda@unige.ch.*

Iryna Fryz

Parastrophic orthogonality of ternary quasigroups

Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine

A ternary operation f defined on Q is called *invertible* and the pair $(Q; f)$ is called a *quasigroup* of the order m, if for every $a,\ b \in Q$ each of the terms $f(x, a, b)$, $f(a, x, b)$, $f(a, b, x)$ defines a permutation of Q.

A triplet of ternary operations f_1, f_2, f_3 is called *orthogonal*, if for all $a_1, a_2, a_3 \in Q$ the system of equations $\{f_i(x_1, x_2, x_3) = a_i\}_{i=1}^{3}$ has a unique solution.

For every permutation σ from the symmetric group S_4, a *σ-parastrophe* ${}^{\sigma}f$ of an invertible ternary operation f is defined by

$$ {}^{\sigma}f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} :\Longleftrightarrow f(x_1, x_2, x_3) = x_4. $$

If $4\sigma = 4$, then a σ-parastrophe is called *principal.* A ternary quasigroup is called *totally self-orthogonal* if its all different principal parastrophes are orthogonal.

A ternary groupoid is called a *group isotope* if it is isotopic to a ternary quasigroup derived from a group. A quasigroup is *medial* if it is decomposed over abelian group with pairwise commuting automorphisms.

$\mathrm{Ps}(f) := \{\sigma \mid {}^{\sigma}f = f\} \leqslant S_4$ is called *parastrophic symmetry group* of f. Let $\mathfrak{P}(H)$ denote the class of all quasigroups whose parastrophic symmetry group includes the subgroup H of the group S_4. Here, the group $S_{22} := \{\iota, (12)(34)\} \leqslant S_4$ is under consideration.

Theorem 1 ([1])**.** *A ternary group isotope $(Q; f)$ belongs to $\mathfrak{P}(\mathrm{S}_{22})$ iff there exists a group $(Q, +, 0)$, its automorphism β, a bijection α and an element $a \in Q$ such that $\beta^2 = \iota$, $\alpha(0) = 0$, $\beta(a) = -a$ and*

$$ f(x_1, x_2, x_3) = \alpha(x_1) - \beta\alpha(x_2) + \beta(x_3) + a. \tag{1} $$

Theorem 2. *A ternary medial quasigroup $(Q; f)$ defined by (1) with parastrophic symmetry group S_{22} is totally self-orthogonal iff*

$$ \alpha + \iota, \quad \beta + \iota, \quad \alpha - \beta, \quad \alpha + \beta - \beta\alpha, \quad 2\alpha^2 + \iota + \beta\alpha^2 + \alpha - \beta\alpha $$

are automorphisms of $(Q; +)$.

[1] Pirus Ye. Classification of ternary quasigroups according to their parastrophic symmetry groups, II. *Bulletin of DonNu. Series A. Natural Sciences*, **1-2** (2019), 66-75.

E-mail: ✉ *iryna.fryz@ukr.net.*

Andriy Gatalevych[1], Mariia Kuchma[2]

Some generalizations of neat range 1 for noncommutative ring

[1] Lviv National Ivan Franko University, Lviv, Ukraine

[2] Lviv Polytechnic National University, Lviv, Ukraine

All rings are associative rings with nonzero identity. If every matrix over R admits a canonical diagonal reduction then R is said to be an *elementary divisor ring*.

Right (left) Bezout rings are rings whose finitely generated right ideals are principal right (left) ideals. Bezout ring is a ring which is a right and left Bezout ring.

A ring R is said to be *a duo ring* if every right or left one-sided ideal in R is two-sided.

A ring R is said to have stable range 1, if for any $a, b \in R$ such that $aR + bR = R$ there exists $t \in R$ such that $(a + bt)R = R$.

A ring R is said to have stable range 2 if for all $a, b, c \in R$ such that $aR+bR+cR = R$, there exists $x, y \in R$ such that $(a+cx)R+(b+cy)R = R$.

Definition A ring R is said to be a ring of neat range 1 if for any elements $a, b \in R$ such that $RaR + RbR = R$ and for any nonzero element $c \in R$ there exist such elements $u, v, t \in R$ that $a + bt = uv$, where $RuR + RcR = R$, $RvR + R(1 - c)R = R$, and $RuR + RvR = R$.

Theorem 1. Let R be a Bezout duo ring of neat range 1. Then R is a ring of stable range 2.

Theorem 2. Bezout duo ring R is an elementary divisor ring iff R is a ring of neat range 1.

[1] Kaplansky I., Elementary divisor rings and modules, Trans. Amer. Math. Soc. 66(1949), 464–491.

[2] Thierrin G., On duo rings, Canad. Math. Bull. **3** (1960), 167–172.

E-mail: ✉[1] *gatalevych@ukr.net,* ✉[2] *markuchma@ukr.net.*

Volodymyr Gavrylkiv

On some functorial extensions of doppelsemigroups

Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine

A family $\mathcal{U}$ of non-empty subsets of a set S is called an *upfamily* if for each set $U \in \mathcal{U}$ any subset $F \supset U$ belongs to $\mathcal{U}$. The set $\upsilon(S)$ of all upfamilies on S is said as *the upfamily extension* of S. It was shown that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$. In this case, the Stone-Čech compactification $\beta(S)$ of a semigroup S is a subsemigroup of the semigroup $\upsilon(S)$. For $k \in \mathbb{N}\setminus\{1\}$, an upfamily $\mathcal{U} \in \upsilon(S)$ is k*-linked* if $\bigcap \mathcal{L} \neq \emptyset$ for any subfamily $\mathcal{L} \subset \mathcal{U}$ with $|\mathcal{L}| \leq k$. The extension $N_k(S)$ of S consists of all k-linked upfamilies on S. Besides the subsemigroup $N_k(S)$, the semigroup $\upsilon(S)$ also contains the subsemigroup $\lambda(S)$ of maximal 2-linked upfamilies on S. The space $\lambda(S)$ is well-known in General and Categorial Topology as the *superextension* of S.

A *doppelsemigroup* is an algebraic structure $(D, \dashv, \vdash)$ consisting of a non-empty set D equipped with two associative binary operations $\dashv$ and $\vdash$ satisfying the axioms $(x \dashv y) \vdash z = x \dashv (y \vdash z)$ and $(x \vdash y) \dashv z = x \vdash (y \dashv z)$. In the talk, we discuss the structure of the doppelsemigroups $(\upsilon(D), \dashv, \vdash)$, $(\lambda(D), \dashv, \vdash)$, $(N_k(D), \dashv, \vdash)$, $(\beta(D), \dashv, \vdash)$ on a doppelsemigroup $(D, \dashv, \vdash)$. In particular, we study right and left zeros and identities, commutativity, the center, ideals of these doppelsemigroup extensions. We introduce the functors υ, λ, N_k, β in the category **DSG** whose objects are doppelsemigroups and morphisms are doppelsemigroup homomorphisms, and show that these functors preserve strong doppelsemigroups, doppelsemigroups with left (right) zero, doppelsemigroups with left (right) identity, left (right) zeros doppelsemigroups. Also we prove that the automorphism groups of the aforementioned functorial extensions of a doppelsemigroup $(D, \dashv, \vdash)$ contain a subgroup, isomorphic to the automorphism group of $(D, \dashv, \vdash)$.

[1] V.M. Gavrylkiv, D.V. Rendziak, Interassociativity and three-element doppelsemigroups, *Algebra Discrete Math.* **28**(2) (2019), 224-247.

[2] V.M. Gavrylkiv, On the upfamily extension of a doppelsemigroup, *Mat. Stud.* **61**(2) (2024), 123–135.

[3] V.M. Gavrylkiv, Superextensions of doppelsemigroups, *Carpathian Math. Publ.* **17** (2025)

[4] V.M. Gavrylkiv, Doppelsemigroups of k-linked upfamilies, *J. Algebra Appl.* **25** (2026), 2650207.

E-mail: ✉ *vgavrylkiv@gmail.com.*

Sergiy Gefter[1], Aleksey Piven'[2]

Ring of copolynomials over a commutative ring

[1] V. N. Karazin Kharkiv National University, Kharkiv, Ukraine, B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, Kharkiv, Ukraine

[2] V. N. Karazin Kharkiv National University, Kharkiv, Ukraine

Let K be a commutative ring with identity and let $K[x_1,\dots,x_n]$ be a ring of polynomials with coefficients in K. By a copolynomial over the ring K we mean a K-linear functional defined on the ring $K[x_1,\dots,x_n]$, i.e. a homomorphism from the module $K[x_1,\dots,x_n]$ into the ring K. We denote the module of copolynomials over K by $K[x_1,\dots,x_n]'$. Let $T_1, T_2 \in K[x_1,\dots,x_n]'$, $\iota = (1,\dots,1) \in \mathbb{N}_0^n$ and $x^\alpha = x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$, where $\alpha \in \mathbb{N}_0^n$. For multi-indexes $\alpha, \beta \in \mathbb{N}_0^n$, the relation $\alpha \le \beta$ means that $\alpha_j \le \beta_j$ for all $j = 1,\dots,n$. Define a *product* of T_1 and T_2 by the following equality:

$$(T_1T_2)(x^\alpha) = \begin{cases} \sum\limits_{\beta \le \alpha-\iota} T_1(x^\beta)T_2(x^{\alpha-\iota-\beta}), & \alpha \ge \iota, \\ 0, & \text{otherwise.} \end{cases}$$

Theorem 1. *The module $K[x_1,\dots,x_n]'$ with the introduced product is an associative commutative ring without identity.*

Denote by $\frac{1}{s_1 s_2 \cdots s_n} K[[\frac{1}{s_1},\dots,\frac{1}{s_n}]]$ the ring of formal Laurent series of the form $\sum\limits_{|\alpha|=0}^{\infty} \frac{c_\alpha}{s^{\alpha+\iota}}$, where $c_\alpha \in K$ and $|\alpha| = \sum\limits_{j=1}^{n} \alpha_j$.

Theorem 2. *The mapping*

$$C : K[x_1,\dots,x_n]' \to \frac{1}{s_1 s_2 \cdots s_n} K[[\frac{1}{s_1},\dots,\frac{1}{s_n}]],\ C(T)(s) = \sum_{|\alpha|=0}^{\infty} \frac{T(x^\alpha)}{s^{\alpha+\iota}}$$

is an isomorphism of the rings.

Corollary 3. *If K is an integral domain, then $K[x_1,\dots,x_n]'$ is also an integral domain.*

E-mail: ✉[1] *gefter@karazin.ua*, ✉[2] *aleksei.piven@karazin.ua.*

Nikolaj (Mykola) Glazunov

On algebraic dynamics and resurgence on Minkowski moduli spaces

Glushkov Institute of Cybernetics NAS of Ukraine, Kyiv, Ukraine

Let $|x|^p + |y|^p \leq 1,\ p > 1$, be the solid Minkowski tube [2] with the surface boundary $\mathcal{M}T$.

Remark 1. For natural $p, d, p = 2d$ the Minkowski curves

$$\mathbb{S}^1_{2d} = C_{2d} : x^{2d} + y^{2d} = 1 \tag{1}$$

are algebraic curves. They define algebraic layers of the moduli space $\mathcal{M}T$ and the bundle of their Jacobians.

The Minkowski–Cohn moduli space $\mathcal{M}$ of admissible lattices of Minkowski balls has the form

$$\Delta(p, \sigma) = (\tau + \sigma)(1 + \tau^p)^{-\frac{1}{p}}(1 + \sigma^p)^{-\frac{1}{p}}, \tag{2}$$

in the domain

$$\mathcal{M}:\ \infty > p > 1,\ 1 \leq \sigma \leq \sigma_p = (2^p - 1)^{\frac{1}{p}}, \tag{3}$$

of the $\{p, \sigma\}$-plane, where σ is some real parameter.

We define and investigate on (1) dynamical Galois groups [3] and Tate modules, and on (2) we define semi complex vector bundle, elliptic curves bundle, bundle of Epstein zeta functions, and investigate their algebraic and dynamical properties [4–7] (resurgence [1] is included).

[1] Jean Ecalle, Six Lectures on Transseries, Analysable Functions and the Constructive Proof of Dulac Conjecture, NATO ASI Bifurcations and Periodic Orbits of Vector Fields (Eds: Dana Schlomiuk), vol. 408, Springer Dordrecht, 75-184.

[2] Minkowski H. Diophantische Approximationen. – Leipzig: Teubner, 1907. Neudruk –Warzburg: Physica-Verlag, 1961, 235 p.

[3] Ferraguti, A. A survey on abelian dynamical Galois groups, Rend. Semin. Mat., Univ. Politec. Torino 80, 41-54 (2022)

[4] Glazunov N. On coverings by Minkowski balls in the plane and a duality, Comptes rendus de Academie bulgare Sci., Tome 77, No 6 (2024).

[5] Glazunov N. Extremal functions on moduli spaces and applications. 2024, arXiv:2411.13671v3 [math.NT].

[6] Glazunov N. On the arithmetic and algebraic properties of Minkowski balls and spheres, arXiv:2407.12048v3 [math.NT]

[7] Glazunov M. On optimal packing of Minkowski spheres, Cybernetics and systems analysis, vol 61, 192–196, Springer, 2025.

E-mail: ✉ *glanm@yahoo.com.*

Rostislav Grigorchuk[1], Dmytro Savchuk[2]

Diagonal actions of groups acting on rooted trees

[1] Texas A&M University, College Station, TX, USA
[2] University of South Florida, Tampa, FL, USA

Any residually finite countable group G embeds in the automorphism group $\mathrm{Aut}(T_{\bar{m}})$ of spherically homogeneous rooted tree $T_{\bar{m}}$ determined by a sequence $\bar{m}$ of integers $m_n \geq 2$ (called the branch index). Also every countably based profinite group Γ embeds into $\mathrm{Aut}(T_{\bar{m}})$ for suitable $\bar{m}$. This leads to the actions $G \curvearrowright V_n$, $\Gamma \curvearrowright V_n, n = 1, 2, \ldots,$ $G \curvearrowright \partial T_{\bar{m}}$, and $\Gamma \curvearrowright \partial T_{\bar{m}}$, where V_n is the set of vertices of the n-th level of the tree $T_{\bar{m}}$ and $\partial T_{\bar{m}}$ is a boundary of this tree supplied with the natural topology that makes it homeomorphic to a Cantor set.

Many important groups (of Burnside type, with intermediate growth, non elementary amenable, of branch type, etc.) were constructed as groups acting on a d-regular rooted tree T_d and the boundary dynamics was often used to study various properties of these groups. One of the first papers in this direction is [2].

Given an action $G \curvearrowright \mathcal{X}$ one can consider the diagonal actions $G \curvearrowright \mathcal{X}^n$ for $n \geq 2$, and study the level of transitivity of the original action and the partition of $\mathcal{X}^n$ into orbits. This leads to a new information about the group as the constructed diagonal actions often have properties that are essentially different from the original system.

In the case when G acts on a topological space, one may study ergodic decompositions of the diagonal actions and the, so called, joinings.

We restrict ourselves to the case when $G = \mathcal{G}$ is a group of intermediate growth constructed by the first author in 1980 [1] and to the case when $\Gamma = \mathrm{Aut}(T_d)$. We introduce the notions of *maximal tree transitivity of diagonal action* and show that $\mathcal{G}$ is $2, 3$ and 4-maximally transitive but not 5-maximally transitive.

Also we describe the partitions into ergodic components for the diagonal actions of $\mathrm{Aut}(T_d)$ on ∂T_d^n.

[1] R. I. Grigorchuk. On Burnside's problem on periodic groups. *Funktsional. Anal. i Prilozhen.*, 14(1):53–54, 1980.

[2] R. I. Grigorchuk, V. V. Nekrashevich, and V. I. Sushchanskiĭ. Automata, dynamical systems, and groups. *Tr. Mat. Inst. Steklova*, 231(Din. Sist., Avtom. i Beskon. Gruppy):134–214, 2000.

E-mail: ✉[1] *grigorch@tamu.edu,* ✉[2] *savchuk@usf.edu.*

Rostislav Grigorchuk[1], Zoran Šunić[2]

Conjugates of the shift map and self-similar groups

[1] Texas A&M University, College Station, TX, USA
[2] Hofstra University, Hempstead, NY, USA
and
CAIR, Ss. Cyril and Methodius University, Skopje, North Macedonia

Consider the action of the group $\mathsf{Aut}(X^*)$ of regular rooted tree automorphisms on the tree boundary ∂X^*, that is, the action on right infinite words. Denote by $\mathsf{SEP}(X)$ the semigroup of tree automorphisms that preserve the set of eventually periodic words in ∂X^*.

For any degree $d \geq 2$, and any rooted d-ary tree automorphisms $\alpha_0, \dots, \alpha_{d-1}$, we may define a transformation of ∂X^* given by $T(xw) = \alpha_x(w)$, for $x \in X$ and $w \in \partial X^*$. The tree automorphism γ, defined by $\gamma = (\gamma\alpha_0, \dots, \gamma\alpha_{d-1})$ conjugates T to the d-ary shift map σ, so that $\gamma T \gamma^{-1} = \sigma$. We prove that if $\alpha_0, \dots, \alpha_{d-1}$ are finite state automophisms, then γ^{-1} is in $\mathsf{SEP}(X)$. On the other hand, we provide concrete examples of finite state automorphisms of the binary tree for which γ is not in $\mathsf{SEP}(X)$, thus showing that $\mathsf{SEP}(X)$ is not a group. Denote by $\mathsf{EP}(X)$ the group of tree automorphisms g such that both g and g^{-1} are in $\mathsf{SEP}(X)$. The group $\mathsf{EP}(X)$ is self-similar, regular branch group, branching over itself, and it is dense in $\mathsf{Aut}(X^*)$.

An interesting example is provided by the Collatz map T, defined on positive integers by $T(2n) = n$ and $T(2n+1) = 3n+2$, which can be extended to the ring $\mathbb{Z}_2$ of dyadic integers. The ring $\mathbb{Z}_2$ can be represented by right infinite binary words, and under this interpretation the map T is given by $T(0w) = w$ and $T(1w) = \mu_2(w)$, where μ_2 is the tree automorphism given by $\mu_2(w) = 3w+2$. Without using either the terminology of rooted trees or dyadic integers, Terras, in 1976, while working on the Collatz $3x+1$ Conjecture, defined a "parity" map γ, which turns out to be exactly the tree automorphism $\gamma = (\gamma, \gamma\mu_2)$ that conjugates T to the binary shift map σ, so that $\gamma T \gamma^{-1} = \sigma$. The Periodicity Conjecture of Lagarias can be stated as the claim that γ belongs to the group $\mathsf{EP}(X)$. The conjecture, if true, would imply that every forward T-orbit of a positive integer eventually enters a cycle.

We provide further results regarding both the general case and the specific example related to the Collatz $3x+1$ Conjecture. In particular, we provide a description of the minimal automaton for the Terras map γ and show that its Moore diagram has exponential growth.

E-mail: ✉[1] *grigorch@tamu.edu*, ✉[2] *zoran.sunic@hofstra.edu.*

Yaroslav Grushka

On representation of changeable sets in the form of a self-multiimage

Institute of Mathematics of National Academy of Sciences of Ukraine,
Kyiv, Ukraine

The present talk is devoted to the problem of representation of a changeable set in the form of multi-image of some its reference frame. Further we use the system of the notions and denotations from theory of changeable sets (see, for example, [1; 2]).

Definition 1. *We say that the changeable set $\mathcal{Z}$ is a **self-multiimage**, iff there exists a reference frame* $\mathfrak{l}_0 \in \mathcal{L}k\,(\mathcal{Z})$, *such that* $\mathcal{Z} = \mathcal{Z}\mathsf{im}\left[\mathfrak{P}^{(e)}_{\langle \mathfrak{l}_0, \mathcal{Z}\rangle}, (\mathfrak{l}_0)\,\hat{}\,\right]$, *where* $\mathfrak{P}^{(e)}_{\langle \mathfrak{l}_0, \mathcal{Z}\rangle}$ *is the evolution multi-projector, defined by the formula* $\mathfrak{P}^{(e)}_{\langle \mathfrak{l}_0, \mathcal{Z}\rangle} := \left(\left(\mathbb{T}\mathbf{m}\left(\mathbf{lk}_\alpha\left(\mathcal{Z}\right)\right), \mathfrak{B}\mathfrak{s}\left(\mathbf{lk}_\alpha\left(\mathcal{Z}\right)\right), \langle !\, \mathbf{lk}_\alpha\left(\mathcal{Z}\right) \leftarrow \mathfrak{l}_0\rangle\right) \mid \alpha \in \mathcal{I}nd\,(\mathcal{Z})\right)$.

From [1, Assertion 3.27.8] it follows that any self-multiimage is an evolutionarily visible changeable set.

Definition 2. *We say that the a precisely changeable set $\mathcal{Z}$ is **partially time-separated** iff there exists the reference frame* $\mathfrak{l}_0 \in \mathcal{L}k\,(\mathcal{Z})$ *such that for each* $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{Z})$ *and for arbitrary* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}\,(\mathfrak{l})$ *the correlations* $\omega_2 \underset{\mathfrak{l}}{\leftarrow} \omega_1$ *and* $\omega_1 \neq \omega_2$ *lead to the correlation* $\mathsf{tm}\left(\langle !\, \mathfrak{l}_0 \leftarrow \mathfrak{l}\rangle\, \omega_1\right) \neq \mathsf{tm}\left(\langle !\, \mathfrak{l}_0 \leftarrow \mathfrak{l}\rangle\, \omega_2\right)$.

The following theorem gives the simple for verification criterion, for evolutionarily visible changeable set to be representable a self-multiimage.

Theorem 1. *Any evolutionarily visible changeable set $\mathcal{Z}$ is a self-multiimage if and only if it is partially time-separated.*

Using Theorem 1 we can prove the existence of changeable sets, which can be represented as a self-multiimage as well as the existence of changeable sets, which cannot be represented as a self-multiimage.

[1] Ya.I. Grushka, *Draft Introduction to Abstract Kinematics (Version 2.0)*. Preprint: ResearchGate, (2017), DOI: 10.13140/RG.2.2.28964.27521.

[2] Ya.I. Grushka, *Set-theoretic methods in relativistic kinematics*. Thesis for the degree of Doctor of Physical and Mathematical Sciences, Kyiv: Institute of Mathematics of NAS of Ukraine, (2023), (in Ukrainian), DOI: 10.13140/RG.2.2.18858.12481.

E-mail: ✉ *grushka@imath.kiev.ua.*

Oleg Gutik

On the bicyclic monoid and bicyclic extensions

Ivan Franko National University of Lviv, Lviv, Ukraine

We shall follow the terminology of [4].

The bicyclic extension $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ for any ω-closed subfamily $\mathscr{F}$ of elements of $\mathscr{P}(\omega)$ and the bicyclic extensions $\mathscr{B}(G)$ and $\mathscr{B}^{+}(G)$ of ordered groups are introduced in [2] and [1; 3], respectively.

In our report we discuss on the following topics concerning the bicyclic extensions $\boldsymbol{B}_{\omega}^{\mathscr{F}}$, $\mathscr{B}(G)$ and $\mathscr{B}^{+}(G)$:

- the algebraic structure;
- topologizations;
- categorical properties;
- the group of automorphisms;
- the semigroup of endomorphisms.

[1] G. L. Fotedar, *On a semigroup associated with an ordered group*, Math. Nachr. **60** (1974), 297–302.

[2] O. Gutik and M. Mykhalenych, *On some generalization of the bicyclic monoid*, Visnyk L'viv. Univ. Ser. Mech.-Mat. **90** (2020), 5–19 (in Ukrainian).

[3] O. Gutik, D. Pagon, and K. Pavlyk, *Congruences on bicyclic extensions of a linearly ordered group*, Acta Comment. Univ. Tartu. Math. **15** (2011), no. 2, 61–80.

[4] M. Lawson, *Inverse semigroups. The theory of partial symmetries*, World Scientific, Singapore, 1998.

E-mail: ✉ *oleg.gutik@lnu.edu.ua.*

Artem Hak[1], Vladyslav Haponenko[2], Sergiy Kozerenko[3], Andrii Serdiuk[4]

Unique eccentric point graphs of diameter at most four

[1,2] National University of Kyiv-Mohyla Academy, Kyiv, Ukraine
[3] Kyiv School of Economics, Kyiv, Ukraine
[4] McGill University, Montreal, Quebec, Canada

The *eccentricity* of a vertex u in a connected graph G is the value $\mathrm{ecc}(u) = \max\{d(u,v) \mid v \in V(G)\}$. A vertex v is an eccentric vertex for u if $\mathrm{ecc}(u) = d(u,v)$. A graph is called a *unique eccentric point graph* (shortly, uep-graph) [2] if every vertex has exactly one eccentric vertex. This is equivalent to saying that the corresponding eccentric digraph $\mathrm{ED}(G)$ is functional.

Generally speaking, characterizing uep-graphs and describing their eccentric digraphs is a non-trivial problem. To construct uep-graphs with various properties, we have applied evolutionary algorithms, which helped us characterize them for small diameters.

The only uep-graph of diameter one is K_2. Those of diameter two are exactly the self-centered $(n-2)$-regular graphs [2]. A uep-graph of diameter thee is either self-centered or upper-diameter critical [2]. In [1], we obtained a characterization of non-self-centered uep-graphs G of $\mathrm{diam}(G) = 3$ as those whose complement $\overline{G}$ is a bi-star.

We also described eccentric digraphs of uep-graphs via the following classification of their weak components: bald, if it is $D_{0,0}$; half-bald, if exactly one of the two vertices on a 2-cycle has in-degree one; full otherwise. A digraph D arises as $\mathrm{ED}(G)$ for some uep-graph G of $\mathrm{diam}(G) = 3$ if and only if D consists of $l \geq 3$ bald components, or $D \simeq D_{m,k}$ for $m, k \geq 1$.
For any non-self-centered uep-graph G of $\mathrm{diam}(G) = 4$, it holds [1]:

1. each eccentric vertex in G lies on a cycle in $\mathrm{ED}(G)$;
2. A 2-cycle $x \leftrightarrow y$ forms a bald weak component in $\mathrm{ED}(G)$ if and only if $d_G(x,y) = 3$;
3. $\mathrm{ED}(G)$ has no half-bald weak components.

[1] A. Hak, V. Haponenko, S. Kozerenko, A. Serdiuk, Unique eccentric point graphs and their eccentric digraphs, *Discrete Math.*, **346(12)**, (2023), 113614.
[2] K.R. Parthasarathy and R. Nandakumar, Unique eccentric point graphs, *Discrete Math.* **46(1)** (1983), 69–74.

E-mail: ✉[1] *artikgak@ukr.net*, ✉[2] *vladyslav.haponenko@gmail.com*, ✉[3] *kozerenkosergiy@ukr.net*, ✉[4] *andrii_serdiuk@outlook.com.*

Waldemar Hołubowski

Endomorphisms of vector spaces of countable and uncountable dimensions

Silesian University of Technology, Gliwice, Poland

Let $\mathfrak{gl}(V)$ be the Lie algebra of endomorphisms and $GL(V)$ the group of automorphisms of a vector space V over a field K.

If $dim(V)$ is finite, then the ideal structure of $\mathfrak{gl}(V)$ and the normal structure of $GL(V)$ are very similar. These results can be extended to infinite dimensions (see [4], [3], [2] for a countable case and [1], [5] for a general case).

In our talk we survey these results.

[1] O. Bezushchak,W. Hołubowski, B. Oliynyk, *Ideals of general linear Lie algebras of infinite-dimensional vector spaces.* Proc. Amer. Math. Soc. 151 (2023), no. 2, 467–473.

[2] W. Hołubowski, M. Maciaszczyk, S. Żurek, *Normal subgroups in the group of column-finite infinite matrices*, J. Group Theory, **25** (2022), 343–353.

[3] W. Hołubowski, S. Żurek, *Lie algebra of column-finite infinite matrices: ideals and derivations.* J. Algebra 619 (2023), 517–537.

[4] I. Penkov, V. Serganova, *T*ensor representations of Mackey Lie algebras and their dense subalgebras, Developments and retrospectives in Lie theory, Dev. Math. Vol. 38, pp. 291–330. Springer, Cham., 2014.

[5] A. Rosenberg,*The structure of the infinite general linear group.* Ann. of Math. (2) 68 (1958), 278–294.

E-mail: ✉ *w.holubowski@polsl.pl.*

Waldemar Hołubowski[1], Bogdana Oliynyk[2], Viktoriia Solomko[3]

On the characterization of unitary Cayley graphs of upper triangular matrix rings

[1] Silesian University of Technology, Gliwice, Poland

[2] Silesian University of Technology, Gliwice, Poland;
National University of Kyiv-Mohyla Academy, Kyiv, Ukraine

[3] National University of Kyiv-Mohyla Academy Kyiv, Ukraine

There are several graphs associated with rings. The unitary Cayley graph of a ring R is a graph whose vertices are elements of the ring, and two elements x and y are adjacent if and only if $x - y$ is a unit of R.

The idea of representing graphs modulo integers was introduced by Erdös and Evans [1], while the concept of unitary Cayley graphs for the rings $\mathbb{Z}_n$, $n \geq 2$, was first presented by Dejter and Giudici [2].

We prove that the unitary Cayley graph $C_{T_n(\mathbb{F})}$ of the ring of all upper triangular matrices $T_n(\mathbb{F})$ over a finite field $\mathbb{F}$ is isomorphic to a semistrong product of complete graph and antipodal graph to Hamming graph. We prove that the clique number and the chromatic number of this graph are equal to the number of elements of the field $\mathbb{F}$ and characterize the domination number of $C_{T_n(\mathbb{F})}$.

Theorem 1. *For any finite field $\mathbb{F}$ and a positive integer n, $n \geq 2$*

$$n + 1 \leq \gamma(C_{T_n(\mathbb{F})}) \leq 2^n.$$

These bounds are tight.

[1] P. Erdös, A.B. Evans, Representations of graphs and orthogonal Latin square graphs, J. Graph Theory **13(5)** (1989) 593–595.

[2] I. J. Dejter, R. E. Giudici, On Unitary Cayley Graphs, J. Combin. Math. Combin. Comput **18** (1995) 121–124.

E-mail: ✉[1] *w.holubowski@polsl.pl*, ✉[2] *bogdana.oliynyk@polsl.pl*,
✉[3] *viktoriia.solomko20@gmail.com.*

Olena Hryniv[1], Yaroslaw Prytula[2]

History of teaching and research in algebra in Lviv

Ivan Franko National University of Lviv, Lviv, Ukraine

The Department of Mathematics at Lviv University was established in 1744. In 1850, Professor Ignacy Lemoch (1802–1875) began teaching higher algebra as a separate course.

The mathematical research of Jakob Kulik (1793–1863), a graduate of Lviv University focused on the theory of algebraic equations, properties of certain transcendental, and number theory. The result of his five years of work was tables of prime divisors for numbers from 3,033,001 to 100,330,201.

Courses on algebraic equations were also taught by Laurentius Żmurko (1824–1889) He attempted to define the analogue of complex numbers in space, which he called spatial numbers. In 1881-1883, Wladyslaw Kretkowski announced lecture courses on the application of determinants, and on Hamilton's theory of quadratic numbers.

Waclaw Sierpiński (1882–1969), a student of the Ukrainian mathematician Georgii Voronoi, taught algebra and number theory at Lviv University from 1908 to 1919. W. Sierpiński and Józef Puzyna (1856–1919) organized advanced seminars where students presented abstracts of mathematical research, including Galois and group theory.

The career of Eustachi Żylinski (1889–1954) is also closely linked with Lviv. He taught algebra at Lviv University from 1919 to 1946. His scientific results are related to number theory, algebra, and logic. Jozef Schreier (1909–1943) and Stanislaw Ulam (1909–1984) also dealt with algebraic issues related to topological groups.

Between 1894 and 1939, Ukrainian mathematicians in the Shevchenko Scientific Society: Volodymyr Levytskyi (1872–1956), Mykola Chaikovskyi (1887–1970) published a dozen scientific articles on algebra.

From 1946 to 1953, Yaroslav Lopatynskyi (1906–1981) headed the Department of Algebra at Lviv University, developing the algebraic theory of linear differential operators. Together with P. Kazymyrskyi in Lviv and S. Berman in Uzhhorod, Lopatynski founded Ukrainian algebraic schools that continue the tradition of algebraic research.

[1] Ya. Prytula Articles on the history of Lviv mathematics. Available at: http://mmf.lnu.edu.ua/istoriia/vydatni-osobystosti.

E-mail: ✉[1] *ohryniv@gmail.com,* ✉[2] *ya.g.prytula@gmail.com.*

Oleksii Ilchuk

Moufang liners

Ivan Franko National University of Lviv, Lviv, Ukraine.

Subject of my work is the structure called Moufang Liners. Liners are the set endowed with ternary relation satisfying two basic geometric properties:

- for every two distinct points exists unique line passing through them;
- for every line there exist two distinct points which belong to it.

With this geometric stucture, all other known properties of various geometries can be seen as a specific subset of all liners, for which certain additional axioms are satisfied. Using notion of liners, I've studied Moufang planes - classical object in geometry and generalized requirements for geometry to be Moufang and established some additional results for Moufang planes as well as preserving already known ones.

Main results in my work are two following theorems:

Theorem 1. *The projective completion of regular Moufang affine liner is the Moufang projective liner.*

Theorem 2. *The affine liner is Moufang if and only if it is a shear liner.*

Shear liners are specific liners possessing many automorphisms which with fixed points on hyperplanes.

E-mail: ✉ *alexilchuk4@gmail.com.*

Volodymyr Ilkiv

On the criterion for extracting a factor of a matrix polynomial

Lviv Polytechnic National University, Lviv, Ukraine

In the works of P. S. Kazimirskii from the 60s and 70s, a criterion for extracting the left unital factor $B = B(z) = B_0 z^r + B_1 z^{r-1} + \cdots + B_r$ from a square matrix polynomial $A = A(z)$ of order n and degree m was established, where

$$A = BD, \qquad \det A = \varphi \cdot \psi, \qquad \det B = \varphi, \qquad B_0 = I,$$

and $D = D(z)$ is also a matrix polynomial, I – identity matrix.

Theorem 1. *To uniquely extract the unitary factor B from the matrix A it is necessary and sufficient to uniquely solve the system of linear algebraic matrix equations*

$$M_{\lfloor A_*, r+1 \rfloor}(\varphi) X = 0, \tag{1}$$

where $M_G(\varphi)$ is the matrix of values of the matrix G on the root system of polynomial φ, $X = \operatorname{col}(X_r, \ldots, X_1, X_0)$, $X_r, \ldots, X_1, X_0$ are square matrices of order n, $X_0 = I$, $\lfloor H, l \rfloor = \big(H(z)\ zH(z)\ \ldots\ z^{l-1}H(z)\big)$ is the accompanying matrix, A_* is the adjugate of A.

Consider the system related to system (1)

$$M_{\left(\lfloor \psi I, r+1 \rfloor\ \lfloor A, q \rfloor\right)}(z^{q+m}) \begin{pmatrix} X \\ Y \end{pmatrix} = 0, \tag{2}$$

where $Y = \operatorname{col}(Y_{(n-1)(m-r)}, \ldots, Y_1, Y_0)$, $q = (n-1)(m-r) + 1$.

The equivalence of systems (1) and (2) is proved, namely: the X-component of the solution of system (2) is the solution of system (1) and, conversely, the solution of system (1) is the X-component of the solution of system (2) and uniquely determines the Y-component of the solution of system (2). From this follows the following alternative criterion for the selection of the factor.

Theorem 2. *For the unique extraction of a left unitary matrix polynomial, it is necessary and sufficient to have a unique solvability of the system of linear algebraic matrix equations* (2).

E-mail: ✉ *ilkivvv@i.ua.*

Mykola Khrypchenko

Quantum upper triangular matrix algebras

Universidade Federal de Santa Catarina, Florianópolis, Brazil.

Let K be a field of characteristic $\neq 2$, $n \geq 2$ integer and $q \in K^*$. We introduce a uniparametric quantization $T_q(n)$ of the K-algebra of upper triangular $n \times n$ matrices. When $\mathrm{char}(K) = 0$ and q is not a root of unity, we describe the derivations and automorphisms of $T_q(2)$.

This is a joint work with Ednei A. Santulo Jr. (Universidade Estadual de Maringá), Érica Z. Fornaroli (Universidade Estadual de Maringá) and Samuel Lopes (Universidade do Porto).

E-mail: ✉ *nskhripchenko@gmail.com.*

Francisco Klock[1], Mykola Khrypchenko[2]

Local confluence and globalizations of partial actions of monoids on semigroups

Federal University of Santa Catarina, Florianopolis, Brazil.

Let M be a monoid, X a semigroup and α a strong partial action of M on X. We can describe a candidate β for a universal globalization of α as the quotient of the free semigroup generated by the set-theoretic universal globalization of α by an equivalence relation generated by a certain abstract rewriting system $\to$. If $\to$ is locally confluent, then β is a universal globalization of α. When $M = G \sqcup \{0\}$ for some group G, and the domain and image of each α_m are ideals of X, we give necessary and sufficient conditions for $\to$ to be locally confluent and find weaker conditions for α to be globalizable.

E-mail: ✉[1]*francisco_gabriel25@hotmail.com.*

Rostyslava V. Kolyada[1], Orest M. Mel'nyk[2], Volodymyr M. Prokip[3]

On solutions of matrix equation $A(\lambda)X(\lambda) + Y(\lambda)B(\lambda) = C(\lambda)$

[1,2] Lviv Polytechnic National University, Lviv, Ukraine
[3] IAPMM NAS of Ukraine, Lviv, Ukraine

Let $\mathbb{F}[\lambda]$ be the ring of polynomials over an infinite field $\mathbb{F}$. Further, let $\mathbb{F}_n[\lambda]$ be the ring of $n \times n$ matrices over $\mathbb{F}[\lambda]$. Consider the matrix equation

$$A(\lambda)X(\lambda) + Y(\lambda)B(\lambda) = C(\lambda), \tag{1}$$

where $A(\lambda), B(\lambda) \in \mathbb{F}_n[\lambda]$ nonsingular matrices, $C(\lambda) \in \mathbb{F}_n[\lambda]$ and $X(\lambda), Y(\lambda)$ unknown matrices. For $A(\lambda)$ and $B(\lambda)$ there exist matrices (see[1]) $P \in GL(n, \mathbb{F})$ and $Q_1(\lambda), Q_2(\lambda) \in GL(n, \mathbb{F}[\lambda])$ such that:

$$PA(\lambda)Q_1(\lambda) = G_A(\lambda) = [g_{ij}(\lambda)],$$

where $g_{ij}(\lambda) = 0$ if $i < j$; $g_{ii}(\lambda) = a_i(\lambda)$ are invariant divisors of $A(\lambda)$ for all $i = j$ and $g_{ij}(\lambda) = a_j(\lambda)\widetilde{g}_{ij}(\lambda)$ for all $i > j$.

$$PB(\lambda)Q_2(\lambda) = H_B(\lambda) = [h_{ij}(\lambda)],$$

where $h_{ij}(\lambda) = 0$ if $i < j$; $h_{ii}(\lambda) = b_i(\lambda)$ are invariant divisors of $B(\lambda)$ for all $i = j$ and $h_{ij}(\lambda) = b_j(\lambda)\widetilde{h}_{ij}(\lambda)$ for all $i > j$.

Thus, the equation (1) is solvable if and only if the equation

$$G_A(\lambda)\widetilde{X}(\lambda) + \widetilde{Y}(\lambda)H_B(\lambda) = PC(\lambda)Q_2(\lambda), \tag{2}$$

is solvable. Put $\widetilde{X}_i(\lambda) = Q_1^{-1}(\lambda)X_i(\lambda)Q_2(\lambda)$ and $\widetilde{Y}_i(\lambda) = PY_i(\lambda)P^{-1}$, $i = 1, 2, \ldots$.

Theorem. *Let pairs of matrices $\widetilde{X}_i(\lambda), \widetilde{Y}_i(\lambda)$, $i = 1, 2$, be solutions of equation (2). Then the following conditions are held:*
1) Matrices $Y_1(\lambda)$ and $Y_2(\lambda)$ are similar if and only if matrices $\widetilde{Y}_1(\lambda)$ and $\widetilde{Y}_2(\lambda)$ are similar.
2) Matrices $X_1(\lambda)$ and $X_2(\lambda)$ are equivalent if and only if matrices $\widetilde{X}_1(\lambda)$ and $\widetilde{X}_2(\lambda)$ are equivalent.

Question: When conditions 1) and 2) of the Theorem are held simultaneously?

[1] Kazimirs'kyi P.S. Decomposition of matrix polynomials into factors. Naukova Dumka, Kyiv; 1981, 226 p. (in Ukrainian).

E-mail: ✉[1] *rostyslavakolyada@gmail.com*, ✉[2] *orest.m.melnyk@lpnu.ua*,
✉[3] *v.prokip@gmail.com*.

Andrii Korzhuk[1], Andriy Oliynyk[2]

Accelerated operations for permutational wreath products

[1] Postgraduate student, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

[2] Doctor of Sciences in Physics and Mathematics, Professor of the Department of Algebra and Computer Mathematics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Operations with permutational wreath products become computationally expensive as structure depth and component group sizes increase.

Definition 1 (Permutational Wreath Product)**.** The wreath product $G \wr H$ of permutation groups (G, X) and (H, Y) consists of pairs $[g, h(x)]$ where $g \in G$ and $h(x) \in H^X$ acting on $X \times Y$ by $(x, y)^{[g,h(x)]} = (x^g, y^{h(x)})$.

Definition 2 (Portrait)**.** A portrait of an element from $\wr_{i=0}^{d-1} G_i$ is a labeled rooted tree of depth d where each node at level l is labeled with elements from G_l, and level l is $|X_l|$-regular with total nodes $|\pi| = 1 + \sum_{l=1}^{d-1} \prod_{k=0}^{l-1} |X_k|$.

We introduce a heap-like data structure for representing portraits – elements of permutational wreath products $\wr_{i=0}^{d-1} G_i$. The proposed data structure allows implementation of optimized algorithms for operations with portraits, such as action, multiplication, inverse, etc.

Theorem 1 (Portrait Operation Complexity)**.** *The portrait multiplication and inverse algorithms have time complexity* $O\left(d \cdot \prod_{i=0}^{d-1} |X_i|\right)$.

Theorem 2 (Cyclic Group Optimization)**.** *When each* G_i *is cyclic, portrait operations only requires determining permutations by their action on generators, reducing complexity to* $O\left(d \cdot \prod_{i=0}^{d-2} |X_i|\right)$.

We validated our approach by identifying all isomorphisms between $\mathbb{Z}_3 \uparrow \mathbb{Z}_3$ and subgroups of $\mathbb{Z}_3 \wr \mathbb{Z}_3 \wr \mathbb{Z}_3$, expanding example from [1]. From 3^{13} possible candidate pairs, our algorithm identified 40,223,304 valid isomorphic pairs, demonstrating the efficiency of our representation with applications to automorphism groups of regular rooted trees.

[1] Oliynyk A., Prokhorchuk V. On exponentiation, p-automata and HNN extensions of free abelian groups. Algebra and Discrete Mathematics Volume 35 (2023). Number 2, pp. 180-190 DOI: 10.12958/adm2132

E-mail: ✉[1] *korzhukandrew@gmail.com,* ✉[2] *aolijnyk@gmail.com.*

Ganna Kudryavtseva

Relating ample and biample topological categories with Boolean restriction and range semigroups

University of Ljubljana, Faculty of Mathematics and Physics and IMFM
Ljubljana, Slovenia

The aim of the talk is to present the results of the paper [1] where the dualities between ample groupoids and Boolean inverse semigroups are extended to the dualities where groupoids are replaced by categories and inverse semigroups are replaced by restriction and range semigroups. The latter are the most well-studied non-regular generalizations of inverse semigroups. We extend the equivalence by Cockett and Garner between restriction monoids and ample categories to the setting of Boolean range semigroups which are non-unital one-object versions of range categories. We show that Boolean range semigroups are equivalent to ample topological categories where the range map r is open, and étale Boolean range semigroups are equivalent to biample topological categories. Our dualities follow from more general adjunctions for the preBoolean case. Our technique builds on the usual constructions relating inverse semigroups with ample topological groupoids via germs and slices.

[1] G. Kudryavtseva, Relating ample and biample topological categories with Boolean restriction and range semigroups, *Adv. Math.*, vol. 474, July 2025, 110313, 48pp.

E-mail: ✉ *ganna.kudryavtseva@fmf.uni-lj.si.*

Ivan Kyrchei

The determinant of the adjacency matrix of a quaternion unit gain graph

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NAS of Ukraine, Lviv, Ukraine

Let $\mathbb{H}$ be the quaternion skew field and $U(\mathbb{H}) = \{q \in \mathbb{H} \mid |q| = 1\}$. Suppose that $\Gamma = (V, E)$ is the simple graph with the set of vertices $V = \{v_1, v_2, \ldots, v_n\}$ and edges in the set E denoted by $e_{ij} = v_i v_j$. Moreover, $\overrightarrow{E} = \overrightarrow{E}(\Gamma)$ is the set of oriented edges of the gain graph. By e_{ij}, we denote the oriented edge from v_i to v_j, and the gain of e_{ij} is $\varphi(e_{ij})$. So, a $U(\mathbb{H})$-gain graph is defined as a triple $G = (\Gamma, U(\mathbb{H}), \varphi)$ consisting of an underlying graph $\Gamma = (V, E)$, the circle group $U(\mathbb{H})$, and the gain function $\varphi : \overrightarrow{E}(\Gamma) \to U(\mathbb{H})$ such that $\varphi(e_{ij}) = \varphi(e_{ji})^{-1} = \overline{\varphi(e_{ji})}$. Using the Moore noncommutative determinant, the research [1] started to study the quaternion unit gain graphs. In [2], a combinatorial description of the determinant of the Laplacian matrix of a quaternion unit gain graph is provided, based on the theory of row-column noncommutative determinants [3]. In this presentation, we extend this line of inquiry by offering a combinatorial description for the determinant of the adjacency matrix of a quaternion unit gain graph.

Theorem Let G be a $U(\mathbb{H})$-gain graph and $\mathbf{A}(G)$ be its adjacency matrix. Then

$$\det \mathbf{A}(G) = \sum_{R \in \mathcal{R}(G)} \left(\prod_{k_1} \det \mathbf{A}\left(C^{(k_1)}\right) \prod_{k_2} \det \mathbf{A}\left(P^{(k_2)}\right) \right),$$

where the sum is taken over the set $\mathcal{R}(G)$ of all needed reductions R of G. Here $R = \bigcup_k S_k = C^{(1)} \cup \ldots \cup C^{(k_1)} \cup P^{(1)} \bigcup \ldots \cup P^{(k_2)}$, where S_i, $(i \in I_k)$, is the component of R, $C^{(i)}$, $(i \in I_{k_1})$, is a unique cycle, and $P^{(i)}$, $(i \in I_{k_2})$, is a path graph. $\det \mathbf{A}(C^{(i)})$ and $\det \mathbf{A}(P^{(j)})$ have simple explicit combinatorial descriptions as well.

[1] F. Belardo, M. Brunetti, N.J. Coble, N. Reff, H. Skogman. Spectra of quaternion unit gain graphs. Linear Algebra Appl. 632, (2022) 15-49.

[2] I.I. Kyrchei, E. Treister, V.O. Pelykh. The determinant of the Laplacian matrix of a quaternion unit gain graph. Discrete Math. 347(6), (2024) 113955.

[3] I.I. Kyrchei. The theory of the column and row determinants in a quaternion linear algebra. In: Baswell A.R. (ed.), Advances in Mathematics Research 15, pp. 301–359. New York, Nova Sci. Publ., 2012.

E-mail: ✉ *ivankyrchei26@gmail.com.*

Nataliia Ladzoryshyn[1], Vasyl' Petrychkovych[2]

(z,k)-equivalence of matrices over quadratic rings

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics
National Academy of Sciences of Ukraine, Lviv, Ukraine

Let $\mathbb{K}$ be a Euclidean quadratic ring, $M(n,\mathbb{K})$ be the ring of $n \times n$ matrices over $\mathbb{K}$. The structure of matrices over quadratic rings was studied only over certain quadratic rings, in particular over Euclidean quadratic rings, the ring of Gaussian integers. In [1], the cyclotomic matrices over quadratic rings are studied, a classification of such matrices is given, and the connection with graphs is indicated. Generalized Klostermann sums are extended over the ring of matrices of Gaussian integers $\mathbb{Z}[i]$ and an estimate of these sums are given [2].

We investigate a special equivalence of matrices over different quadratic rings. Matrices $A, B \in M(n,\mathbb{K})$ are called (z, k)-equivalent if there exist invertible matrices S over $\mathbb{Z}$ and Q over $\mathbb{K}$, such that $A = SBQ$. It is established that each matrix A over the Euclidean quadratic ring is reduced by (z,k)–equivalent transformations to the following standard form T^A with invariant factors on the main diagonal: $T^A = SAQ = \left\| \begin{matrix} \mu_1 & 0 & \dots & 0 \\ t_{21}\mu_1 & \mu_2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ t_{n1}\mu_1 & t_{n2}\mu_2 & \dots & \mu_n \end{matrix} \right\|$, where $t_{ij} = 0$, if $\mu_i = 1$ and $\varepsilon(t_{ij}) < \frac{\varepsilon(\mu_i)}{\varepsilon(\mu_j)}$, if $t_{ij} \neq 0$, $i, j = 1, ..., n, j < i$, $\varepsilon(\cdot)$ is the Euclidean norm of an element from $\mathbb{K}$ [3]. The standard form T^A of a matrix A is defined ambiguously. It is established that the number of matrices over imaginary Euclidean quadratic rings is finite and given an estimate this number. The classes of matrices over these rings are selected for which the standard form is uniquely defined and equal to the Smith normal form. Standard form of matrices are used for solving matrix linear equations over quadratic rings and for descriptions of the structure of solutions these equations.

[1] Taylor G. *Cyclotomic matrices and graphs over the ring of integers of some imaginary quadratic fields.* J. Algebra., 2011, **331**, P. 523—545.

[2] Velichko I.N. *Generalized Kloosterman sum over the matrix ring.* Visn. Odes. Nats. Univ., Ser. Mat. and Mekh., 2010, **1**, No. 19., P. 9—20.

[3] Ladzoryshyn N.B., Petrychkovych V.M. *Standard form of matrices over quadratic rings with respect to the (z,k)-equivalence and the structure of solutions of bilateral matrix linear equations* J. Math. Sci., 2021, **253**, No. 1., P. 54—62.

E-mail: ✉[1] *natalja.ladzoryshyn@gmail.com,* ✉[2] *vas_petrych@yahoo.com.*

Tetiana Lukashova[1], Maryna Drushlyak[2], Anastasiia Pidopryhora[3]

On the norm of non-cyclic *pd*-subgroups in torsion locally nilpotent groups

Sumy State Pedagogical University named after A. S. Makarenko, Sumy, Ukraine

The authors study the norm of non-cyclic *pd*-subgroups, which is one of the generalizations of the classical concept of the norm of a group in group theory (see [1]). Recall (see, for example, [2]) that a *pd*-group is a group that contains elements of order p for some prime p. The intersection of the normalizers of all non-cyclic *pd*-subgroups of a group G (provided that the system of such subgroups is nonempty) is called the norm of non-cyclic *pd*-subgroups of a group and is denoted by $N_G^{pd\overline{H}}$. In the case when a group G does not contain non-cyclic *pd*-subgroups, let's assume that $G = N_G^{pd\overline{H}}$.

It is clear that in the class of primary groups the norm of non-cyclic *pd*-subgroups coincides with the non-cyclic norm N_G of a group (see [3]). Therefore, in this case all the results got in [3] for the norm N_G hold for the norm $N_G^{pd\overline{H}}$.

In this paper the authors consider the properties of the norm of non-cyclic *pd*-subgroups in the class of torsion locally nilpotent *pd*-groups.

Proposition. The norm $N_G^{pd\overline{H}}$ of a torsion *pd*-group G is Dedekind if at least one of the following conditions is satisfied:

- a group G contains a non-cyclic *pd*-subgroup or a non-cyclic p'-subgroup that does not intersect with $N_G^{pd\overline{H}}$;
- a group G contains an elementary Abelian subgroup of order p^3.

Theorem. Let G be a torsion non-primary locally nilpotent *pd*-group. If its norm $N_G^{pd\overline{H}}$ is non-Dedekind, then any Sylow p'-subgroup of a group G is cyclic and $G = G_p \times \langle x \rangle$, where $(|x|, p) = 1$ and $N_G^{pd\overline{H}} \cap G_p$ is a non-Dedekind p-group in which all non-cyclic subgroups are normal.

[1] Baer R. Der Kern, eine Charakteristische Untergruppe. Comp. Math. 1935. 1. 254- 283.
[2] Lyman F. N. Groups with some systems of invariant *pd*-subgroups. Groups and systems of their subgroups. Kyiv: Institute of Mathematics of the Academy of Sciences of the Ukrainian SSR, 1983. 100-118.
[3] Lukashova T. D. On locally-finite *p*-groups with non-Dedekind non-cyclic norm. Mathematychui Studii. 2002. 17 (1). 18-22.

E-mail: ✉[1] *tanya.lukashova2015@gmail.com*, ✉[2] *marydru@fizmatsspu.sumy.ua*, ✉[3] *anastasialogvin2@gmail.com*.

Volodymyr Lyubashenko

Symmetric weak multicategories and biprops

Institute of Mathematics of National Academy of Sciences of Ukraine,
Kyiv, Ukraine.

Symmetric multicategories (over sets) can be defined in the style of Leinster's book. They can be enriched over categories, which means that we are given a set of objects; for each tuple $\big((X_i)_{i\in I}, Y\big)$ of objects of C (I is finite totally ordered set) we are given not a set $\mathsf{C}\big((X_i)_{i\in I}; Y\big)$, but a category; compositions

$$\mu_\phi \colon \Big[\prod_{j\in J} \mathsf{C}\big((X_i)_{i\in\phi^{-1}j}; Y_j\big)\Big] \times \mathsf{C}\big((Y_j)_{j\in J}; Z\big) \to \mathsf{C}\big((X_i)_{i\in I}; Z\big)$$

are functors, where $\phi\colon I \to J$ is a mapping, not necessarily preserving the order; etc. In strict category-enriched case the composition functors satisfy an associativity equation. For weak multicategories these equations are replaced with a functorial isomorphisms which, in turn, satisfy the associativity pentagon equations. In particular, weak multicategories are bicategories. Examples of symmetric weak multicategories come from symmetric monoidal bicategories.

Just as ordinary symmetric multicategory C gives rise to a colored prop $\mathbb{F}\mathsf{C}$ with the same set of objects as C, the set of operations

$$\mathbb{F}\mathsf{C}\big((X_1,\dots,X_n),(Y_1,\dots,Y_m)\big) = \coprod_{\varphi\colon \{1,\dots,n\}\to\{1,\dots,m\}} \prod_{j=1}^{m} \mathsf{C}\big((X_i)_{i\in\varphi^{-1}(j)}; Y_j\big),$$

($\varphi\colon \{1,\dots,n\} \to \{1,\dots,m\}$ – any map), weak symmetric multicategory C gives rise to a biprop $\mathbb{F}\mathsf{C}$, whose categories of operations are given by the above formula. We conjecture the coherence result for weak symmetric multicategories similar to the coherence result for bicategories.

E-mail: ✉ *lub@imath.kiev.ua.*

Sergiy Maksymenko[1], Mykola Lysynskyi[2]

Double coset classes and differentiable structures on non-Hausdorff one-dimensional manifolds

Institute of Mathematics of National Academy of Sciences of Ukraine, Kyiv, Ukraine

Fix $k = 1, \ldots, \infty$ and let $\mathbb{R}_{neg} = (-\infty; 0)$ denote the set of negavtive reals. Let $\mathcal{D} := \mathcal{D}^{+}(\mathbb{R}_{neg})$ be the group of preserving orientation C^k-diffeomorphisms of $\mathbb{R}_{neg}$, and $\mathcal{E} := \mathcal{E}_{\mathbb{R}}((\mathbb{R}_{neg}))$ be its subgroup consisting of diffeomorphisms h which can be extended to some C^k-diffeomorphism of all $\mathbb{R}$.

Let also $\mathbb{Z}_2 = \{\pm 1\}$ be the cyclic group of order 2, and $\mathcal{E} \wr \mathbb{Z}_2$ be the wreath product of $\mathcal{E}$ and $\mathbb{Z}_2$. By definition, $\mathcal{E} \wr \mathbb{Z}_2$ is the Cartesian product of sets $\mathcal{E} \times \mathcal{E} \times \mathbb{Z}_2$ with the following operation:

$$(a, b, \delta)(c, d, 1) := (ac, bd, \delta), \qquad (a, b, \delta)(c, d, -1) := (bc, ad, -\delta)$$

for $a, b, c, d \in \mathcal{E}$ and $\delta = \pm 1$.

Then there is a natural action of the wreath product $\mathcal{E} \wr \mathbb{Z}_2$ on $\mathcal{D}$ given by

$$(a, b, \delta) \cdot g = (bga^{-1})^{\delta},$$

for $a, b, \in \mathcal{E}$, $g \in \mathcal{D}$, and $\delta = \pm 1$. The corresponding set of orbits of this action will be denoted by $\mathcal{E} \setminus \mathcal{D}^{\pm} / \mathcal{E}$.

Note that set of orbits under the induced action of the subgroup $\mathcal{E} \times \mathcal{E} \times 1$ is denoted by $\mathcal{E} \setminus \mathcal{D} / \mathcal{E}$ and called *double $\mathcal{E}$-cosets.*

Further let $\mathbb{Y} = \big(\mathbb{R} \times \{0, 1\}\big)/_{(x,0)\sim(x,1), x<0}$ be the topological space obtained by gluing two copies of $\mathbb{R}$ by the identity homeomorphism. Then $\mathbb{Y}$ is a non-Hausdorff one-dimensional manifold.

It is well known that the real line $\mathbb{R}$ admits a unique up to a diffeomorphism C^k-structures.

Theorem 1. *There is a canonical bijection between the set of all C^k-structures on $\mathbb{Y}$ and the set $\mathcal{E} \setminus \mathcal{D}^{\pm} / \mathcal{E}$. In particular, $\mathbb{Y}$ has uncountably many pair-wise non-diffeomorphic C^k-structures.*

In the talk we will also discuss a formalism behind the proof of this theorem, which allows to do similar classifications in more general contexts.

E-mail: ✉[1] *maks@imath.kiev.ua,* ✉[2] *m.lysynskyi@imath.kiev.ua.*

Marta Maloid-Hliebova

About finitely-generated weakly-second submodules

Ivan Franko National University of Lviv, Lviv, Ukraine

Let R be associative ring with $1 \neq 0$, M be right R-module. We write $N \leq M$ to indicate that N is submodule of M.

Definition 1. Module M is called weakly-second module if for all the ideals A, B of R and every submodule $K \leq M$, $MAB \subseteq K$ implies either $MA \subseteq K$ or $MB \subseteq K$.

Definition 2. By weakly-second submodule of a module we mean a submodule, which is also a weakly-second module itself.

Theorem 1. *Let N be finitely-generated submodule of R-module M, such, that $Ann_R(N) = Ann_R(M)$. If N is weakly-second submodule, then M is weakly-second module.*

Theorem 2. *Let N be finitely-generated submodule of module M, and $Ann_R(M)$ is a prime ideal. If N is weakly-second submodule, then $Ann_R(M) = (N : M)$.*

[1] S. Annin, "Associated and Attached Primes Over Noncommutative Rings", Ph.D.Thesis, Univ. of Baghdad, (2002).

[2] H. Ansari-Toroghy, F. Farshadifar "The dual notion of some generalizations of prime submodules", Comm. Algebra, (2011), Vol. 39, No. 7, pp. 2396-2416

[3] S. Çeken, M. Alkan, "On second submodules", Contemporary Mathematics, 634 (2015), 67-77.

[4] S. Yassemi, "The dual notion of prime submodules", Arch. Math (Brno) 37 (2001), 273-278.

E-mail: ✉ *martamaloid@gmail.com.*

Alex Martsinkovsky[1], Blas Torrecillas Jover[2]

Stabilization of adjoints

[1] Northeastern University, Boston, MA, USA

[2] University of Almería, Almería, Spain

The term "stabilization" in the title refers to the stabilization of both categories and functors. Paraphrasing Eilenberg and Mac Lane, one might say that stable categories are necessary to accommodate stable functors – and in this lecture, we will indeed focus on stable functors.

Numerous examples of such functors exist, the most prominent being the higher (i.e., degree ≥ 1) Ext and Tor, which are familiar to everyone. In fact, all higher derived functor are stable. A simple yet surprising fact is that these higher derived functors are entirely determined by their zeroth counterparts. Given any additive functor F on a module category (or more generally, on an abelian category with enough injectives or projectives), one can canonically construct a stable functor, called the stabilization of F. Figuratively speaking, the stabilization process takes place in degree zero.

In recent years, it has become evident that many results, concepts, techniques, and theories can be explained, clarified, and expanded through the lens of stable functors. The goal of this talk is to demonstrate how applying this philosophy to the stabilization of adjoined functors allows us to recover Auslander–Reiten theory for finitely generated modules over artin algebras. Moreover, the generality of our approach also enables us to establish the existence of Auslander–Reiten sequences for finite-dimensional comodules over semiperfect coalgebras.

The first author was supported in part by the Shota Rustaveli National Science Foundation of Georgia Grants NFR-18-10849 and FR-24-8249

E-mail: ⌧[1] *a.martsinkovsky@northeastern.edu,* ⌧[2] *btorreci@ual.es.*

Oles Mazurenko[1], Taras Banakh[2], Olesia Zavarzina[3]

Dense plastic subgroups in strictly convex normed spaces

[1,2] Ivan Franko National University of Lviv, Lviv, Ukraine
[3] V.N. Karazin Kharkiv National University, Kharkiv, Ukraine

Definition. A metric space X is *plastic* if every non-expanding bijection $f : X \to X$ is an isometry.

We shall discuss the existence of a dense plastic subspace of a metric space, and a dense plastic subgroup of a normed space, as described in our following results.

Definition. A metric space (X, d) is *strictly convex* if for all $a, b \in X$ and $r_1, r_2 \in \mathbb{R}_+$ such that $d(a, b) = r_1 + r_2$, the intersection of the closed balls $B[a, r_1] \cap B[b, r_2]$ contains exactly one element.

Definition. A topological space X is *k-crowded* if every non-empty open set in X contains an uncountable compact subset.

Theorem 1. *A k-crowded separable metric space contains a dense plastic subspace.*

Theorem 2. *A strictly convex separable normed vector space contains a dense plastic subgroup.*

Theorem 3. *Every countable dense subspace of a normed vector space is not plastic.*

[1] J. van Mill, *A topological group having no homeomorphisms other than translations*, Trans. Amer. Math. Soc. **280** (1983), no. 2, 491–498.
[2] S. A. Naimpally, Z. Piotrowski, and E. J. Wingler, *Plasticity in metric spaces*, J. Math. Anal. Appl. **313** (2006), 38–48.
[3] W. Hurewicz and H. Freudenthal, *Dehnungen, Verkurzungen, Isometrien*, Fundamenta Mathematicae **26** (1936), 120–122.

E-mail: ✉[1] *oles.mazurenko@lnu.edu.ua*, ✉[2] *t.o.banakh@gmail.com*,
✉[3] *olesia.zavarzina@yahoo.com.*

Ivanna Melnyk[1], Andriy Andrushko[2]

On prime subsemimodules of multiplication semimodules

Ivan Franko National University of Lviv, Lviv, Ukraine

We consider left semimodules over semirings, not necessarily commutative.

Theorem 1. *Let M be a left R-semimodule. Let P be a subsemimodule of M with $P \neq M$. The following statements are equivalent:*

1. *P is prime;*
2. *For any $r \in R$ and $m \in M$, if $(r)(m) \subseteq P$, then either $m \in P$ or $r \in (P : M)$;*
3. *For any $r \in R$ and $m \in M$, if $rRm \subseteq P$, then either $m \in P$ or $r \in (P : M)$.*

Let S be an arbitrary m-system of a semiring R. A non-empty subset X of the semimodule M is called an *Sm-system* of the semimodule M if for any $s \in S$ and any $x \in X$, there exists an element $r \in R$ such that $srx \in X$.

Theorem 2. *Let R be a semiring, M be a left R-semimodule, and N be a subsemimodule of M such that $N \cap X = \emptyset$, where X is an Sm-system. Then there exists a maximal subsemimodule P, with $P \subseteq N$, of the semimodule M among those subsemimodules satisfying $P \cap X = \emptyset$. This subsemimodule P is prime.*

A left R-semimodule M is called a multiplication module if for every submodule N of M, there exists an ideal I of R such that $N = IM$.

Theorem 3. *The following statements are equivalent for a proper k-subsemimodule N of M:*

1. *N is a prime k-subsemimodule of M;*
2. *$(N : M)$ is a prime k-ideal of R;*
3. *$N = PM$ for some prime k-ideal P of R.*

[1] *Golan J. S.* Semirings and their Applications. Kluwer Academic Publishers, 1999. 460 p.

[2] *R. E. Atani, S. E. Atani*, On subsemimodules of semimodules. Bul. Acad. Stiinte Repub. Moldova. Matematica. 2010. No. 2 (63). P. 20–30.

[3] *R. E. Atani.* Prime Subsemimodules of Semimodules. International Journal of Algebra, 4(26), 2010, pp. 1299–1306.

E-mail: ✉[1] *ivannamelnyk@yahoo.com*, ✉[2] *andrii.andrushko@lnu.edu.ua.*

Yurii Merkushev

Twisted Cayley machines for finite groups: implementation and analysis

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Twisted Cayley machines are Mealy-type automata whose states correspond to elements of a finite group and whose input symbols are ordered pairs from that group. Such automata generate lamplighter-type dynamics and are known to produce groups of the form $A \wr \mathbb{Z}$.

In this work, we present a universal Python-based implementation for constructing such machines over arbitrary finite groups, including abelian products and non-abelian structures like the dihedral group D_4. The tool computes transition tables and supports graphical visualization.

We experimentally compare automata built from different groups using graph-theoretic characteristics such as the number of unique transitions and cycles of fixed length. The results show that automata structures differ substantially even for groups of equal order. This implementation serves as a practical tool for studying automaton groups and their properties.

[1] Dominik Francoeur, *Bireversible automata generating lamplighter groups*, 2022. Available at: https://arxiv.org/abs/2206.04633

E-mail: ✉ *yuri_merkushev@knu.ua.*

Oksana Mykytsei

Compatibility between continuous semilattices

Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine

We consider the concept of compatibility between continuous semilattices and its connection to adjunctions.

Definition. Let S and S' be continuous semilattices with bottom elements 0 and $0'$, respectively. A mapping $P : S \times S' \to \{0, 1\}$ is called a compatibility between S and S' if:

(1) P preserves bottom elements in both arguments:

$$P(0, y) = P(x, 0') = 0 \quad \text{for all } x \in S,\ y \in S'.$$

(2) P is Scott-continuous in both arguments.

We denote the set of all such compatibilities by $\mathcal{C}_{\mathrm{w}}(S, S')$.

In [1] we consider various subclasses of $\mathcal{C}_{\mathrm{w}}(S, S')$ and investigate their connections to a wide range of structures, such as monotone predicates, Galois connections, and completely distributive lattices.

In particular, for a completely distributive lattice L, the subclass $\mathcal{C}^{\bullet}_{\vee}(S, S')$ of compatibilities that preserve pairwise suprema in the second argument corresponds to the set $\mathcal{M}_{[L]}S$ of L-valued normalized monotone predicates.

The class $\mathcal{C}_{\vee\vee}(S, S')$ of compatibilities that preserve pairwise suprema in both arguments is, as a partially ordered set, anti-isomorphic to the lattice of contravariant Galois connections between the continuous lattices L and L'.

We define the subclass $\mathcal{C}_{\nearrow}(S, S')$ by the condition:

$$P(x_1 \wedge x_2, y) = P(x_1, y) \wedge P(x_2, y) \quad \text{for } y \le y_1 \vee y_2.$$

It is shown that

$$\mathcal{C}_{\nearrow}(S, S') \cong \mathcal{M}^{\wedge}_{[L]}S \subseteq \mathcal{M}_{[L]}S,$$

where $\mathcal{M}^{\wedge}_{[L]}S$ consists of all L-valued normalized monotone predicates on S that map pairwise infima to pairwise suprema.

[1] Mykytsei O.Ya., Koporkh K.M. Compatibilities between continuous semilattices. Carpathian Math. Publ., 2021, Vol. 13, No. 1, P. 5–14.

E-mail: ✉ *oksana.mykytsei@pnu.edu.ua.*

Mikhail Neklyudov

Orthogonalization and polarization of Yangians

Universidade Federal do Amazonas (UFAM), Manaus, Brazil

For every family of orthogonal polynomials, we define a new realization of the Yangian of $gl(n)$. Except in the case of Chebyshev polynomials, the new realizations do not satisfy the RTT relation. We obtain an analogue of the Christoffel-Darboux formula. Similar construction can be made for any family of functions satisfying certain recurrence relations, for example, q-Pochhhammer symbols and Bessel functions. Furthermore, using an analogue of the Jordan-Schwinger map, we define the ternary Yangian for an arbitrary finite dimensional Lie algebra as a flat deformation of the current algebra of a certain ternary extension of the given Lie algebra. The talk is based on a joint work [1] in progress with V. Futorny, W. Bock, and J. Zhang.

[1] W. Bock, V. Futorny, M. Neklyudov, J. Zhang, Orthogonalization and polarization of Yangians, arXiv:2504.00259

E-mail: ✉ *misha.neklyudov@gmail.com.*

Mykola Nikitchenko

Program algebras and logics over nominative data

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

The widespread use of artificial intelligence actualizes the question of its theoretical foundations to which logic belongs. Mathematical logic is based on algebras of various types. Interpreting algebras as models of activity in subject domains, we propose an approach that allows to describe algebras (and logics based on them) which take into account such specific domain features as intensionality, nominativity, compositionality, and partiality.

Intensionality. It specifies the basic behavioral characteristics of subject domains and is complementary to their extensional characteristics. Intensional characteristics can be classified according to the following levels of activity: pure, becoming, determinate, real, and actual levels. The pure level means that no activity is identified, the becoming level means that transitions are allowed, the determinate level is related to the qualities of things, the real level is based on properties of things, and actual level is associated with actions in the domain. Each of the levels specifies some action algebras, in particular, program algebras.

Nominativity. The models of states of the subject domains can be specified by classes of nominative data [1]. These data are based on the name-value relation. Various data structures used in subject domain models can be represented by hierarchical nominative data.

Compositionality. The main means of program constructions are formalized as compositions. We identify compositions with respect to the levels of activity. Thus, the considered algebras can be called composition-nominative algebras.

Partiality. In conventional algebras and logics functions and predicates are considered as total mappings. But in applications we often encounter situations with partiality. This requires constructing algebras with partiality both in data and mappings.

Here we define composition-nominative program algebras and logics with partial predicates and functions over hierarchical nominative data. We investigate such algebras and construct for the corresponding logics sequent calculi in the style of multimodal logics.

[1] Nikitchenko, M. Composition-Nominative Methods and Models in Program Development. SN COMPUT. SCI. 3, 507 (2022).

E-mail: ✉ *mykola.nikitchenko@gmail.com.*

Andriy Oliynyk

Lamplighter groups and reversible automata

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Let G be a finite group. The lamplighter group $\mathcal{L}_G$ is defined as the restricted wreath product $G \wr \mathbb{Z}$, i.e. as $\bigoplus_{\mathbb{Z}} G \rtimes \mathbb{Z}$, where $\mathbb{Z}$ acts on $\bigoplus_{\mathbb{Z}} G$ by shifts. We discuss approaches to constructing finite automata such that their automaton groups are lamplighter groups. For any nontrivial finite abelian group G, we construct a reversible automaton whose automaton group is $\mathcal{L}_G$.

The talk is based on the paper [1].

[1] Piotr W. Nowak, Andriy Oliynyk, Veronika Prokhorchuk *On reversible automata generating lamplighter groups*, Journal of Algebra, V.661, 2025, P.578-594.

E-mail: ✉ *aolijnyk@gmail.com.*

Sher-Ali Penza[1], Oleg Gutik[2]

On monoid endomorphisms of the semigroup $\mathscr{C}_+(a,b)$

Ivan Franko National University of Lviv, Lviv, Ukraine

We shall follow the terminology of [1; 2].

The bicyclic monoid $\mathscr{C}(p,q)$ is the semigroup with the identity 1 generated by two elements p and q subjected only to the condition $pq = 1$. The semigroup operation on $\mathscr{C}(p,q)$ is determined as follows:

$$q^k p^l \cdot q^m p^n = q^{k+m-\min\{l,m\}} p^{l+n-\min\{l,m\}}.$$

In [4] the following subsemigroup

$$\mathscr{C}_+(a,b) = \left\{ b^i a^j \in \mathscr{C}(a,b) \colon i \leqslant j,\ i,j \in \omega \right\}$$

of the bicyclic monoid is studied.

In [3] we describe monoid endomorphisms of the semigroup $\mathscr{C}_+(a,b)$ which are generated by the family of all congruences of the bicyclic monoid and all injective monoid endomorphisms of $\mathscr{C}_+(a,b)$.

In our report we describe difference types of monoid endomorphisms of the semigroup $\mathscr{C}_+(a,b)$ and their semigroup structures.

[1] A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. I., Amer. Math. Soc. Surveys 7, Providence, R.I., 1961.
[2] A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. II., Amer. Math. Soc. Surveys 7, Providence, R.I., 1967.
[3] O. Gutik and Sh.-A. Penza, *On the semigroup of monoid endomorphisms of the semigroup $\mathscr{C}_+(a,b)$*, Algebra Discr. Math. **38** (2024), no. 2, 233–247.
[4] S. O. Makanjuola and A. Umar, *On a certain subsemigroup of the bicyclic semigroup*, Commun. Algebra **25** (1997), no. 2, 509–519.

E-mail: ✉[1] *sher-ali.penza@lnu.edu.ua,* ✉[2] *oleg.gutik@lnu.edu.ua.*

Anton Petrov[1], Oleksandr Pypka[2]

Automorphism group of some non-nilpotent Leibniz algebras

Oles Honchar Dnipro National University, Dnipro, Ukraine

Let L be an algebra over a field F with the binary operations $+$ and $[,]$. Then L is called a (*left*) *Leibniz algebra* if it satisfies the left Leibniz identity:

$$[[a,b],c] = [a,[b,c]] - [b,[a,c]]$$

for all elements $a, b, c \in L$ [1,2].

A linear transformation f of L is called an *endomorphism* of L, if

$$f([a,b]) = [f(a), f(b)]$$

for all elements $a, b \in L$. A bijective endomorphism of L is called an *automorphism* of L. We note that the set $\mathrm{Aut}_{[,]}(L)$ of all automorphisms of L is a group by a multiplication.

Consider the following type of 3-dimensional non-nilpotent Leibniz algebras:

$$\begin{gathered} L = Fa_1 \oplus Fa_2 \oplus Fa_3, \text{ where } [a_1,a_1] = [a_1,a_3] = a_3, \\ [a_1,a_2] = -a_2, [a_2,a_1] = a_2, \\ [a_2,a_2] = [a_2,a_3] = [a_3,a_1] = [a_3,a_2] = [a_3,a_3] = 0. \end{gathered}$$

Thus, $\mathrm{Leib}(L) = \zeta^{\mathrm{left}}(L) = Fa_3$, $[L,L] = Fa_2 \oplus Fa_3$.

Theorem 1. *Let G be the automorphism group of a Leibniz algebra L. Then G is isomorphic to a subgroup of $\mathrm{GL}_3(F)$ consisting of matrices of the following form:*

$$\begin{pmatrix} 1 & 0 & 0 \\ \alpha_2 & \beta_2 & 0 \\ \alpha_3 & \beta_3 & 1+\alpha_3 \end{pmatrix},$$

where $\alpha_2, \alpha_3, \beta_2, \beta_3 \in F$, $\beta_2 \neq 0$, $1+\alpha_3 \neq 0$.

[1] Blokh A. On a generalization of the concept of Lie algebra. Dokl. Akad. Nauk SSSR, 1965, 165(3), 471–473.

[2] Loday J.-L. Cyclic homology. – New York: Springer-Verlag, 1992, 451 p.

E-mail: ✉[1] *antondne86@gmail.com*, ✉[2] *sasha.pypka@gmail.com.*

Andrii Plaksin[1], Oleh Romaniv[2]

A ring right (left) almost stable range 1

Ivan Franko National University of Lviv, Lviv, Ukraine

Let R be an associative ring with a nonzero identity element. We say that a ring R is a ring of stable range 1 if for any elements $a, b \in R$ the equality $aR + bR = R$ implies $(a + b\lambda)R = R$ for some $\lambda \in R$. We say that a ring R is a ring of stable range 2 if for any elements $a, b, c \in R$ the equality $aR + bR + cR = R$ implies $(a + c\lambda)R + (b + c\mu)R = R$ for some $\lambda, \mu \in R$. Element $a \in R\backslash\{0\}$ is said to be an element of a right (left) almost stable range 1 if for any elements $b, c \in R$ such that $aR + bR + cR = R$ $(Ra + Rb + Rc = R)$ we have $aR + (b + c\lambda)R = R$ $(Ra + R(b + \mu c) = R)$ for some $\lambda, \mu \in R$. A ring R is a ring right (left) almost stable range 1 if every nonzero element R is an element of right (left) almost stable range 1. We will remind that a right (left) Bezout ring it is a ring in which every right (left) finitely generated ideal is principal right (left) ideal. A Bezout ring is a ring which is a right Bezout and left Bezout an one time.

Theorem 1. *Let R be a right Bezout ring of stable range 2. Then for any $a, b \in R$, there exist elements $d, a_1, b_1 \in R$ such that $a = da_1$, $b = db_1$ and $a_1R + b_1R = R$.*

Theorem 2. *A right Bezout ring R of stable range 1 is a ring of right almost stable range 1.*

Theorem 3. *A right (left) Bezout ring of a right (left) almost stable range 1 is a ring of stable range 2.*

Theorem 4. *A ring R is a ring of right almost stable range 1 if and only if $R/J(R)$ is a ring of right almost stable range 1, where $J(R)$ is the Jacobson radical of R.*

Theorem 5. *Let R be a right Bezout ring in which for any elements $a, b \in R$ such that $aR + bR = R$ there exists an element $\lambda \in R$ such that $a + b\lambda$ is an element of right almost stable range 1. Then R is a ring of stable range 2.*

[1] H. Bass, K-theory and stable algebra, Publ. Inst. Hautes Etude Sci., **22** (1964).

[2] W. McGovern, Bezout rings with almost stable range 1, *J. Pure Appl. Algebra* **212** (2007) 340–348.

[3] B. V. Zabavsky, A. M. Romaniv, S. I. Bilavska, Adequate properties of the elements with almost stable range 1 of a commutative elementary divisor domain, *Applied Problems of Mechanics and Mathematics* **16** (2018) 33–35.

E-mail: ✉[1] *andrii.plaksin@lnu.edu.ua*, ✉[2] *oleh.romaniv@lnu.edu.ua*.

Roman Popovych

On the construction of high order elements in finite fields given by binomial

Lviv Polytechnic National University, Lviv, Ukraine.

It is well known that the multiplicative group of a finite field is cyclic. A generator of this group is called primitive element. The problem of constructing efficiently a primitive element for a given finite field is notoriously difficult in the computational theory of finite fields. That is why on considers less restrictive question: to find an element with high multiplicative order [4]. We are not required to compute the exact order of the element. It is sufficient in this case to obtain a lower bound on the order. High order elements are needed for a number of applications, including cryptography and coding theory.

F_q denotes a finite field of q elements, where q is a power of a prime number. The extension of the field given by binomial is of the form $F_q[x]/(x^m - a)$. It is shown in [3] how to construct high order elements in such extension with the condition that m divides $q-1$. The lower bound $5,8^m$ is obtained in this case. High order elements are constructed in [2] for extensions F_{q^m} ($m = 2^t$, $q \equiv 1 \ (mod\ 4)$, lower bound $2^{(m^2+3m)/2+ord_2(q-1)}$) and ($m = 3^t$, $q \equiv 1 \ (mod\ 3)$, $q \neq 4$, lower bound $3^{(m^2+3m)/2+ord_3(q-1)}$) without the division condition. For arbitrary m and without the division condition, the best known results are: the lower bound $2^{\sqrt[3]{2m}} = 2,3^{\sqrt[3]{m}}$ [5] and the refined bound $5^{\sqrt[3]{m/2}} = 3,5^{\sqrt[3]{m}}$ [1].

We consider any extension of the form $F_q[x]/(x^m - a)$ and explicitly construct in it elements with the multiplicative order at least $2^{\sqrt{2m}}$.

[1] V. Bovdi, A. Diene, R. Popovych, Elements of high order in finite fields specified by binomials, Carpathian Math. Publ. 14 (1) (2022) 238-246.

[2] J.F. Burkhart et al., Finite field elements of high order arising from modular curves, Des. Codes Cryptogr. 51 (3) (2009) 301-314.

[3] Q. Cheng, On the construction of finite field elements of large order, Finite Fields Appl. 11 (3) (2005) 358-366.

[4] G.L. Mullen, D. Panario, Handbook of Finite Fields, CRC Press (2013).

[5] R. Popovych, Elements of high order in finite fields of the form $F_q[x]/(x^m - a)$, Finite Fields Appl. 19 (1) (2013) 86-92.

E-mail: ✉ *rombp07@gmail.com.*

Inna Pozdniakova[1], Oleg Gutik[2]

On endomorphisms of the semigroup $\boldsymbol{B}_{\mathbb{Z}}^{\mathscr{F}}$

Ivan Franko National University of Lviv, Lviv, Ukraine

We shall follow the terminology of [1; 2].

On the set $\boldsymbol{B}_{\mathbb{Z}} \times \mathscr{F}$, where $\mathscr{F}$ is an ω-closed subfamily of $\mathscr{P}(\omega)$, we define the semigroup operation "$\cdot$" by the formula

$$(i_1, j_1, F_1)\cdot(i_2, j_2, F_2)=\begin{cases} (i_1-j_1+i_2, j_2, (j_1-i_2+F_1) \cap F_2), & \text{if } j_1\leqslant i_2;\\ (i_1, j_1-i_2+j_2, F_1 \cap (i_2-j_1+F_2)), & \text{if } j_1\geqslant i_2.\end{cases}$$

In [3] it is proved that $(\boldsymbol{B}_{\mathbb{Z}} \times \mathscr{F}, \cdot)$ is a semigroup. Moreover, if an ω-closed family $\mathscr{F} \subseteq \mathscr{P}(\omega)$ contains the empty set $\varnothing$ then the set $\boldsymbol{I} = \{(i, j, \varnothing) \colon i, j \in \mathbb{Z}\}$ is an ideal of the semigroup $(\boldsymbol{B}_{\mathbb{Z}} \times \mathscr{F}, \cdot)$. For any ω-closed family $\mathscr{F} \subseteq \mathscr{P}(\omega)$ the following semigroup

$$\boldsymbol{B}_{\mathbb{Z}}^{\mathscr{F}} = \begin{cases} (\boldsymbol{B}_{\mathbb{Z}} \times \mathscr{F}, \cdot)/\boldsymbol{I}, & \text{if } \varnothing \in \mathscr{F};\\ (\boldsymbol{B}_{\mathbb{Z}} \times \mathscr{F}, \cdot), & \text{if } \varnothing \notin \mathscr{F}\end{cases}$$

is defined and studied in [3].

A subset A of ω is said to be *inductive*, if $i \in A$ implies $i+1 \in A$. In [4] we study automorphisms of the semigroup $\boldsymbol{B}_{Z}^{\mathscr{F}}$ with the family $\mathscr{F}$ of inductive nonempty subsets of ω and prove that the group $\mathbf{Aut}(\boldsymbol{B}_{\mathbb{Z}}^{\mathscr{F}})$ of automorphisms of the semigroup $\boldsymbol{B}_{Z}^{\mathscr{F}}$ is isomorphic to the additive group of integers.

In our report we describe all endomorphisms of the semigroup $\boldsymbol{B}_{Z}^{\mathscr{F}^2}$ for a two-element family $\mathscr{F}^2$ of inductive nonempty subsets of ω.

[1] A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. I., Amer. Math. Soc. Surveys 7, Providence, R.I., 1961.

[2] A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. II., Amer. Math. Soc. Surveys 7, Providence, R.I., 1967.

[3] O. V. Gutik and I. V. Pozdniakova, *On the semigroup generating by extended bicyclic semigroup and an ω-closed family*, Mat. Metody Fiz.-Mekh. Polya **64** (2021), no. 1, 21–34 (in Ukrainian).

[4] O. Gutik and I. Pozdniakova, *On the group of automorphisms of the semigroup $\boldsymbol{B}_{Z}^{\mathscr{F}}$ with the family $\mathscr{F}$ of inductive nonempty subsets of ω*, Algebra Discrete Math. **35** (2023), no. 1, 42–61.

E-mail: ✉[1] *pozdnyakova.inna@gmail.com*, ✉[2] *oleg.gutik@lnu.edu.ua.*

Mykola Pratsiovytyi, Sofiia Ratushniak

Numeral systems with redundant alphabet and their applications in geometry of numerical series and fractal analysis

Dragomanov Ukrainian State University, Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine

Let s and r be fixed natural parameters such that $2 \le s \le r$; let $A_r \equiv \{0, 1, \ldots, r\}$ be an alphabet, and let $L_r \equiv A_r \times A_r \times \ldots$ be the set of all sequences of elements from the alphabet A_r.

For every $x \in [0; \frac{r}{s-1}]$ there exists a sequence $(\alpha_n) \in L_r$ such that

$$x = \sum_{n=1}^{\infty} \alpha_n s^{-n} = \Delta^{r_s}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}. \tag{1}$$

The expansion of x in the series (1) is called the r_s-expansion of the number, and the notation $\Delta^{r_s}_{\alpha_1\alpha_2\ldots\alpha_n\ldots}$ is called the r_s-representation of the number. The digits α_n are called the r_s-digits of the r_s-expansion.

In general, numbers from the interval $[0; \frac{r}{s-1}]$ may admit more than one r_s-expansion. Each pair of parameters r and s defines a specific geometry, which is effectively revealed through the study of the properties of cylinder sets.

Fractal analysis, as a component of the theory of fractals, includes the study of structural, topological, and metric properties of sets, functions, measures, and dynamical systems, based on the theory of fractal dimensions.

The term geometry of number series generally refers to the topological, metric, and fractal analysis of sets of partial sums of absolutely convergent number series.

Given a convergent series $S = a_1 + a_2 + \ldots + a_n + \ldots$ the set of its partial sums is defined as

$$E(a_n) = \{x : \sum_{n=1}^{\infty} \varepsilon_n a_n, (\varepsilon_n) \in L_2\} \subset [0; S].$$

This talk focuses on the number of r_s-representations of a real number, their relationship with r_s-expansions in terms of partial sums of corresponding series, and further applications. Special attention will be devoted to the case $s = r$ that is, to systems with one redundant digit.

E-mail: ✉ *prats4444@gmail.com, ratush404@gmail.com.*

Volodymyr M. Prokip

On the uniqueness of solutions of a matrix equation $AX - YB = C$ over the ring of integers

IAPMM NAS of Ukraine, Lviv, Ukraine

Let $\mathbb{Z}_{m,n}$ be the set of $m \times n$ matrices over the ring of integers $\mathbb{Z}$. Consider the matrix equation

$$AX - YB = C, \tag{1}$$

where $A \in \mathbb{Z}_{m,m}$, $B \in \mathbb{Z}_{n,n}$, $C \in \mathbb{Z}_{m,n}$ and X, Y are unknown $m \times n$-matrices over $\mathbb{Z}$.

The following question naturally arises: Under which conditions does equation (1) have a unique solution? See also [1]–[3] and references therein. The purpose of this report is to present the following statement.

Theorem. *Let $A \in \mathbb{Z}_{m,m}$ and $B \in \mathbb{Z}_{n,n}$ be nonsingular matrices and $C \in \mathbb{Z}_{m,n}$. Further, let $W \in GL(m, \mathbb{Z})$ be such that*

$$AW = H_A = \begin{bmatrix} h_1 & 0 & \dots & \dots & 0 \\ h_{21} & h_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ h_{m1} & h_{m2} & \dots & h_{m,m-1} & h_m \end{bmatrix} \in \mathbb{Z}_{m,m}$$

is the Hermitian normal form of the matrix A, i.e., $h_i > 0$ for all $1 \le i \le m$ and $0 \le h_{ij} < h_i$ for all $j < i$, $1 \le j < i$.

The matrix equation $AX + YB = C$ has a unique solution

$$X_0, Y_0 = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \dots & \dots & \dots & \dots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} \in \mathbb{Z}_{m,n}$$

such that $0 \le y_{ij} < h_i$ for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$ if and only if $(\det A, \det B) = 1$.

[1] Prokip V.M. About the uniqueness solution of the matrix polynomial equation $A(\lambda)X(\lambda) - Y(\lambda)B(\lambda) = C(\lambda)$. Lobachevskii J. Math. 29.3 (2008): 186–191.
[2] Prokip V.M. On the divisibility of matrices with remainder over the domain of principal ideals. J. Math. Sciences. 243, N.1 (2019): 45–55.
[3] Prokip V.M., Mel'nyk O.M., Kolyada R.V. On divisibility with remainder of polynomial matrices over an arbitrary field. Math. methods and phys.-mech. fields. 66.1-2 (2023): 23–39.

E-mail: ✉ *v.prokip@gmail.com.*

Santiago Radi

Classification of groups of finite type by isomorphism

Texas A& M University of Texas, United States

Groups of finite type (also known as finitely constrained groups) are closed subgroups of Aut(T), the automorphism group of a regular rooted tree T, whose action locally around every vertex is determined by a finite group of allowed actions of a certain depth D. They were introduced in 2005 by Grigorchuk in [3], who proved that the closure of regular branch groups belongs to this class. In 2006, Sunic proved the converse in [5]. In 2014, Bondarenko and Samoilovich gave results to check if a group of finite type is topologically finitely generated and calculated all the groups of finite type in the binary tree with depth 2, 3 and 4 (see [1]).

It is of interest to know whether two groups of finite type are isomorphic or not. This can be used as a tool to prove whether two abstract groups whose closure in Aut(T) is of finite type are not isomorphic (if the corresponding closures are not isomorphic) or it can also be used to prove that the groups are not profinite rigid (if the closures are isomorphic).

In my talk, I will present theorems that allow the classification of groups of finite type. With these results, we will classify groups of finite type in the binary tree for depth 2, 3 and 4 and in the ternary tree for depths 2 and 3. With another theorem, we will also compute the closure of some famous groups in the literature such as Grigorchuk groups [2] and GGS groups [4], and we will derive conclusions comparing their closures.

[1] I. Bondarenko and I. Samoilovych, On finite generation of self-similar groups of finite type, *International Journal of Algebra and Computation*, 23 (1) (2013), 69–79.

[2] R. Grigorchuk, On Burnside's problem on periodic groups, *Akademiya Nauk SSSR. Funktsionalnyĭ Analiz i ego Prilozheniya*, 14 (1) (1980), 53–54.

[3] R. Grigorchuk, Solved and Unsolved Problems Around One Group, *Birkhäuser Basel*, (2005), 117–218.

[4] N. Gupta and S. Sidki, On the Burnside Problem for Periodic Groups, *Mathematische Zeitschrift*, 182 (1983), 385–388.

[5] Z. Šunić, Hausdorff dimension in a family of self-similar groups, *Geometriae Dedicata*, 124 (2007), 213–236.

E-mail: ✉ *santiradi@tamu.edu.*

Iryna Raievska

On direct products of metacyclic Miller–Moreno p-groups and cyclic p-groups as additive groups of local nearrings

Institute of Mathematics of National Academy of Sciences of Ukraine,
Kyiv, Ukraine

A nearring R with an identity is called local if the set of all non-invertible elements of R forms a subgroup of the additive group of R. In paper [1] it was given a full classification of the metacyclic Miller–Moreno p-groups which appear as the additive groups of finite local nearrings. Moreover, if G is such an additive group, then we describe all possible multiplications "$\cdot$" on G for which the system $(G, +, \cdot)$ is a local nearring. In the report we consider the direct products of Miller–Moreno p-groups and cyclic p-groups as additive groups of nearrings with identity and local nearrings.

In what follows we use the following notation: $F(p^m, p^n, p^k)$ denotes an additively written group with generators a, b and c of orders p^m, p^n and p^k, respectively, so that $-b + a + b = a(1 + p^{m-1})$, $c + a = a + c$ and $c + b = b + c$, where $m \geq 2$, $n \geq 1$ and $k \geq 1$.

We will give examples of local nearrings.

Lemma 1. *Let R be a local nearring whose additive group of R^+ is isomorphic to $F(p^m, p^n, p^k)$, $|R : L| = p$, $m \geq n$ and $m \geq k$. If $x = ax_1 + bx_2 + cx_3$, $y = ay_1 + by_2 + cy_3$, then the mappings $\alpha : R \to \mathbb{Z}_{p^m}$, $\beta : R \to \mathbb{Z}_{p^n}$, $\gamma : R \to \mathbb{Z}_{p^k}$, $\phi : R \to \mathbb{Z}_{p^m}$, $\psi : R \to \mathbb{Z}_{p^n}$ and $\xi : R \to \mathbb{Z}_{p^k}$ can be the following: $\phi(x) = 0 \pmod{p^m}$, $\psi(x) = 0 \pmod{p^m}$, $\alpha(x) = 0 \pmod{p^m}$, $\gamma(x) = 0 \pmod{p^k}$, $\xi(x) = 1 \pmod{p^k}$,*

$$\beta(x) = \begin{cases} 1, & if\ x_1 \not\equiv 0 \pmod{p}; \\ 0, & if\ x_1 \equiv 0 \pmod{p}. \end{cases}$$

Theorem 2. *For each odd prime p, $m \geq n$ and $m \geq k$ there exists a local nearring R whose additive group R^+ is isomorphic to $F(p^m, p^n, p^k)$.*

[1] Raievska I. Yu., Sysak Ya. P. Finite local nearrings on metacyclic Miller–Moreno p-groups. Algebra Discrete Math., 2012, 13, No. 1, 111–127.

E-mail: ✉ *raeirina@imath.kiev.ua.*

Maryna Raievska[1], Yaroslav Sysak[2]

p-Groups with cyclic subgroup of index p and local nearrings

Institute of Mathematics of National Academy of Sciences of Ukraine,
Kyiv, Ukraine

p-Groups with cyclic subgroup of index p as the additive groups and the multiplicative groups of local nearrings are investigated. Furthermore, the classifications of such nearrings are given.

However, it is not true that any finite group is the additive group of a nearring with identity. Therefore the determination of the non-abelian finite p-groups which are the additive groups of local nearrings is an open problem (see [1]).

In present report we study p-groups with cyclic subgroup of index p (see, for example, [2]) as the additive groups and the multiplicative groups of local nearrings. Furthermore, the classifications of such nearrings are given.

Theorem 1. *Let G be a non-cyclic p-group with cyclic subgroup of index p. There exist 501 local nearrings whose multiplicative groups are isomorphic to G.*

[1] Feigelstock S. Additive groups of local near-rings. Com. in Alg., 2006, 34, 743–747.
[2] Hall M. Jr. The Theory of Groups. The Macmillan Company, New York, 1959.

E-mail: ✉[1] *raemarina@imath.kiev.ua*, ✉[2] *sysak@imath.kiev.ua.*

Maryna Rassadkina (Styopochkina)

Oversupercritical partially ordered sets

Polissia National University, Zhytomyr, Ukraine

Using the idea of minimax equivalence of posets [1], the author together with V. M. Bondarenko described in 2005 [2], up to isomorphism, all (finite) posets S critical with respect to posets with positive Tits quadratic form $q_S(z) := z_0^2 + \sum_{i \in S} z_i^2 + \sum_{i<j, i,j \in S} z_i z_j - z_0 \sum_{i \in S} z_i$, which are called P-critical posets. In 2009 [4], we described all posets S critical with respect to posets with nonnegative Tits quadratic form, which are called NP-critical posets. In this situation the next question arises: is it possible to continue the series "P-critical posets — NP-critical posets" taking into account that the quadratic forms that are not nonnegative do not have a natural gradation?

The author (together with V. M. Bondarenko) have been proposed the following solution:

1) describe the (pairwise nonisomorphic) posets that differ from the Nazarova supercritical posets in the same extend as the supercritical posets differ from the Kleiner critical posets; they are called oversupercritical (in more detail see, e.g., [5]);

2) take all posets minimax isomorphic to supercritical (see [2]).

The motivation for this decision are the following facts (presented in terms of the paper [6]): the Kleiner critical posets form a minimax system of generators for the P-critical posets [2] and the Nazarova supercritical posets — for the NP-critical posets [3].

[1] V. M. Bondarenko. On (min, max)-equivalence of posets and applications to the Tits forms. *Bulletin of Taras Shevchenko Universityof Kyiv (series: Physics & Mathematics)*, (1):24–25, 2005.

[2] V. M. Bondarenko, M. V. Styopochkina. (Min, max)-equivalence of partially ordered sets and the Tits quadratic form. *Collection of works of Inst. of Math. NAS Ukraine – Problems of Analysis and Algebra*, 2(3):18–58, 2005.

[3] V. M. Bondarenko, M. V. Styopochkina. (Min, max)-equivalence of posets and nonnegative Tits forms. *Ukr. Math. J.*, 60(9):1349–1359, 2008.

[4] V. M. Bondarenko, M. V. Styopochkina. Description of posets critical with respect to the nonnegativity of the quadratic Tits form. *Ukr. Math. J.*, 61(5):734–746, 2009.

[5] Bondarenko V. M., Styopochkina M. V. Classification of the posets of minmax types which are symmetric oversupercritical posets of the eighth order. *Mathematical methods and physicomechanical fields*, 66(1-2):5–15, 2023.

[6] Bondarenko V. M. Minimax equivalence method: initial ideas, first applications and new concepts. *Algebra Discrete Math.*, 38(1):1–22, 2024.

E-mail: ✉ *stmar@ukr.net.*

Andriy Regeta

Rationality and solvable subgroups in the Cremona group

University of Padua, Padua, Italy

In this talk I will present the following two results about the group of birational transformations (which we denote by $\mathrm{Bir}(X)$) of an irreducible variety X:

The first one: If $\mathrm{Bir}(X)$ is isomorphic to $\mathrm{Bir}(\mathbb{P}^n)$, then X is rational.

The second one: a closed connected solvable subgroup of $\mathrm{Bir}(X)$ has derived length less than or equal to $2\dim X$ and the equality holds iff X is rational. Moreover, all Borel subgroups (maximal connected solvable subgroups) of $\mathrm{Bir}(\mathbb{P}^n)$ of derived length $2n$ are conjugate. Furthermore, Borel subgroups of $\mathrm{Bir}(\mathbb{P}^n)$ of derived length strickly smaller than $2n$ exist. This talk is based on the joint work with C. Urech and I. Van Santen.

E-mail: ✉ *andriyregeta@gmail.com.*

Andrii Romaniv

On reduction of invertible matrices to simpler forms

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, Lviv, Ukraine

Let R be a commutative Bezout domain of stable range 1.5 [2]. According to Theorem 1 from [3], R is an elementary divisor domain [1], i.e., for every nonsingular matrix $D \in M_n(R)$, there exist invertible matrices P_D and Q_D of appropriate sizes, such that

$$P_D D Q_D = diag(\varphi_1, \varphi_2, \ldots, \varphi_n) =: \Phi, \text{ where } \varphi_i | \varphi_{i+1},\ i = 1, \ldots, n-1.$$

The matrices P_D and Q_D are the left and right transforming matrices for D, respectively. We denote by $\mathbf{P_D}$ the set of all left transforming matrices of the matrix D. It is known that $\mathbf{P_D} = \mathbf{G}_\Phi P_D$ where, the set $\mathbf{G}_\Phi$ is a multiplicative group (Zelisko group).

Consider a diagonal matrix

$$diag(\psi_1, \psi_2, \ldots, \psi_n) =: \Psi, \text{ where } \psi_i | \psi_{i+1},\ i = 1, \ldots, n-1.$$

Let $\alpha \in R$. We denote by $K(\alpha)$ the set of representatives of the residue classes of the factor ring $R/R\alpha$. Let $MLT_n(R)$ be the ring of lower triangular matrices over R.

Theorem. Let $S = \|s_{ij}\|_1^n \in MLT_n(R)$. Then there exist lower unitriangular matrices $H \in \mathbf{G}_\Phi$ and $L \in \mathbf{G}_\Psi$, such that

$$HSL = \left\| \begin{array}{ccccc} 1 & 0 & \ldots & 0 & 0 \\ k_{21} & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ k_{n-1.1} & k_{n-1.2} & \ldots & 1 & 0 \\ k_{n1} & k_{n2} & \ldots & k_{n.n-1} & 1 \end{array} \right\| = K,$$

where K is the canonical form of the matrix S, in which $k_{ij} \in K(\nu_{ij})$, $\nu_{ij} = \left(\frac{\varphi_i}{\varphi_j},\ \frac{\psi_i}{\psi_j}\right)$, for $i = 2, \ldots, n$, $j = 1, \ldots, n-1$, $i > j$.

[1] Kaplansky I. Elementary divisor and modules. *Trans. Amer. Math. Soc.*, 66, 464–491, 1949.

[2] Shchedryk V.P. Bezout rings of stable range 1.5. *Ukrainian Math J.*, 67, 960–967, 2015. doi.org:10.1007/s11253-015-1126-9.

[3] Shchedryk V.P. Bezout rings of stable range 1.5 and the decomposition of a complete linear group into the product of its subgroups *Ukrainian Math J.*, 69, 138–147, 2017. doi.org/10.1007/s11253-017-1352-4.

E-mail: ✉ *romaniv_a@ukr.net.*

Oleh Romaniv

Rings with Dubrovin condition

Ivan Franko National University of Lviv, Lviv, Ukraine

We will consider elementary divisor rings satisfying the Dubrovin condition, i.e., the condition that for every nonzero element $a \in R$, there exists an element $a^* \in R$ such that $RaR = a^*R = Ra^*$. A ring R is called a ring of almost stable range 1 if from the condition $RaR + RbR + RcR = R$, with $c \neq 0$, it follows that there exists $\lambda \in R$ such that $R(\lambda a + b)R + RcR = R$.

Let R be a Bezout domain with Dubrovin condition. Let $a \in R$, $a \neq 0$ and suppose $RaR = a^*R = Ra^* \neq R$. The element a is called one-sided adequate if for every element $b \in R$ the following conditions hold: 1) $a = rs$ for some $r, s \in R$ and $RrR = r^*R = Rr^*$, $RsR = s^*R = Rs^*$, $RbR = b^*R = Rb^*$; 2) $r^*R + b^*R = R$; 3) for every nontrivial divisor s'^* of s^* we have $s'^*R + b^*R \neq R$ where $Rs'R = s'^*R = Rs'^*$. A ring R is called simultaneously adequate if every nonzero element a such that $RaR \neq R$ is one-sided adequate.

Theorem 1. *A simultaneous adequate Bezout domain with Dubrovin condition is a ring of almost stable range 1.*

Theorem 2. *Let R be a Hermite ring of almost stable range 1. Then for any matrix $\left(\begin{smallmatrix} a & 0 \\ b & c \end{smallmatrix}\right)$ over R, where $RaR + RbR + RcR = R$, there exist invertible matrices P and Q such that $PAQ = \left(\begin{smallmatrix} z & 0 \\ * & * \end{smallmatrix}\right)$, where $RzR = R$.*

Theorem 3. *Let R be a Hermite ring with Dubrovin condition of almost stable range 1. Then R is an elementary divisor ring if and only if every matrix of the form $\left(\begin{smallmatrix} a & 0 \\ b & c \end{smallmatrix}\right)$, where $RaR = R$, admits a canonical diagonal reduction.*

[1] H. Bass, K-theory and stable algebra, Publ. Inst. Hautes Etude Sci., **22** (1964).
[2] W. McGovern, Bezout rings with almost stable range 1, *J. Pure Appl. Algebra* **212** (2007) 340–348.
[3] I. Kaplansky, Elementary divisors and modules, *Trans. Amer. Math. Soc.* **66** (1949) 464–491.
[4] B. V. Zabavsky, Diagonal reduction of matrices over finite stable range rings, *Mat. Stud.* **41** (2014) 101–108.
[5] B. V. Zabavsky, Conditions for stable range of an elementary divisor rings, *Comm. Algebra* **45** (2017) 4062–4066.
[6] B. V. Zabavsky, *Diagonal reduction of matrices over rings* (Mathematical Studies, Monograph Series, v. XVI, VNTL Publishers, Lviv, 2012).
[7] N. I. Dubrovin, On rings with elementary divisors, *Soviet Math.* **30**(11) (1986) 16–24.

E-mail: ✉ *oleh.romaniv@lnu.edu.ua.*

Oleh Romaniv[1], Andriy Sagan[2]

Clear conditions of $R(X)$ and $R\langle X\rangle$

Ivan Franko National University of Lviv, Lviv, Ukraine

Throughout this paper we suppose R is an associative ring with nonzero unit and $U(R)$ its group of units.

According to Ehrlich [2], an element $a \in R$ is *unit-regular* if $a = aua$ for some unit $u \in U(R)$. An element a of a ring R is *clear* if $a = r + u$, where r is a unit-regular element and $u \in U(R)$. A ring R is *clear* if every element is clear [4].

The *content ideal* of a polynomial $f(x) = a_0 + a_1X + \ldots + a_nX^n \in R[X]$ is the ideal of R generated by $a_0, a_1, \ldots, a_n$. We denote the content ideal of by $c(f)$. Let $U = \{f \in R[X] : c(f) = R\}$.

The *Nagata ring* [3] over $R[X]$, denoted $R(X)$, is the localization of $R[X]$ with respect to U, that is, $R(X) = U^{-1}R[X]$. Thus every element of $R(X)$ has the form f/g where $f, g \in R[X]$ and $c(g) = R$. We also consider another interesting localization $R[X]$ over a multiplicative closed subset $S = \{f \in R[X] | f \text{is monic polynomial}\}$. We denoted such a factor ring by $R\langle X\rangle = S^{-1}R[X]$.

Theorem 1. *A ring R is clear if and only if $R(X)$ is clear.*

We say that an element r is a ring R is *zero dimensional* [1] if either of the following equivalent conditions holds.

- We can write $R = A \times B$ where r is nilpotent in A and a unit in B.
- There is n such that $r^n \in Rr^{n+1}$.

Theorem 2. *The following statements are equivalent for a ring R.*

(1) *R is zero dimensional*;
(2) *$R\langle X\rangle$ is zero dimensional*;
(3) *$R\langle X\rangle$ is clear ring*;
(2) *$R\langle X\rangle = R(X)$.*

1. Anderson D. D., Anderson D. F., Markanda R. The rings $R(X)$ and $R\langle X\rangle$. J. Algebra, 1985, 95, No. 1, 96–115.
2. Ehrlich G. Units and one-sided units in regular rings. Trans. Amer. Math. Soc., 1976, 216, 81–90.
3. McGovern W. Wm., Richman F. When $R(X)$ and $R\langle X\rangle$ are clean: a constructive treatment. Commun. Alg., 2015, 43, 3389–3394.
4. Zabavsky B. V., Domsha O. V., Romaniv O. M. Clear rings and clear elements. Mat. Stud., 2021, 55, No. 1, 3–9.

E-mail: ✉[1] *oromaniv@franko.lviv.ua,* ✉[2] *andrij.sagan@gmail.com.*

Volodymyr Rubanenko[1], Andriy Oliynyk[2]

Refinement of the Troika hash function

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine.

This work presents a further evolution of the architecture of the Troika hash function [1], emphasizing its adaptability and efficiency for arbitrary prime fields $\mathbb{F}_n$. The main focus of this version is the redesign and generalization of its architecture, which allows increased flexibility and opens up previously closed research opportunities, such as, but not limited to, merging the architecture of Troika with that of Kupyna [2].

The primary advancement is the thorough parameterization of the hash function for arbitrary prime fields $\mathbb{F}_n$. As a result, the nonlinear transformation (Subtrytes) and all state manipulation routines now function seamlessly regardless of the chosen field.

The column parity linear layer has been significantly optimized. The previous neighbor-dependent parity computation has been replaced by a vectorized method that more efficiently computes column-wise sums. This reduces computational complexity to $O(n^2)$ per round, which is particularly beneficial as the state size increases for larger fields.

The empirical evaluation for n = 5 demonstrates that despite the increase in state size, implementation remains practical, with 24 rounds executed in less than a second on standard hardware. Preliminary security analysis suggests that the original Troika properties are preserved, but this statement requires further testing, while the increased algebraic complexity for larger n may enhance resistance to certain classes of attacks.

In conclusion, several promising directions remain for future work on this generalized Troika hash function. Completing a comprehensive cryptanalysis will be essential to fully understand the security and efficiency implications of the new design. Additionally, exploring interoperability between Troika and Kupyna presents an intriguing avenue - specifically, investigating whether structural elements or nonlinear layers can be adapted or integrated into the Kupyna framework.

[1] Stefan Kolbl, Elmar Tischhauser, Patrick Derbez, Andrey Bogdanov, Troika: a ternary cryptographic hash function, 2019, https://link.springer.com/article/10.1007/s10623-019-00673-2

[2] A. Boyko, R. Olijnykov, I. Horbenko, Cryptographic informational security. Hash function. DSTU 7564:2014, https://usts.kiev.ua/wp-content/uploads/2020/07/dstu-7564-2014.pdf

E-mail: ✉[1] *rubanenkovp@knu.ua*, ✉[2] *a.oliynyk@knu.ua.*

Andriy Russyev

Almost all automata over a binary alphabet generate infinite groups

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine.

Let X be a finite non-empty set. Denote by X^* the set of all words over the alphabet X, including the empty word Λ. The length of a word $w \in X^*$ is denoted by $|w|$.

A finite invertible automaton A over the alphabet X is a tuple $A = (X, Q, \varphi, \lambda)$, where Q is a finite set of states, $\varphi : Q \times X \to Q$ is a transition map, $\lambda : Q \times X \to X$ is an output map, and for each state $q \in Q$, the map $\pi_q : X \to X$ given by $\pi_q(x) = \lambda(q, x)$ is a permutation. The automaton A is called degenerate if there exists a permutation ρ such that $\pi_q = \rho$ for all states $q \in Q$.

The transition and output maps of the automaton A can be naturally extended to the set $Q \times X^*$. The extension of the output map defines a map $f_q : X^* \to X^*$ for every state $q \in Q$, given by $f_q(w) = \lambda(q, w)$. The group generated by the set $\{f_q : q \in Q\}$ is denoted by $G(A)$ and is called the group generated by the automaton A.

The nucleus of the automaton A is a subset of the set Q defined by

$$\mathcal{N}(A) = \bigcap_{n \geq 0} \big\{\varphi(q, w) : q \in Q, w \in X^*, |w| \geq n\big\}.$$

Theorem 1 ([1])**.** *Let $A = (Q, X, \varphi, \lambda)$ be an invertible automaton over a binary alphabet X such that $\mathcal{N}(A) = A$. Suppose that there exist states $q_1, q_2 \in Q$ and letters $x_1, x_2 \in X$ such that $\varphi(q_1, x_1) = \varphi(q_2, x_2)$ and $\pi_{q_1} \neq \pi_{q_2}$. Then $G(A)$ is infinite.*

Theorem 2. *Let $\mathcal{A}_n$ be the set of invertible automata with n states, $n \geq 2$, and a non-degenerate nucleus over the binary alphabet $\{0, 1\}$, and let $\mathcal{F}_n$ be its subset of automata A such that $\mathcal{N}(A)$ does not satisfy the conditions of Theorem 1. Then*

$$\lim_{n\to\infty} \frac{|\mathcal{A}_n|}{2^n n^{2n}} = 1 \qquad \textit{and} \qquad \lim_{n\to\infty} \frac{|\mathcal{F}_n|}{|\mathcal{A}_n|} = 0.$$

[1] Russyev A. Infiniteness of groups of automata over a binary alphabet. International Mathematical conference: Abstracts of talks. — Mykolayiv: 2012. — June 13–19. — P. 75.

E-mail: ✉ *andriyrussyev@knu.ua.*

Nataliia Samaruk

SO(3)-quasimonomial families of Appell polynomials

Khmelnytskyi National University, Khmelnytskyi, Ukraine

Definition. A family of polynomials $\{B_{m,n,k}(x,y,z)\}$ is called quasimonomial with respect to H if the group operators in two different bases $\{x^m y^n z^k\}$ and $\{B_{m,n,k}(x,y,z)\}$ have the same matrices. The polynomials $\{B_{m,n,k}(x,y,z)\}$ are called quasimonomials.

Let us consider the two polynomial families $\{V^{(s)}_{m,n,k}(x,y,z)\}$ and $\{U^{(s)}_{m,n,k}(x,y,z)\}$ defined by the exponential generating functions:

$$\frac{1}{(1-2(xu+yv+zw)+u^2+v^2+w^2)^{\frac{2+s}{2}}} = \sum_{m,n=0}^{\infty} V^{(s)}_{m,n,k}(x,y,z)\frac{u^m}{m!}\frac{v^n}{n!}\frac{w^k}{k!},$$

$$\frac{1}{((1-(ux+vy+wz))^2-(u^2+v^2+w^2)(x^2+y^2+z^2-1))^{\frac{s}{2}}} =$$

$$= \sum_{m,n=0}^{\infty} U^{(s)}_{m,n,k}(x,y,z)\frac{u^m}{m!}\frac{v^n}{n!}\frac{w^k}{k!}.$$

These polynomials are called Appell polynomials of type V and U. These polynomials first appeared in the works of Hermite, Didon, Appell, and Campe de Ferrier, see [1], [2]. These families of polynomials are quasimonomials.

The following theorem presents a simple criterion for the quasimonomiality of a polynomial family in terms of its exponential generating function.

Theorem. The polynomial family $\{B_{m,n,k}(x,y,z)\}$ defined by the exponential generating function

$$G = G(x,y,z,u,v,w) = \sum_{m,n,k=0}^{\infty} B_{m,n,k}(x,y,z)\frac{u^m}{m!}\frac{v^n}{n!}\frac{w^k}{k!}$$

is quasipolynomial with respect to $SO(3)$ if and only if G is a function of the three variables $ux+vy+wz, x^2+y^2+z^2$ and $u^2+v^2+w^2$.

[1] Appell P., Kampé de Fériet J. *Fonctions Hypergéométriques et Hypersphériques, Polynomes d'Hermite.* — Gauthier-Villars, 1926.
[2] Kampé de Fériet J. *Sur les fonctions hypersphériques.* Théses de l'entre-deux-guerres. — Paris, 1915.

E-mail: ✉ *samaruk.nat@khmnu.edu.ua.*

Dmytro Savchuk

Simultaneous conjugacy search problem in contracting self-similar groups

University of South Florida, Tampa, FL, USA

Many modern group-based cryptographic protocols are based on the variants of conjugacy search problem. We study the simultaneous conjugacy search problem (SCSP) in the class of self-similar contracting groups [2]. This class of groups contains extraordinary examples like Grigorchuk group [1], which is known to be non-linear as a group of intermediate growth, thus has a potential to withstand certain cryptanalytic attacks. The groups in this class admit a natural normal form based on the notion of a nucleus portrait and admit a fast polynomial time algorithm solving the word problem. While for some groups in the class the conjugacy search problem has been studied, there are many groups for which no such algorithms are known. We discuss benefits and drawbacks of using these groups in cryptography and provide computational analysis of variants of the length-based attack on SCSP for some groups in the class, including Grigorchuk group. Additionally, we discuss another effective heuristic attack on SCSP for contracting groups acting on a binary tree. The talk is based on two projects joint with Delaram Kahrobaei, Arsalan Malik, and Luciana Scuderi and Kerry Seekamp.

[1] R. I. Grigorchuk. On Burnside's problem on periodic groups. *Funktsional. Anal. i Prilozhen.*, 14(1):53–54, 1980.

[2] Volodymyr Nekrashevych. *Self-similar groups*, volume 117 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2005.

E-mail: ✉ *savchuk@usf.edu.*

Marko Serivka[1], Oleg Gutik[2]

On the semigroup of endomorphisms of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}^2}$ with the two-element family $\mathscr{F}^2$ of inductive nonempty subsets of ω

Ivan Franko National University of Lviv, Lviv, Ukraine

We shall follow the terminology of [5]. By ω we denote the set of all non-negative integers. For any $a \in \omega$ we denote $[a) = \{x \in \omega \colon x \geqslant a\}$. The semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ is introduced in [1] for any ω-closed subfamily $\mathscr{F}$ of elements of $\mathscr{P}(\omega)$. Monoid endomorphisms of the monoid $\boldsymbol{B}_{\omega}^{\mathscr{F}^2}$ for two-elements ω-closed family $\mathscr{F}^2$ of inductive nonempty elements of $\mathscr{P}(\omega)$ studied in [2–4]. Without loss of generality we may assume that $\mathscr{F}^2 = \{[0), [1)\}$ (see [1]).

We define the endomorphism $\varpi \colon \boldsymbol{B}_{\omega}^{\mathscr{F}^2} \to \boldsymbol{B}_{\omega}^{\mathscr{F}^2}$ by the formula

$$(i, j, [p))\varpi = \begin{cases} (i, j, [1)), & \text{if } p = 0; \\ (i+1, j+1, [0)), & \text{if } p = 1. \end{cases}$$

Theorem. *For any endomorphism ε of $\boldsymbol{B}_{\omega}^{\mathscr{F}^2}$ there exists the unique monoid endomorphism $\varepsilon_1 \colon \boldsymbol{B}_{\omega}^{\mathscr{F}^2} \to \boldsymbol{B}_{\omega}^{\mathscr{F}^2}$ and the unique non-negative integer n such that $\varepsilon = \varepsilon_1 \varpi^n$.*

We describe all endomorphisms of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}^2}$

[1] O. Gutik and M. Mykhalenych, *On some generalization of the bicyclic monoid*, Visnyk L'viv. Univ. Ser. Mech.-Mat. **90** (2020), 5–19 (in Ukrainian).

[2] O. Gutik and I. Pozdniakova, *On the semigroup of injective monoid endomorphisms of the monoid $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ with the two-elements family $\mathscr{F}$ of inductive nonempty subsets of ω*, Visnyk L'viv. Univ. Ser. Mech.-Mat. **94** (2022), 32-55

[3] O. Gutik and I. Pozdniakova, *On the semigroup of non-injective monoid endomorphisms of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ with the two-elements family $\mathscr{F}$ of inductive nonempty subsets of ω*, Visnyk L'viv. Univ. Ser. Mech.-Mat. **95** (2023), 14–27.

[4] O. Gutik and I. Pozdniakova, *On the semigroup of all monoid endomorphisms of the semigroup $\boldsymbol{B}_{\omega}^{\mathscr{F}}$ with the two-elements family $\mathscr{F}$ of inductive nonempty subsets of ω*, Visnyk L'viv. Univ. Ser. Mech.-Mat. **96** (2024), 5–24.

[5] M. Lawson, *Inverse semigroups. The theory of partial symmetries*, World Scientific, Singapore, 1998.

E-mail: ✉[1]*marko.serivka@lnu.edu.ua*, ✉[2]*oleg.gutik@lnu.edu.ua*.

Ihor Shapochka[1], Andrii Shapochka[2]

On tensor products of matrix representations of a cyclic p-group of order p over the ring $\mathbb{Z}_p[[x]]$

Uzhhorod National University, Uzhhorod, Ukraine

Let $G = \langle a \rangle$ be a cyclic p-group of order p, and let $\mathbb{Z}_p[[x]]$ be the ring of formal power series in one variable with p-adic integer coefficients. In [1], all pairwise non-equivalent indecomposable matrix representations of the group G over the ring $\mathbb{Z}_p[[x]]$ are described. They are exhausted by the following representations:

$$\Delta_0 : a \to 1, \quad \Delta_1 : a \to \tilde{\varepsilon}, \quad \Gamma_0 : a \to \begin{pmatrix} \tilde{\varepsilon} & \langle 1 \rangle \\ 0 & 1 \end{pmatrix},$$

$$\Gamma_i : a \to \begin{pmatrix} \tilde{\varepsilon} & \langle x^i \rangle \\ 0 & 1 \end{pmatrix}, \Gamma_j' : a \to \begin{pmatrix} 1 & \langle x^j \rangle^T \\ 0 & \tilde{\varepsilon} \end{pmatrix},$$

where $\tilde{\varepsilon}$ is the companion matrix of the cyclotomic polynomial $\Phi_p(x)$, $\langle x^i \rangle$ denotes a column vector with x^i as its first entry and zeros elsewhere, and $\langle x^j \rangle^T$ denotes the transpose of the column vector $\langle x^j \rangle$, and i, $j \in \mathbb{N}$. Based on the ideas of V. P. Rud'ko and P. M. Gudivok (see [2]), we derived the decomposition of tensor products of the aforementioned indecomposable matrix $\mathbb{Z}_p[[x]]$-representations of the group G into indecomposable components. In particular, it is shown that

$$\Delta_1 \otimes \Gamma_i \cong \Gamma_i' \oplus (p-2)\Gamma_0, \quad \Delta_1 \otimes \Gamma_i' \cong \Gamma_i \oplus (p-2)\Gamma_0$$

for every natural i.

[1] Gudivok P. M., Oros V. M., Roiter A. V., On representations of finite p-groups over a ring of formal power series with p-adic integer coefficients, *Ukrain. Mat. Zh.*, **44**, 1992, pp. 753–765.
[2] Gudivok P. M., Rud'ko V. P., Tensor products of representations of finite groups, Uzhgorod. Univ., 1985, 118 p.

E-mail: ✉[1] *ihor.shapochka@uzhnu.edu.ua,* ✉[2] *shapochkaandrij@gmail.com.*

Bohdan Shavarovskii

Oriented by characteristic roots polynomial matrices of simple structure

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine, Lviv, Ukraine

We consider matrices with elements from the ring of polynomials over the field of complex numbers (polynomial matrices). We investigate the semiscalar equivalence of polynomial matrices of simple structure. By definition [1], in polynomial matrices of simple structure, all elementary divisors are linear. Two matrices are called semiscalar equivalent if one of them is transformed into the other by multiplication on the left and right by a non-singular numerical and invertible polynomial matrix, respectively [1]. The problem of classifying matrices up to semiscalar equivalence is posed. The triangular form with invariant factors on the main diagonal established in [2] does not solve the problem due to the ambiguity of its definition. In the author's work [3], the reducibility using semiscalar equivalent transformations of a polynomial matrix of simple structure to the so-called triangular form oriented by characteristic roots is proved. The matrix of the specified form is defined more precisely than the mentioned triangular matrix established in [2]. This is confirmed by the block-triangular form of the left transformation matrix when going from one matrix oriented by characteristic roots to another such matrix (oriented by the same characteristic roots). The invariance of the placement of zero subrows below the main diagonal of a matrix oriented by characteristic roots is proved. Other invariants of such a matrix with respect to semiscalar equivalence are established. The obtained result has applications to the problem of classifying polynomial matrices with respect to semiscalar equivalence, and through it to the problem of classifying sets of numerical matrices with accuracy up to similarity. This result is also applicable to solving polynomial matrix equations.

[1] Kazimirskii P. S. *Factorization of Matrix Polynomials*, Naukova Dumka, Kyiv, Ukraine, 1981.

[2] Kazimirskii P. S. and Petrychkovych V. M *On the equivalence of polynomials matrices, In Theoretical and Applied Problems in Algebra and Differential Equations*, Naukova Dumka, Kyiv, Ukraine, 1977, 61–66.

[3] Shavarovskii B.Z. *On the triangular form of a polynomial matrix of simple structure and its invariants with respect to semi-scalar equivalence* Mat. Met. Fiz. Mekh.-Polya, 2023, 66, no. 1–2, 16-22.

E-mail: ✉ *bshavarovskii@gmail.com.*

Volodymyr Shchedryk

Canonical form of low-dimensional matrices with respect to one-sided transformations

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, L'viv, Ukraine

The Hermite and Smith normal forms play a fundamental role in matrix theory and have numerous applications in both pure and applied mathematics. For instance, they are used in constructive proofs of the structure theorem for finitely generated Abelian groups, in computing invariant polynomials and elementary divisors of polynomial matrices. The majority of numerical studies concerning these normal forms focus on optimizing the algorithms for their computation. However, several theoretical issues related to the structural properties of these forms remain unresolved. One such issue is the description of all right (or left) non-associated matrices with a given Smith normal form. The Hermite normal form, which serves as a tool for verifying one-sided equivalence of matrices, is generally a rather coarse method for addressing such a "delicate" problem, as it only allows for distinguishing non-associated matrices with a fixed determinant. The following example illustrates this statement. We describe all 2×2 matrices over $\mathbb{Z}$ with determinant 8 that are not right-associated with each other. Using the right Hermite normal form, it is easy to verify that such matrices are:

$$\begin{bmatrix} 1 & 0 \\ a_8 & 8 \end{bmatrix}, \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ a_4 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ a_2 & 2 \end{bmatrix},$$

where $a_i \in \{0,1,\ldots,i-1\}$, $i = 8,4,2$. This set consists of right non-associated matrices with Smith forms diag(1,8), diag(2,4). In particular, the matrices with the Smith normal form diag(1,8) are:

$$\begin{bmatrix} 1 & 0 \\ a_8 & 8 \end{bmatrix}, \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}.$$

These matrices are distributed across all four listed classes, and their identification requires an analysis of all elements within these classes.

We construct a new canonical form for third order matrices with respect to one-sided transformations. Unlike the classical triangular Hermite normal form, our canonical form is expressed as the product of two matrices: $P\Phi$, where P is invertible and Φ is in Smith normal form.

E-mail: ✉ *shchedrykv@ukr.net.*

Markiian Simkiv[1], Kateryna Makarova[2]

Genome as a metric space: statistical properties of genetic sequences

Ivan Franko National University of Lviv, Faculty of Mechanics and Mathematics, Lviv, Ukraine

This research applies mathematical methods to genome analysis by representing genetic sequences as metric spaces. We formalize genomes using statistical distance between k-mers (all possible substrings of length k in a sequence) and Levenshtein distance (the minimum number of single-character edits required to change one string into another) to reveal hidden patterns in genetic material and develop efficient analysis algorithms. The k-mer distance is calculated using the formula: $d_{mer}(x,y) = \sum_{i=1}^{l} \frac{1}{2^k} d(P_x^{(k)}, P_y^{(k)})$, where $P_x^{(k)}(i) = \frac{f_x^{(k)}(i)}{\sum_j f_x^{(k)}(j)}$ represents the normalized frequency of the i-th k-mer in sequence x.

Our methodology compares distance metrics, calculates fractal and entropy dimensions, and analyzes organisms comparatively.

Our key findings reveal that statistical distance (complexity $O(l^2)$) offers computational advantages over Levenshtein distance (complexity $O(mn)$) for large genomic datasets. Additionally, metric spaces constructed on genomes have specific quantitative characteristics distinguishing them from random sequences.

Organism	**Mean**	**Diam.**	**Fractal D.**	**Entropy D.**
E. coli	0.077	0.38	(2.84,-3.42)	(3.73,-8.69)
B. subtilis	0.082	0.39	(2.87,-3.47)	(3.76,-8.63)
S. solfat.	0.083	0.4	(2.98,-4.34)	(3.44,-7.85)
H. salin.	0.089	0.33	(2.98,-4.19)	(3.64,-8.21)
S. cerev.	0.077	0.45	(2.89,-3.45)	(3.31,-7.54)
Drosdofila	0.07	0.46	(2.68,-2.56)	(3.07,-7.09)
Random seq.	0.046	0.25	(3,-4.75)	(2.71,-6.74)

Table 1: Comparative metrics across different organisms.

Our approach provides an effective mathematical framework for genome analysis, enabling quantitative characterization of genetic sequences and taxonomic classification. The model's computational efficiency and dimensional parameters serve as powerful markers of genomic structural complexity.

Keywords: genome, metric space, statistical distance, Levenshtein distance, fractal dimension, entropy dimension, k-mers.

E-mail: ✉[1] *markiian.simkiv@lnu.edu.ua*, ✉[2] *kateryna.makarova@lnu.edu.ua.*

Oksana Skyhar

Fano and Boolean liners

Ivan Franko National University of Lviv, Lviv, Ukraine

In the talk we shall discuss some properties of Fano and Boolean affine and projective planes.

A *liner* is a set of points X endowed with a family of subsets $\mathcal{L}$ called lines, such that any distinct points $x, y \in X$ belong to a unique line $\overline{x\,y} \in \mathcal{L}$, and there exist three points that do not belong to a single line.

A liner X is *k-long* if every line in X contains at least k points.

A liner $(X, \mathcal{L})$ is

- a *projective plane* if it contains no disjoint lines;
- an *affine plane* if for any line L and point $x \in X \setminus L$ there exists a unique line that contain the point x and is disjoint with the line L.

It is known that every 4-long affine plane Π is a subliner of a unique projective plane, called the *projective completion* of Π.

A liner X is called

- *Boolean* if every parallelogram in X has parallel diagonals;
- *Fano* if for every quadrangle $abcd$ in X, the set $(\overline{a\,b} \cap \overline{c\,d}) \cup (\overline{a\,c} \cap \overline{b\,d}) \cup (\overline{a\,d} \cap \overline{b\,c})$ is contained in a line and is not a singleton.

It is easy to see that Boolean liners are Fano.

Theorem 1. *The projective completion of any 4-long affine Fano plane is a projective Fano plane.*

The question of algebraization of Fano and Boolean linears gives rise to two hypotheses:

- A projective plane is Fano iff it is coordinatized by a skew-field of characteristic 2.
- An affine plane is Boolean iff it is coordinatized by a quasifield of characteristic 2.

E-mail: ✉ *oksana.skyhar@lnu.edu.ua.*

Agata Smoktunowicz

On certain interactions between noncommutative algebra, algebraic geometry, and pre-Lie rings

School of Mathematics, University of Edinburgh, Edinburgh, UK

In recent years, rich and deep connections have emerged between noncommutative algebra and algebraic geometry.

Notably, Will Donovan and Michael Wemyss introduced contraction algebras – noncommutative algebras arising in connection with invariants of flops. More recently, Gavin Brown and Michael Wemyss provided a purely algebraic description of these algebras by presenting explicit generators and defining relations.

In this talk, we discuss and partially address several questions posed by Michael Wemyss concerning contraction algebras. Although these questions are motivated by links to Gopakumar–Vafa (GV) invariants, they are formulated in a purely algebraic context. We further offer a perspective on extending the classical Baker–Campbell–Hausdorff (BCH) formula beyond Lazard's correspondence, incorporating pre-Lie algebraic structures, in the context of finite pre-Lie rings. Finally, we pose an open question regarding potential generalisations to the setting of Lie rings.

E-mail: ✉ *a.smoktunowicz@ed.ac.uk.*

Fedir Sokhatsky[1], Bohdan Buniak[2]

Semisymmetric Anticommutative Loops up to order 15

[1] Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of NASU, Ukraine.

[2] Department of Information Security of Vinnytsia National Technical University, Ukraine.

Although loops are a generalization of groups, only some generalizations of group study methods are effective in loop theory. However, in classes of loops in which some original properties of groups are preserved, such a transfer of methods is possible. For example, classes of loops in which the isotopy relation and the isomorphy relation coincide. Any loop isotope of a group is isomorphic to the group. The class of G-loops is characterized by this property. In the variety of IP-loops, there is a weaker property: if two commutative IP-loops are isotopic, then they are isomorphic, but isotopism is not always an isomorphism [1]. Here, we continue the study of SA-loops in which any isotopism is an isomorphism [2], [3]; any autotopism of an SA loop is its automorphism.

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1	1	0	3	4	2
2	2	4	0	1	3
3	3	2	4	0	1
4	4	3	1	2	0

SA loop $(\mathbb{Z}_8;\circ,0)$
Semisymmetricity: $x\circ(y\circ x)=y$;
Anticommutativity:
$x\circ y=y\circ x\Rightarrow(x=0\vee y=0\vee x=y)$.

Theorem. *SA loop of order* $3m$ *does not exist.*

Using computer calculations, the spectrum of existence of SA loops up to order 15 was established: SA loops of orders 1, 2, 3, 4, 6, 7, 9, 12, 15 do not exist; SA loops of orders 5, 8, 10, 11, 13, 14 exist. Moreover up to isomorphism, there exists exactly one SA loop of each of orders 5 and 8, and there are exactly 22 SA loops of order 10.

[1] Sokhatsky Fedir M. On pseudoisomorphy and distributivity of quasigroups Bul. Acad. Ştiinţe Repub. Moldova, Mat., 2016, No. 2(81), 125–142.
[2] Fedir Sokhatsky, Quasigroups and loops up to order 5. Abstracts of ConfQRS-2025, July 2-4, 2025.
[3] Fedir Sokhatsky, Bohdan Buniak. Formulas for determining some quasigroups of the order 8. Abstracts of ConfQRS-2025, July 2-4, 2025.

E-mail: ✉[1] *fmsokha@ukr.net*, ✉[2] *bbuniak@ukr.net*.

Fedir Sokhatsky[1], Halyna Krainichuk[2], Volodymyr Luzhetsky[3]

Canonical and matrix figuration of quasigroups of order 4

[1] Ya. S. Pidstryhach Institute of Applied Problems of Mechanics and Mathematics, NAS of Ukraine, Lviv, Ukraine

[2,3] Vinnytsia National Technical University, Vinnytsia, Ukraine

A quasigroup is called a 0-loop if it has a neutral element denoted by 0. There are four 0-loops of order four, one of which is a Klein group, others are isomorphic cyclic groups. The obtained results for order four: 1) every quasigroup has a unique canonical decomposition over exactly one of these 0-groups; 2) every quasigroup has a unique matrix canonical decomposition over either the cyclic group or Klein group; 3) the respective formulas and examples for applying are given [1].

Theorem 1. *Every quasigroup operation f on $Z_2^2 := \{00; 01; 10; 11\}$ is determined exactly by one of the formulas $f(\overline{x}, \overline{y}) = \overline{x}A \oplus \overline{y}B \oplus \overline{a}$, $f(\overline{x}, \overline{y}) = (\overline{x}A + \overline{y}B + \overline{a})C$ where $(\oplus), (+)$ are addition of the rings $Z_2 \times Z_2$ and Z_4 respectively, i.e.*

+	00	01	10	11
00	00	01	10	11
01	01	10	11	00
10	10	11	00	01
11	11	00	01	11

$\oplus$	00	01	10	11
00	00	01	10	11
01	01	00	11	10
10	10	11	00	01
11	11	10	01	00

where $\overline{a} \in Z_2^2$, $C \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$,

$A, B \in \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\}$

Theorem 2. *The quasigroup $(Z_2^2; \circ)$ satisfies $(\overline{x} \circ \overline{y}) \circ (\overline{x} \circ \overline{x}) = \overline{y}$ over $(Z_2^2; \oplus, \overline{0})$ if and only if it has one of the following decomposition $\overline{x} \circ \overline{y} = \overline{x}A \oplus \overline{y}A^2 \oplus \overline{a}$, $\quad \overline{x} \circ \overline{y} = \overline{x}A^2 \oplus \overline{y}A \oplus \overline{a}$, where $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $\overline{a} \in \{00, 01, 10, 11\}$.*

[1] F. M. Sokhatsky, H. V. Krainichuk, V. A. Luzhetsky. Canonical and matrix figuration of quasigroups of the fourth order *Applied problems of mechanics and mathematics*, (2024), Issue 22. P. 95–105.

E-mail: ✉[1]*fmsokha@ukr.net*, ✉[2]*krainichuk@ukr.net*, ✉[3]*lva.kzi2002@gmail.com.*

Yaryna Stelmakh

The automorphism group of the natural and integral Kirch spaces

Ivan Franko National University of Lviv, Lviv, Ukraine

The *integral Kirch space* is the set $\mathbb{Z}^\bullet$ of nonzero integers endowed with the *Kirch topology*, which is generated by arithmetic progressions $a + b\mathbb{Z}$ where $a, b \in \mathbb{Z}^\bullet$, where a, b are coprime numbers with b square-free. The *natural Kirch space* is the set $\mathbb{N}$ of positive integers endowed with the subspace topology inherited from the integer Kirch space.

Theorem 1. *The homeomorphism group of the natural Kirch space is trivial.*

Theorem 2. *The homeomorphism group of the integral Kirch space contains exactly* 2 *elements: the identity homeomorphism and the envolution* $j : \mathbb{Z}^\bullet \to \mathbb{Z}^\bullet$, $j : z \mapsto -z$.

The proofs are not trivial and exploit some deep results of Number Theory, in particular, the famous Dirichlet theorem (on primes in arithmetic progressions), the Zsigmondy theorem on primitive prime divisors, Mihăilescu theorem (on neighbour prime powers).

[1] T. Banakh, Ya. Stelmakh, S. Turek, *The Kirch space is topologically rigid*, Topology Appl. **304** (2021), 107782.

[2] Ya. Stelmakh, *Homeomorphisms of the space of nonzero integers with the Kirch topology*, Visnyk Lviv Univ. Ser. Mech. Math. **89** (2020), 33–53.

E-mail: ✉ *yarynziya@ukr.net.*

Olena Toichkina

Green's relations on the weak endomorphism semigroup of a partial equivalence relation

Luhansk Taras Shevchenko National University, Poltava, Ukraine

Let X be a nonempty set and $\rho \subseteq X \times X$. A transformation f of X is called an *endomorphism* of ρ if $(x, y) \in \rho$ implies $(xf, yf) \in \rho$ for all $x, y \in X$. A transformation f of X is called a *weak endomorphism* of ρ if $(x, y) \in \rho$ implies that $xf = yf$ or $(xf, yf) \in \rho$ for all $x, y \in X$ (see, e.g., [1]). The set of all weak endomorphisms of $p \subseteq X \times X$ with respect to the operation of the composition of transformations is a semigroup and it is denoted by $\mathrm{WEnd}(X, \rho)$.

A binary relation on a set X is called a *partial equivalence relation* on X if it is symmetric and transitive. By $PEq(X)$, we denote the set of all partial equivalences on X. Some properties of the weak endomorphism semigroup $WEnd(X, \rho)$, $\rho \in PEq(X)$, were described in [2]. We continue to study the structure of the semigroup mentioned above and consider Green's relations on it. Note that the description of Green's relations on the endomorphism semigroup of an arbitrary equivalence was obtained in [3].

[1] Knauer, U., Pipattanajinda, N.A.: A formula for the number of weak endomorphisms on paths. Algebra Discrete Math. **25**(2), 270–279 (2018)
[2] Zhuchok, Yu., Toichkina, O.: The semigroup of weak endomorphisms of a partial equivalence relation. Ukr. Math. J. **76**(12), 1727–1737 (2024)
[3] Pei, H.: Regularity and Green's relations for semigroups of transformations that preserve an equivalence. Comm. Algebra **33**(1), 109–118 (2005)

E-mail: ✉ *toichkina.e@gmail.com.*

Anatolii Tushev

On induced modules over group rings of soluble groups of finite rank

Justus-Liebig University of Giessen, Giessen, Germany

Let G be an abelian group and $t(G)$ be the torsion subgroup of G. Let $p \in \pi(t(G))$ and G_p be the Sylow p-subgroup of $t(G)$. Then we can define the total rank $r_t(G)$ of G by the following formula: $r_t(G) = r(G/t(G)) + \sum_{p\in\pi(t(G))} r(G_p)$. A soluble group has finite abelian total rank, or is a soluble $FATR$-group, if it has a finite series in which each factor is abelian of finite total rank.

Theorem 1. *Let G be a nilpotent $FART$-group and let D be a normal subgroup of G such that the quotient group G/D is polycyclic. Let k be a finitely generated field such that $char\, k \notin Sp(G)$ and let M be a faithful kG-module. Suppose that the subgroup D contains an isolated in D abelian G-invariant subgroup A such that $P = ann_{kA}(M)$ is a maximal G-invariant faithful ideal of kA. If the module M is kD/PkD-torsion-free then for any nonzero element $0 \neq a \in M$ there is a finitely generated subgroup $H \leq G$ such that $akG = akH \otimes_{kH} kG$.*

Let G be a group, let k be a field and let M be a kG -module. The module M is said to be primitive if it is not induced from any kH-submodule for any subgroup $H < G$. The module M is said to be semiprimitive if it is not induced from any kH-submodule for any subgroup $H < G$ such that $|G : H| < \infty$. A representation φ of G over k is said to be primitive (semiprimitive) if the module of the representation φ is primitive (semiprimitive). The above theorem allows us to obtain the following result.

Theorem 2. *Let k be a finitely generated field and let G be a nilpotent $FATR$-group of nilpotency class 2 such that the torsion subgroup T of G is contained in the centre Z of G and $char\, k \notin Sp(G)$. Suppose that the group G admits a faithful semiprimitive irreducible representation φ over the field k. Then the group G is finitely generated.*

The case of minimax groups of nilpotency class 2 and primitive representations was considered in [1].

[1] A.V. Tushev, On primitive representations of minimax nilpotent groups, Math. Notes (1-2) 72 (2002) 117-128.

E-mail: ✉ *anavlatus@gmail.com.*

Vasyl Ustimenko

On geometries over diagrams, symbolic computations and their applications

Royal Holloway University of London, Egham, United Kingdom

Let K be commutative ring with unity. Jordan-Gauss graph $J(K)$ is the special case of linguistic incidence structure of type (s, r, m) given by the following way. We identify points with tuples of kind $(x) = (x_1, x_2, \ldots, x_{s+m}) \in P$ and lines with tuples $[y] = [y_1, y_2, \ldots, y_{r+m}] \in L$. Brackets and parenthesis are convenient to distinguished type of the vertex of the graph. Elements (x) and $[y]$ are incident $(x)I[y]$ if and only if the following relations hold.

$a_1 x_{s+1} - b_1 y_{r+1} = f_1(x_1, x_2, \ldots, x_s, y_1, y_2, \ldots, y_r)$,
$a_2 x_{s+2} - b_2 y_{r+2} = f_2(x_1, x_2, \ldots, x_{s+1}, y_1, y_2, \ldots, y_{r+1})$,
$\ldots$,
$a_m x_{s+m} - b_m y_{r+m} = f_m(x_1, x_2, \ldots, x_{s+m-1}, y_1, y_2, \ldots, y_{r+m-1})$

where a_i and b_j, $j = 1, 2, \ldots, m$ are not zero divisors, quadratic polynomials f_j define the bilinear maps of K^{s+j-1} and K^{r+j-1} onto K.

We assume that f_j are given in their standard form, i. e. the list of monomial terms ordered lexicographically and say that two Jordan graphs $J_1(K)$ and $J_2(K')$ of the same type (r, s, m) are symbolically equivalent if they are given by the systems as written above over commutative rings K and K' where quadratic polynomials f_j and f'_j respectively have the lists of monomial terms with nonzero coefficients such they differ only by coefficients of monomial terms.

Proposition 1. *Let $(\Gamma(G(K)), I, t)$ be the geometry of Chevalley group G with Coxeter-Dynkin diagram X_n over the field F and Borel subgroup B. Then the restriction of the incidence relation I on two orbits of $(B, \Gamma(G))$ is a Jordan-Gauss graph or empty set.*

The change of each Jordan-Gauss graph over the field F appeared in the proposition for the symbolically equivalent Jordan-Gauss graph over commutative ring K can be used for the construction of the incidence system over K. These systems were used for the constructions od special elements, groups and semigroups of affine Cremona group $End(K[x_1, x_2, \ldots, x_n])$. Main results in this direction will be presented during the talk.

E-mail: ✉ *vasyl.ustymenko@rhul.ac.uk.*

Pavel Varbanets[†1], Sergey Varbanets[2], Yakov Vorobyov[3]

Kloosterman-weighted arithmetic sums over the Gaussian integers

[1,2] I.I. Mechnikov Odesa National University,
Odesa, Ukraine

[3] Izmail State University of Humanities,
Izmail, Ukraine

Let G denote the ring of Gaussian integers. For $\gamma \in G$, by G_γ we denote the ring of residue classes modulo γ, and by G_γ^* the multiplicative group of this ring. For $\alpha, \beta, \gamma \in G$, the Kloosterman sum $K(\alpha, \beta; \gamma)$ is defined by the equality

$$K(\alpha, \beta; \gamma) = \sum_{x \in G_\gamma^*} \exp\left(2\pi i \mathrm{Re}\left(\frac{\alpha x + \beta x^{-1}}{\gamma}\right)\right)$$

where x^{-1} denotes the multiplicative inverse of x modulo γ.
In this sequence of investigations, we study the arithmetic function $\tau(\omega)$ weighted by Kloosterman sum defined above. We consider the distribution of the values of $\tau(\omega)$ in arithmetic progressions with increasing differences over the ring $\mathbb{R}(d)$ of integral values in imaginary quadratic field $\mathbb{Q}(\sqrt{-d})$, $d > 0$. We derive an asymptotic formula for the mean value of the divisor function weighted by the Kloosterman sum and its analogues.

E-mail: ✉[2] *svarbanets@onu.edu.ua*, ✉[3] *yashavoro@gmail.com.*

Tetiana Voloshyna

Closed inverse subsemigroups of the finitary inverse semigroup

Lesya Ukrainka Volyn National University, Lutsk, Ukraine

Since every transitive permutation representation of an inverse semigroup is given by some closed inverse subsemigroup [1], an important question is to describe all its closed inverse subsemigroups.

Let $IS(X)$ is a symmetric inverse semigroup on the set X. For partial permutation $\tau \in IS(X)$ we denote its domain by *dom* τ. Let ω is the natural partial order on the inverse semigroup [1, 2]:

$$\alpha\,\omega\,\beta \quad\Leftrightarrow\quad \alpha\beta^{-1} = \alpha\alpha^{-1}.$$

A subset $T \subseteq S$ of the inverse semigroup S is called closed with respect to the natural partial order ω, if it satisfies the equality

$$\{s \in S \mid \exists t \in T \ : t\,\omega\,s\} = T.$$

A description of all closed inverse subsemigroups of the finite symmetric inverse semigroup and the structure of its right ω-cosets by the closed inverse semigroup are given in [3].

Let $\mathbb{N}$ denote the set of natural numbers. The set of all such partial permutations $\tau \in IS(\mathbb{N})$ for which the condition $|dom\ \tau| < \infty$ is satisfied, forms an inverse semigroup, which we will call a finitary inverse semigroup, and denote by $FI(\mathbb{N})$.

Theorem. For every finite subset $M \subseteq \mathbb{N}$ and every subgroup G of the symmetric group $S(M)$ the subsemigroup $H = G \oplus FI(\mathbb{N}\backslash M)$ is a closed inverse subsemigroup of $FI(\mathbb{N})$. On the other hand, every closed inverse subsemigroup of $FI(\mathbb{N})$ has this form.

[1] Clifford A.H., Preston G.B. The Algebraic Theory of Semigroups. V.1. Math. Surveys, No. 7. Rhode Islands: Amer. Math. Society, 1964. 224 p.
[2] Clifford A.H., Preston G.B. The Algebraic Theory of Semigroups. V.2. Math. Surveys, No. 7. Rhode Islands: Amer. Math. Society, 1967. 352 p.
[3] T. Voloshyna. Effective transitive representations of the finite symmetric inverse semigroup. Bulletin of Kyiv University. Mathematics and Mechanics. 1998. Issue.2. P. 16–21. (In Ukrainian)

E-mail: ✉ *tetianavoloshyna@gmail.com.*

Oksana Yakimova

A bi-Hamiltonian nature of the Gaudin algebras

Friedrich-Schiller-Universität Jena, Germany

Let h be a direct sum of n copies of a simple Lie algebra g. In 1994, Feigin, Frenkel, and Reshetikhin constructed a large commutative subalgbera of the enveloping algebra U(h). This subalgebra, which is an image of the Feigin—Frenkel centre, contains quadratic Gaudin Hamiltonians and therefore is known as a Gaudin subalgebra. By now it has been studied from various points of view and numerous generalisations have been obtained. We look at the 'classical' version of a Gaudin algebra, i.e., at its image in the symmetric algebra S(h). This image, say C, is Poisson-commutative and can be obtained from a suitable pair of compatible Poisson brackets on S(h) via the Lenard—Magri scheme. An advantage of the Lenard—Magri approach is a well-developed geometric machinery. For example, it allows us to show that C is algebraically closed in S(h). We will discuss also a generalisation to a non-reductive setting.

E-mail: ✉ *oksana.yakimova@uni-jena.de.*

Davyd Zashkolnyi

Computing self-replicating degrees of plane groups

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine.

An action of a group on the space of words over a finite alphabet is called self-similar if, roughly speaking, it acts on smaller subsets in the same way as on the entire set. Self-similar groups arise in different areas of mathematics, such as dynamical systems, fractal geometry, finite automata, and have interesting properties related to the growth, amenability, word problem etc.

A group G admits a (transitive) self-similar action if and only if there exists a homomorphism ϕ from its subgroup of finite index to G, such that there is no nontrivial ϕ-invariant normal subgroup of G. The size of an alphabet of the associative self-similar action equals to the index of this subgroup and is called a self-similar degree. An action is called self-replicating if the homomorphism ϕ is surjective.

We study self-replicating degrees of crystallographic groups, since every crystallographic group admits a self-replicating action. Surjective virtual endomorphisms of crystallographic groups are induced by conjugation on an affine transformation (A, t). The determinant of the matrix A equals to the degree of the respective self-replicating action. The determination of possible degrees leads to certain diophantine equations together with a nonlinear conditions on the characteristic polynomial of A^{-1}.

We design an algorithm for computing possible self-replicating degrees of the plane groups. Minimal possible degrees are presented in the tables below.

Group num (ITA)	1	2	3	4	5	6	7	8	9
Minimal Degree	2	2	4	6	4	2	6	3	3

Group num (ITA)	10	11	12	13	14	15	16	17
Minimal Degree	2	2	9	3	4	4	3	3

Note to the tables: Group num (ITA) is a sequential number of a group as given in the International Tables for Crystallography, Vol. A.

E-mail: ✉ *davendiy@gmail.com.*

Halyna Zelisko

On strongly prime monoids with zero

Ivan Franko National University of Lviv, Lviv, Ukraine

Recently, the properties of semigroups, monoids and acts have been studied quite actively, many of which are analogous to the properties of rings and modules. The concept of a strongly prime module was introduced in [1] and [2], and strongly prime rings were studied in [3].

Let S be a monoid with zero. Let $Act-S$ be a category of unitary and centered right acts over monoid S. In the category $Act-S$ a torsion preradical r is defined if each act $M \in Act-S$ is assigned its subact $r(M)$ such that for any $S-$homomorphism $f : M \to N$ $f(r(M)) \subseteq r(N)$. It is clear that r is subfunctor of the identity functor in the category $Act-S$. The torsion preradical r is called a torsion radical if in addition $r(M/r(M)) = 0$ for all $M \in Act-S$. A right act M is called $r-$torsion if $r(M) = M$ and $r-$torsionfree if $r(M) = 0$.

A nonzero right act M is called strongly prime if M is a prime act and for each nonzero right subact $N \subseteq M$ and for each element $y \in M$ there exist elements $x_1,\ x_2,\ \ldots,\ x_n \in N$ such that

$$Ann(x_1,\ x_2,\ \ldots,\ x_n) \subseteq Ann(y).$$

A monoid S is called right strongly prime if the act S_S is strongly prime.

Theorem. *For monoid with zero S the following conditions are equivalent:*

(1) S s a right strongly prime monoid;

(2) $r(S) = 0$ for each proper preradical in the category $Act-S$;

(3) the injective envelope $E(S)$ of right act S_S does not contain nontrivial completely invariant subacts;

(4) if $a \in S$, $a \neq 0$, then there exist $s_1,\ s_2,\ \ldots,\ s_n \in S$ such that from $as_ib = 0$ for all i follows that $b = 0$.

[1] Beachy J. Some aspects of noncommutative localization. Lecture Notes in Math, 545(1976), P. 2-31.

[2] Handelman D., Lawrence J. Strongly prime rings // Trans. Amer. Math. Soc. 211(1975), P. 209-223.

[3] Kaucikas A., Wisbauer R. On strongly prime rings and ideals // Comm. Algebra, 28(2000), P. 5461-5473.

E-mail: ✉ *zelisko_halyna@yahoo.com.*

Efim Zelmanov

On Jordan and Lie homomorphisms

Shenzhen International Center for Mathematics, Southern University of Science and Technology, Shenzhen, China

Let A and B be associative algebras. A linear map

$$\varphi\colon A \to B$$

is called a *Jordan homomorphism* if

$$\varphi(a^2) = \varphi(a)^2, \quad \varphi(aba) = \varphi(a)\,\varphi(b)\,\varphi(a) \quad \text{for all } a, b \in A.$$

A linear map φ is called a *Lie homomorphism* if

$$\varphi\big([a, b]\big) = \big[\varphi(a), \varphi(b)\big] \quad \text{for all } a, b \in A.$$

We will discuss both the classical and the recent results concerning such mappings.

E-mail: ✉ *efim.zelmanov@gmail.com.*

Anatolii Zhuchok

On the determinability of free strict n-tuple semigroups by their endomorphism semigroups

University of Potsdam, Potsdam, Germany; Luhansk Taras Shevchenko National University, Poltava, Ukraine

Following [1], a nonempty set G equipped with n binary operations denoted by $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ is called a *strict n-tuple semigroup* if it satisfies the axioms $\left(x \boxed{r} y\right) \boxed{s} z = x \boxed{i} \left(y \boxed{j} z\right)$ for all $x, y, z \in G$ and $r, s, i, j \in \{1, 2, \ldots, n\}$. Let X be an arbitrary nonempty set and $F[X]$ the free semigroup on X. Let further $\mathbb{N}$ denote the set of all positive integers and $n \in \mathbb{N}\backslash\{1\}$. We denote the union of $n-1$ disjoint copies of $X \times X$ by $(X \times X)_{n-1}$. For every pair $(x_1, x_2) \in X \times X$, denote by $(x_1, x_2)_i$ with $2 \leq i \leq n$ the i-th copy of (x_1, x_2). For all $h = (x_1, x_2)_i \in (X \times X)_{n-1}$, where $x_1, x_2 \in X$ and $2 \leq i \leq n$, assume that $[h] = x_1 x_2 \in F[X]$. Define n binary operations $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ on $F[X] \cup (X \times X)_{n-1}$ by

$$a_1 \ldots a_m * b_1 \ldots b_s = \begin{cases} (a_1, b_1)_i, \text{ if } * = \boxed{i},\ m = s = 1 \neq i, \\ a_1 \ldots a_m b_1 \ldots b_s \text{ otherwise,} \end{cases}$$

$$w * h = w[h], \quad h * w = [h]w, \quad h * f = [h][f]$$

for all $a_1 \ldots a_m$, $b_1 \ldots b_s$, $w \in F[X]$, $h, f \in (X \times X)_{n-1}$ and $* \in \left\{\boxed{1}, \boxed{2}, \ldots, \boxed{n}\right\}$. The algebra $(F[X] \cup (X \times X)_{n-1}, \boxed{1}, \boxed{2}, \ldots, \boxed{n})$ is denoted by $X^\sharp(n)$.

Theorem 1. *([1], Theorem 3) For every $n > 1$, $X^\sharp(n)$ is the free strict n-tuple semigroup.*

An algebra A of some class Σ *is determined* by its endomorphism semigroup in the class Σ if for any algebra $B \in \Sigma$ the condition $\mathrm{End}(A) \cong \mathrm{End}(B)$ implies $A \cong B$.

Theorem 2. *Let $X^\sharp(n)$ and $Y^\sharp(n)$ be free strict n-tuple semigroups with $n > 1$, and suppose that* $\mathrm{End}(X^\sharp(n)) \cong \mathrm{End}(Y^\sharp(n))$. *Then $X^\sharp(n)$ and $Y^\sharp(n)$ are isomorphic.*

The author has been supported by a Philipp Schwartz Fellowship of the Alexander von Humboldt Foundation.

[1] Zhuchok, A.V.: Free strict n-tuple semigroups. Semigroup Forum 109, 753–758 (2024). https://doi.org/10.1007/s00233-024-10471-5

E-mail: ✉ *zhuchok.av@gmail.com.*

Anatolii Zhuchok[1], Yuliia Zhuchok[2]

On the automorphism group of the endomorphism semigroup of a free strict n-tuple semigroup of rank 1

[1] University of Potsdam, Potsdam, Germany and
Luhansk Taras Shevchenko National University, Poltava, Ukraine

[2] Luhansk Taras Shevchenko National University, Poltava, Ukraine

A nonempty set G equipped with n binary operations denoted by $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ is called a *strict n-tuple semigroup* [1] if it satisfies the axioms $\left(x \boxed{r} y\right) \boxed{s} z = x \boxed{i} \left(y \boxed{j} z\right)$ for all $x, y, z \in G$ and $r, s, i, j \in \{1, 2, \ldots, n\}$.

Let $\mathbb{N}$ denote the set of all positive integers. We add $n-1$ $(n > 1)$ arbitrary elements $x \notin \mathbb{N}$ to $\mathbb{N}$. Each added i-th element will be conveniently denoted by 2_i and imagined as a copy of the number 2. Define n binary operations $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ on $\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$ by

$$m * s = \begin{cases} 2_i, \text{ if } * = \boxed{i}, \ m = s = 1 \neq i, \\ \quad m + s \ \text{ otherwise,} \end{cases}$$

$$m * 2_i = 2_i * m = m + 2, \quad 2_i * 2_j = 4$$

for all $m, s \in \mathbb{N}$, $i, j \in \{2, \ldots, n\}$ and $* \in \left\{\boxed{1}, \boxed{2}, \ldots, \boxed{n}\right\}$. The algebra $(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \boxed{1}, \boxed{2}, \ldots, \boxed{n})$ is denoted by $\mathbb{N}^\sharp(n)$. By Corollary 4 of [1], for every $n > 1$, $\mathbb{N}^\sharp(n)$ is the free strict n-tuple semigroup of rank 1.

We describe the endomorphism semigroup of $\mathbb{N}^\sharp(n)$ and the automorphism group of the endomorphism semigroup of $\mathbb{N}^\sharp(n)$ $(n > 1)$.

The first named author has been supported by a Philipp Schwartz Fellowship of the Alexander von Humboldt Foundation.

[1] Zhuchok, A.V.: Free strict n-tuple semigroups. Semigroup Forum 109, 753–758 (2024). https://doi.org/10.1007/s00233-024-10471-5

E-mail: ✉[1] *zhuchok.av@gmail.com*, ✉[2] *yulia.mih1984@gmail.com*.

Yu.V. Zhuchok

On some classes of trioids defined by semigroups

Luhansk Taras Shevchenko National University, Poltava, Ukraine;
Johannes Kepler University Linz, Linz, Austria

A nonempty set T with binary associative operations $\dashv$, $\vdash$, and $\perp$ is called *a trioid* if for all $x, y, z \in T$ the following conditions hold:

$$\begin{gathered}(x \dashv y) \dashv z = x \dashv (y \vdash z), \quad (x \vdash y) \dashv z = x \vdash (y \dashv z),\\ (x \dashv y) \vdash z = x \vdash (y \vdash z), \quad (x \dashv y) \dashv z = x \dashv (y \perp z),\\ (x \perp y) \dashv z = x \perp (y \dashv z), \quad (x \dashv y) \perp z = x \perp (y \vdash z),\\ (x \vdash y) \perp z = x \vdash (y \perp z), \quad (x \perp y) \vdash z = x \vdash (y \vdash z).\end{gathered}$$

The notion of a trioid was first appeared in [1] at the study of ternary planar trees. Trioids are a basis of trialgebras as well a generalization of semigroups and dimonoids (see, e.g., [2]). A trioid (dimonoid) we call *trivial* if all trioid (dimonoid) operations coincide and *non-trivial* otherwise. If $\mathcal{S} = (S, *)$ is a semigroup, we refer to $\mathcal{S}$ as a trivial dimonoid $(S, *, *)$ or a trivial trioid $(S, *, *, *)$.

According to Theorem 4 of [3], for an arbitrary group H there exists a non-trivial digroup such that the group part of this digroup coincides with H. A similar statement holds for dimonoids: for an arbitrary semigroup there exists a non-trivial dimonoid containing it as a subdimonoid in which operations coincide (see Theorem 1 of [3]). In connection with this, it is natural to consider the following question: is there for an arbitrary semigroup a non-trivial trioid with three pairwise different operations containing it as a subtriod in which operations coincide? We consider the mentioned problem and some related questions.

The author has been supported by the Joint Excellence in Science and Humanities program (JESH) of the Austrian Academy of Sciences.

[1] Loday J.-L., Ronco M.O., Trialgebras and families of polytopes, *Contemp. Math.* **346** (2004), 369–398.

[2] Zhuchok Yu.V., Automorphisms of the category of free dimonoids, *J. Algebra* **657** (2024), no. 1, 883–895.

[3] Zhuchok Yu.V., Pilz G.F., Zhuchok A.V., On embedding groups into digroups, *Algebra Discrete Math.* **38** (2024), no. 2, 270–287.

E-mail: ✉ *zhuchok.yu@gmail.com.*

List of Authors

15th Ukraine Algebra Conference
Lviv, Ukraine,
July 8–12, 2025

Technical editors:
I. Raievska, M. Raievska

Editors:
T. Banakh, O. Bezushchak, A. Oliynyk,
I. Raievska, M. Raievska