BLAST 2025

Lattices
Universal Algebra
Boolean Algebras
Set Theory
Topology

University of Colorado, Boulder, May 19–23, 2025

Celebrating J. Donald Monk, Ágnes Szendrei, and Walter Taylor

ABSTRACTS

University of Colorado
Boulder

Contents

Kira Adaricheva 1
Ana Belén Avilez Garcia 3
Richard N. Ball 5
Jennifer Brown 6
Filippo Calderoni 7
Todd Eisworth 8
Matt Evans 9
Jonathan Feigert 10
Stefano Fioravanti 11
Nick Galatos 12
Volodymyr Gavrylkiv 13
Valeria Giustarini 15
Temitope Jaiyeola 16
Nicholas Jamesson 17
Mashiath Khan 18
Michael Kompatscher 19
Sangman Lee 20
Francesco Manfucci 21
Miklós Maróti 23
Marcos Mazari-Armida 24
Sebastian Melzer 25
Andrew Moorhead 26
Mateo Muro 28

John Nicholson 29
Sebastian Pardo Guerra 30
Ranjitha Raviprakash 31
Branimir Šešelja 32
Ricardo Suarez 34
Joanne Walters-Wayland 35
Jonathan Weinberger 36
Ross Willard 37
Patrick Wynne 38

The E-base of finite semidistributive lattices

Kira Adaricheva (kira.adaricheva@hofstra.edu)
Hofstra University

Implicational bases (IBs) are a well-known representation of finite closure spaces and their closure lattices. Implications go by many names in a broad range of fields, e.g., attribute implications in Formal Concept Analysis, functional dependencies in database theory, or Horn clauses in propositional logic.

In lattice theory, implications translate into join covers, i.e., sets of relations $j \leq j_1 \vee \cdots \vee j_k$, $k \geq 1$, on the set of join-irreducible elements of a lattice.

The representation by an IB is not unique, and a closure space usually admits multiple IBs. Among these, the canonical base, the canonical direct base as well as the D-base aroused significant attention due to their structural and algorithmic properties.

The study of free lattices was influential in bringing up an IB of a new sort, which was called the E-base. It is a refinement of the D-base that, unlike the aforementioned IBs, does not always accurately represent its associated closure space. This leads to an intriguing question: for which classes of (closure) lattices do closure spaces have a valid E-base?

Finite lower bounded lattices are known to form such a class. In recent publication `https://arxiv.org/abs/2502.04146`, we prove that for semidistributive lattices, the E-base is both valid and minimum.

Among other results, we look into E-base in a few classes of closures spaces with the Exchange Axiom, establish the complexity of recognizing that two elements of closure space defined by some IB are

E-related, and show that D-geometries—closure spaces corresponding to lower bounded lattices with the Anti-Exchange axiom—can be recognized from any IB in polynomial time, using the help from the E-base.

This is a joint work with Simon Vilmin, Aix-Marseille Université, CNRS, France.

What are coz-inclusions?

Ana Belén Avilez Garcia (avilezgarcia@chapman.edu)
Chapman University

The framework categories in point-free topology are the category of frames and its dual, the category of locales. The former has an algebraic nature, while the latter has a more geometrical flavor. Together, they provide a richer way to study spaces. Locales are referred to as generalized spaces, and sublocales (or frame quotients) are the generalized versions of subspaces in the category of locales. We will study a special class of sublocales in which one can observe how complex the interaction between the lattice-theoretic part of frames and the topological aspect of sublocales can be.

The cozero part of a frame is a sub-σ-frame inside it that plays an important role under complete regularity, since it join-generates the frame. The relationship between the cozero part of a sublocale and its ambient frame is generally not well-behaved; in fact, it is quite chaotic. Nevertheless, there are several types of embeddings that help control the behavior of the cozero part of a sublocale.

In this talk, we will study the new notion of a *coz-included* sublocale: a special kind of embedding that describes how the cozero part of a sublocale sits with respect to the cozero part of the ambient frame. This new notion will be compared with other well-known forms of embedding, such as C-, C^*-, and z-embeddings. Motivated by non-trivial spatial examples, this new notion and its weakenings—such as casi coz-included and relative zero sublocales—will be explored and characterized. As a consequence, new characterizations of several classes of frames, such as perfectly normal and Oz frames, among others, will be provided.

This is a joint work with Oghenetega Ighedo and Joanne Walters-Wayland.

A construction of the assembly of a frame

Richard N. Ball (rball@du.edu)
University of Denver

We offer here a particularly simple and direct construction of the congruence frame of a given frame, aka its assembly, from the bounded meet semilattice of differences of frame elements. The construction enables economical proofs of two of the assembly's known attributes, namely ultranormality and ultraparacompactness, as well as a proof that the assembly is an essential extension of the frame with the same essential closure as the frame. The major new result is that the assembly is free over its meet semilattice of differences.

The finite condensation and the lattice of ordinals of finite degree

Jennifer Brown (jennifer.brown@csuci.edu)
CSU Channel Islands

For linear orders L and M, we define an operation $\cdot_{\mathrm{F}}$ by $L \cdot_{\mathrm{F}} M = (LM)/\sim_F$, where LM is the lexicographic product of L and M, and $\sim_F$ is the finite condensation. Using this product, we define a set of weakly order-preserving maps on the ordinals of finite degree in Cantor normal form. One of these maps is related to the Cantor-Bendixson derivative on a linear order equipped with the order topology, and another projects an ordinal of finite degree d onto the ordinal ω^d. We show how these maps can be used to navigate a generalized lattice representing the ordinals of finite degree in Cantor normal form.

Groups, orders, and classification

Filippo Calderoni (filippo@calderoni@rutgers.edu)
Rutgers

In this talk I will discuss the classification problem for various classes of ordered groups. We will see how such Borel classifications are related to one of the most famous problem in mathematical logic: the Vaught's conjecture. Moreover, we will discuss more closely the isomorphism relation for countable Archimedean ordered groups, which represents an interesting a rather unusual example of Borel complexity.

Generic Elementary Embeddings and Cardinal Arithmetic

Todd Eisworth (eisworth@ohio.edu)
Ohio University

In this tutorial we will develop the basic theory of generic elementary embeddings with a focus on applications to traditional cardinal arithmetic. We begin Silver's original proof that $\aleph_{\omega_1}$ cannot be the first counterexample the Generalized Continuum Hypothesis, move through work of Shelah in the 1980s on extending Silver's techniques, and finally look at some recent applications.

Commuting degrees of BCK-algebras

Matt Evans (mevans@washjeff.edu)
Washington & Jefferson College

Given a finite algebraic structure A and first order formula in k-free variables, what is the probability that a randomly selected k-tuple over A satisfies that formula? This type of question has been investigated in the context of groups, rings, semigroups, and more recently Heyting algebras. In this talk, we discuss the probability that two elements in a finite BCK-algebra commute, which we call the *commuting degree* of that algebra. We will show that, for each $n \geq 2$, there is a BCK-algebra of order n realizing each possible commuting degree. In fact, more generally, every rational number in $(0, 1]$ is the commuting degree of some finite BCK-algebra.

Pure Embeddings in Acts: Stability and Cofibrant Generation

Jonathan Feigert (jonathan_feigert1@baylor.edu)
Baylor University

Acts — sets with a monoid action — form a natural generalization of modules. In this talk, we study the class of S-acts over a fixed monoid S with pure embeddings. This setting fits into the framework of an abstract elementary class (AEC), a generalization of elementary model theory designed to study classes of structures that are not axiomatizable in first-order logic. In contrast to module theory, where all first-order theories are stable, it is known that some first-order theories of acts are unstable. This makes the study of stability particularly interesting in this setting. We show that the AEC of S-acts with pure embeddings has a stable independence relation if and only if the monoid S satisfies that, for every $s, t \in S$, either $s \in St$ or $t \in Ss$. We use this result to show that pure embeddings in acts are cofibrantly generated — that all pure monomorphisms can be constructed from a *set* via pushouts, transfinite composition, and retracts.

This is joint work with S. Cox, M. Kamsma, M. Mazari-Armida, J. Rosický.

Varieties of MV-monoids and positive MV-algebras

Stefano Fioravanti (stefano.fioravanti66@gmail.com)
Charles University Prague

We investigate MV-monoids and their subquasivarieties. MV-monoids are algebras $\langle A, \vee, \wedge, \oplus, \odot, 0, 1\rangle$ where $\langle A, \vee, \wedge, 0, 1\rangle$ is a bounded distributive lattice, $\langle A, \oplus, 0\rangle$ and $\langle A, \odot, 1\rangle$ are commutative monoids, and some further connecting axioms are satisfied. Every MV-algebra in the signature $\{\oplus, \neg, 0\}$ is term equivalent to an algebra that has an MV-monoid as a reduct, by defining, as standard, $1 := \neg 0$, $x \odot y := \neg(\neg x \oplus \neg y)$, $x \vee y := (x \odot \neg y) \oplus y$ and $x \wedge y := \neg(\neg x \vee \neg y)$. Particular examples of MV-monoids are positive MV-algebras, i.e. the $\{\vee, \wedge, \oplus, \odot, 0, 1\}$-subreducts of MV-algebras. Positive MV-algebras form a peculiar quasivariety in the sense that, albeit having a logical motivation (being the quasivariety of subreducts of MV-algebras), it is not the equivalent quasivariety semantics of any logic. We study the lattice of subvarieties of MV-monoids and describe the lattice of subvarieties of positive MV-algebras. We characterize the finite subdirectly irreducible positive MV-algebras. Furthermore, we axiomatize all varieties of positive MV-algebras.

Joint work with Marco Abbadini (School of Computer Science, University of Birmingham, UK) and Paolo Aglianò (DIISM, Università di Siena, Italy).

Partially supported by the Austrian Science Fund FWF P33878 and by the PRIMUS/24/SCI/008.

Decidability and undecidability in substructural logics

Nick Galatos (ngalatos@du.edu)
University of Denver

Substructural logics constitute resource-sensitive generalizations of classical and of intuitionistic logic and include linear logic, relevance logic, and many-valued logic; their study goes back to the 1920's. Their algebraic semantics, residuated lattices, have their independent history (starting in the 1930's) and include structures such as lattice-ordered groups, lattices of ideals of rings, Heyting and Boolean algebras, and relation algebras. Moreover, the deductive systems associated with substructural logics connect to linguistics (both applied and mathematical) and they have applications to computer science (for example in pointer management and memory allocation in concurrent programming).

We will survey decidability and undecidability results for substructural logics and also discuss the computational complexity of some of them. The tools for getting undecidability and lower complexity bounds include encoding counter machines. Methods for decidability and for upper bounds include a proof-theoretic analysis of the derivational systems and also the use of well quasi-ordered sets. On the way we will mention rational semantics for these logics and use them both to establish the correctness of the encodings, as well as the extraction of finite countermodels from the proof-theoretic systems.

On the structure of superextensions of doppelsemigroups

Volodymyr Gavrylkiv (vgavrylkiv@gmail.com)
Vasyl Stefanyk Precarpathian National University, Ivano-Frankivsk, Ukraine

A family $\mathcal{U}$ of non-empty subsets of a set D is called an *upfamily* if for each set $U \in \mathcal{U}$ any set $F \supset U$ belongs to $\mathcal{U}$. An upfamily $\mathcal{L}$ of subsets of D is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily $\mathcal{M}$ of subsets of D is *maximal linked* if $\mathcal{M}$ coincides with each linked upfamily $\mathcal{L}$ on D that contains $\mathcal{M}$. The *superextension* $\lambda(D)$ of D consists of all maximal linked upfamilies on D. Any associative binary operation $* : D \times D \to D$ can be extended to an associative binary operation

$$\begin{aligned} &* : \lambda(D) \times \lambda(D) \to \lambda(D), \\ &\mathcal{M} * \mathcal{L} = \Big\langle \bigcup_{a \in M} a * L_a : M \in \mathcal{M},\ \ \{L_a\}_{a \in M} \subset \mathcal{L} \Big\rangle. \end{aligned}$$

A *doppelsemigroup* is an algebraic structure $(D, \dashv, \vdash)$ consisting of a non-empty set D equipped with two associative binary operations $\dashv$ and $\vdash$ satisfying the following axioms:

$$(x \dashv y) \vdash z = x \dashv (y \vdash z) \text{ and } (x \vdash y) \dashv z = x \vdash (y \dashv z).$$

In the talk, we discuss the structure of the doppelsemigroup $(\lambda(D), \dashv, \vdash)$ of maximal linked upfamilies on a doppelsemigroup $(D, \dashv, \vdash)$. In particular, we describe right and left zeros and identities, commutativity, the center, ideals of the superextension $(\lambda(D), \dashv, \vdash)$ of a doppelsemigroup $(D, \dashv, \vdash)$. We introduce *the superextension functor* λ in the category **DSG** whose objects are doppelsemigroups and morphisms are doppelsemigroup homomorphisms, and show that this functor preserves strong doppelsemigroups, doppelsemigroups with left (right) zero, doppelsemigroups with left (right) identity, left (right) zeros doppelsemigroups. On the other hand, the functor λ does not preserve commutative doppelsemigroups and group doppelsemigroups. Also we show that the automorphism group of the superextension of a doppelsemigroup $(D, \dashv, \vdash)$ contain a subgroup, isomorphic to the automorphism group of $(D, \dashv, \vdash)$.

Amalgamation failures in semilinear residuated lattices

Valeria Giustarini (valeria.giustarini@gmail.com)
IIIA-CSIC

In this contribution we present some new results concerning the amalgamation property in varieties of residuated lattices. These are structures that play a crucial role in the field of algebraic logic; indeed, their equivalent algebraic semantics a la Blok-Pigozzi encompass classical logic and many important nonclassical logics, such as: intuitionistic logic, intermediate logics, many-valued logics, and relevance logics to name a few. The algebraic investigation of residuated lattices is then crucial in the systematic and comparative study of such logics. As a consequence of algebraizability, a most relevant bridge theorem between the logical and algebraic perspective allows one to study the interpolation property of a logic via the study of the amalgamation property in the corresponding class of algebras. Precisely, if a logic L has a variety V as its equivalent algebraic semantics, and V satisfies the congruence extension property, L has the deductive interpolation property if and only if V has the amalgamation property. We focus on the study of the amalgamation property in semilinear varieties of residuated lattices (i.e., residuated lattices that are subdirect products of chains), solving some long-standing open problems; most importantly, we establish that semilinear commutative integral residuated lattices and their 0-bounded version do not have the amalgamation property. As a consequence, the corresponding logics fail to have the deductive interpolation property.

Analysis of Weak Associativity in Some Hyper-Algebraic Structures that Represent Dismutation Reactions

Temitope Jaiyeola (tjayeola@oauife.edu.ng)
Obafemi Awolowo University

In this paper, some chemical systems of Tin (Sn), Indium (In) and Vanadium (V) which are represented by hyper-algebraic structures $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ were studied. The analyses of their algebraic properties and the probabilities of elements in dismutation reactions were carried out with the aid of computer codes in Python programming language. It was shown that in the dismutation reactions, the left nuclear (N_λ)-probability, middle nuclear (N_μ)-probability and right nuclear (N_ρ)-probability for each of the hyper-algebraic structures $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ is less than 1.000. This implies that, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ are non-associative hyper-algebraic structures. Also, from the results obtained for FLEX-probability, it was shown that, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ have flexible elements because the values of their FLEX-probabilities are 1.000 each. Hence, $(S_{Sn}, \oplus)$, $(S_{In}, \oplus)$ and $(S_V, \oplus)$ are flexible. Overall, $(S_V, \oplus)$ exhibited the lowest measure of weak-associativity, $(S_{Sn}, \oplus)$ exhibited lower measure of weak-associativity, and $(S_{In}, \oplus)$ exhibited a low measure of weak-associativity.

Complexity of Solving Promise Systems of Equations Over Algebras

Nicholas Jamesson (nicholas.jamesson@colorado.edu)
University of Colorado Boulder

Larrauri and Živný (2024) started the investigation of the computational complexity of the following promise system of equations problem for two semigroups **A** and **B** such that **A** maps homomorphically into **B**: Is a given system of equations solvable in **A** or is it not even solvable in **B**? The solvability of a system of equations over an algebra **A** is known to be either in **P** or **NP**-complete by the famous CSP dichotomy theorem of Bulatov and Zhuk (2017). The complexity of promise systems of equations is still open. I will give a brief introduction to promise systems of equations and state complexity results generalizing the results of Larrauri and Živný.

The Monadic Grzegorczyk Logic

Mashiath Khan (mashyk@nmsu.edu)
New Mexico State University

We develop a semantic criterion for determining whether a given monadic modal logic axiomatizes the one-variable fragment of a predicate modal logic. We show that the criterion applies to the monadic Grzegorczyk logic MGrz, thus establishing that MGrz axiomatizes the one-variable fragment of the predicate Grzegorczyk logic QGrz. This we do by proving the finite model property of MGrz, which is achieved by strengthening the notion of a maximal point of a descriptive MGrz-frame and by refining the existing selective filtration methods.

Clonoids and uniform generation by minors

Michael Kompatscher (kompatscher@karlin.mff.cuni.cz)
Charles University Prague

Let $\mathbf{A}$ and $\mathbf{B}$ be two algebraic structures with universes A and B. Then, a *clonoid from* $\mathbf{A}$ *to* $\mathbf{B}$ is a set of finitary operations from A to B that is closed under composition with the term operations of $\mathbf{A}$ (on the domain side) and $\mathbf{B}$ (on the codomain side). In recent years there has been a number of classification results of clonoids for fixed $\mathbf{A}$ and $\mathbf{B}$.

In the first part of my talk, I would like to discuss the notion of "uniform generation" of operations by n-ary $(\mathbf{A}, \mathbf{B})$-minors that was introduced by Mayr and Wynne in 2024, and show how it can be used to simplify several of the know classification results. In the second part, I will use these techniques to show that all clonoids from a finite vector space to a module of coprime order are finitely generated.

This is joint work with Stefano Fioravanti and Bernardo Rossi.

The Subpower Intersection Problem for semigroups

Sangman Lee (sangman.lee@colorado.edu)
University of Colorado Boulder

The Subpower Intersection Problem (SIP) for a fixed finite algebra A asks whether two subalgebras of A^n given by their sets of generators have a nonempty intersection. While the SIP for finite monoids is trivial because of the identity element, the problem becomes nontrivial for semigroups. Building upon the work of Bulatov, Kozik, Mayr, and Steindl (2016) on the related Subpower Membership Problem (SMP), we discuss the computational complexity of SIP for special classes of semigroups, in particular on commutative semigroups and normal bands.

Algebras of conditionals

Francesco Manfucci (fmanfucci@gmail.com)
No affiliation

In this contribution we introduce and study a logico-algebraic notion of conditional operator. A conditional statement is a hypothetical proposition of the form "If [antecedent] is the case, then [consequent] is the case", where the antecedent is assumed to be true. Such a notion can be formalized by expanding the language of classical logic by a binary operator a/b that reads as "a given b". A most well-known approach in this direction comes from a philosophical perspective developed first by Stalnaker, and further analyzed by Lewis, that in order to axiomatize the operator "/" ground their investigation on particular Kripke-like structures. The novel approach we propose here is grounded in the algebraic setting of Boolean algebras, where we show that there is a natural way of formalizing conditional statements starting from the algebraic notion of "quotient". Given a Boolean algebra B and an element b in B, one can define a new Boolean algebra, say B/b, intuitively obtained by assuming that b is true. More in details, one considers the congruence collapsing b and the truth constant 1, and then B/b is the corresponding quotient. Then the idea is to define a conditional operator "/" such that a/b represents the element a as seen in the quotient B/b, mapped back to B. The particular structural properties of Boolean algebras allow us to do so in a natural way. Translating this intuition using Stone duality, we define a class of standard models and then we analyze the variety QA generated by them. In this work, we menage to axiomatize QA and we prove that it is a subvariety of the variety of Lewis variably strict conditional algebras VA. We then provide an

algebraic study of this variety, which in particular turns out to be a discriminator variety. This fact in particular entails that the classes of subdirectly irreducible, directly indecomposable, and simple algebras in QA coincide, and in this case they are exactly the class of (isomorphic copies of) our standard models.

Prime Maltsev conditions and compatible digraphs

Miklós Maróti (mmaroti@math.u-szeged.hu)
University of Szeged

In 1984 Garcia and Taylor initiated a systematic study of Maltsev conditions, certain sets of identities used to describe classes of varieties of similar behavior, and introduced the lattice of interpretability types of varieties. One of the main questions investigated is what classical Maltsev conditions determine a prime filter in this lattice. Prime filters identify properties of varieties that are not implied by any two strictly weaker conditions and thus can be considered as the most fundamental properties of varieties. We have recently proved that Taylor varieties and Hobby-McKenzie varieties form prime filters.

Using the developed tools we can characterize these varieties by their compatible reflexive digraphs. A relational structure $\mathbb{G}$ is compatible with a variety if the variety has an algebra $\mathbf{A}$ on the same universe as that of $\mathbb{G}$ and all relations of $\mathbb{G}$ are subpowers of $\mathbf{A}$. We prove that a variety has a Taylor term if and only if all reflexive, antisymmetric compatible digraphs are cycle-free. We also show that a variety has Hobby-McKenzie terms if and only if in all reflexive compatible digraphs strongly connected elements are also extremely connected (with back and forth edges).

A Baer-like criterion for relative injective modules via model theory

Marcos Mazari-Armida (marcos_mazari@baylor.edu)
Baylor University

Baer criterion is a classical result from module theory that asserts that to determine if a module is injective it is enough to test for homomorphisms coming from ideals of the ring. In this talk, we show that relative injective modules satisfy a similar criterion using model theory. More precisely, the result is obtained using independence relations which generalize Shelah's non-forking to abstract elementary classes. This result is one of the first purely algebraic applications of independence relations to algebra in the context of abstract elementary classes. We will introduce all the abstract elementary classes notions used in the talk and in parallel give a quick introduction to abstract elementary classes of modules.

The main result of the talk is joint work with J. Rosicky.

Local compactness does not always imply spatiality

Sebastian Melzer (smelzer@nmsu.edu)
New Mexico State University

In frame theory, every continuous frame is spatial. Whether this result extends to McKinsey–Tarski (MT) algebras was an open problem in the theory. In this talk, we construct a locally compact sober MT-algebra that is not spatial, thereby resolving the problem in the negative. We also revisit Nöbeling's largely overlooked approach to pointfree topology from the 1950s and compare his separation axioms and local compactness condition to those arising in the MT setting. While most coincide, we show that the MT-versions of Hausdorff and locally compact strictly strengthen Nöbeling's conditions.

This is joint work with Guram Bezhanishvili, Ranjitha Raviprakash, and Anna Laura Suarez.

Fine grained analysis of conservative Maltsev CSP

Andrew Moorhead (apmoorhead@gmail.com)
TU Dresden

Feder and Vardi famously conjectured in the nineties that a finite domain constraint satisfaction problem whose constraint relations are chosen from some predetermined template is either NP-complete or solvable in polynomial time. Bulatov, Jeavons, and Krokhin suggested a general algebraic criterion for determining the tractability of a finite domain CSP in terms of the polymorphism algebra of its template. The CSP dichotomy conjecture and the stronger algebraic CSP dichotomy conjecture initiated a fertile area of research connecting universal algebra to complexity theory.

While the algebraic dichotomy conjecture was proven true independently by Bulatov and Zhuk in 2017, there are still questions concerning the fine-grained classification of the fixed template CSPs within P, up to logspace reductions. For example, while the famous Bulatov-Dalmau algorithm does show every Maltsev CSP is tractable, it is still an open question whether every Maltsev CSP belongs to the complexity class NC.

By providing a new algorithm, we are able to show that every finite domain fixed template CSP possessing a conservative Maltsev polymorphism belongs to the complexity class Parity L (which is contained in NC), which consists of all problems that can be solved by a Turing machine with a work tape of logarithmic size in the sense that the positive instances of the problem are precisely those where the machine has an even number of accepting computation paths. In fact, conservative Maltsev CSPs are either in L or they are complete for Parity L, so we now have a full complexity classification up to logspace reductions for conservative Maltsev CSP.

This is joint work with Manuel Bodirsky.

A Characterization of Finitely Based Abelian Mal'cev Algebras

Mateo Muro (mateo.muro@colorado.edu)
University of Colorado Boulder

The Finite Basis Problem is a classical problem in universal algebra. In this talk, I will introduce the problem and necessary notions. Then I will present my recent contribution: an abelian Mal'cev variety is finitely based if and only if its ring of binary idempotent terms is finitely presented and its module of unary terms is finitely presented.

Simulation classes and aperiodicity

John Nicholson (nichoj6@mcmaster.ca)
McMaster University

I will describe a framework for classifying the computational power of a finite algebra via a construction known as a simulation class, originally studied by VanderWerf. This perspective connects naturally with tame congruence theory, and I will highlight several simulation classes that admit clean characterizations in those terms. The focus of the talk will be the aperiodic simulation class, the class of all finite algebras that cannot simulate any finite group. I will conclude with recent joint work with my supervisor, Matt Valeriote, towards a description of this class.

The category of linear modular lattices: exploring the lattice-theoretic counterparts of modules

Sebastian Pardo Guerra (seb.pardog@gmail.com)
Center for Engineered Natural Intelligence

In this talk, we explore lattice-theoretic counterparts of notions and results from the category R-Mod of R-modules. To this end, we consider the category of linear modular lattices, whose objects are complete modular lattices and whose morphisms are linear morphisms.

The Funayama envelope as the T_D-hull of a frame

Ranjitha Raviprakash (prakash2@nmsu.edu)
New Mexico State University

A topological space X is T_D provided each point is locally closed, and a frame L is T_D-spatial provided L is isomorphic to the frame of opens of a T_D-space. While there is no satisfactory pointfree generalization of T_D-spatiality in the language of frames, the richer formalism of MT-algebras does afford such a generalization. Indeed, an MT-algebra M is T_D provided the set of locally closed elements of M join-generates M. By utilizing the Funayama embedding of a frame into a complete Boolean algebra, with each frame L we may associate the MT-algebra $\mathcal{F}(L)$—the Funayama envelope of L—which always satisfies the T_D-separation axiom. We regard $\mathcal{F}(L)$ as the T_D-hull of L, and show that $L \mapsto \mathcal{F}(L)$ extends to an equivalence between the category of frames and frame homomorphisms and the category of T_D-algebras and special morphisms between them that preserve proximity-like structure of MT-algebras. As a consequence, we generalize the T_D-duality of Banaschewski and Pultr to the setting of MT-algebras, yielding a pointfree version of the T_D-coreflection of T_0-spaces.

This is joint work with G. Bezhanishvili, A. L. Suarez, J. Walters-Wayland.

Algebraic properties of groups in lattice framework

Branimir Šešelja (seselja@dmi.uns.ac.rs)
University of Novi Sad, Serbia

In the last few years, we have been investigating groups in the framework of their lattices of weak congruences. They can be understood as lattices of all normal subgroups of all subgroups of the given group, ordered by inclusion. These lattices are algebraic, and they offer information about numerous algebraic properties of groups, formulated in lattice terms. Using these lattice properties, we have characterized many classes of groups. Usually, we were given necessary and sufficient conditions that should be satisfied by the weak congruence lattice of a group G in order that G belongs to a particular class.

In this talk, we systematically present those lattice properties, explaining their equivalent algebraic nature. We show that all basic features of classes and varieties of groups (formulated algebraically as operators S, H, and P) can be analyzed and investigated in lattice terms. E.g., among subgroups properties, we define a lattice relation of being a normal subgroup; dealing with operator H, we identify all homomorphic images of the group and its subgroups within the same weak congruence lattice; we also give the lattice description of the internal direct product of subgroups. All kinds of chains and series of (normal) subgroups or systems have equivalent descriptions in lattice terms, etc. We describe the center of a group, the commutator subgroup, and its usage, closure operators among subgroups, etc.

In addition to the systematic description of the lattice terms related to algebraic properties of groups, we give in this context several new weak congruence lattice characterizations of some group classes.

A question remains: which group properties are essentially lattice properties?

- J. Jovanović, B. Šešelja, A. Tepavčević, *Lattice characterization of finite nilpotent groups*, Algebra Univers. 2021 82(3) 1–14.
- J. Jovanović, B. Šešelja, A. Tepavčević, *Lattices with normal elements* Algebra Univers. 2022 83(1) 1–28.
- M.Z. Grulović, J. Jovanović, B. Šešelja, A. Tepavčević, *Lattice Characterization Of Some Classes Of Groups By Series Of Subgroups*, International Journal of Algebra and Computation, 2023 33(02) 211–235.
- J. Jovanović, B. Šešelja, A. Tepavčević, *Nilpotent groups in lattice framework*, Algebra univers. 2024 85(4) 40.
- M.Z. Grulović, J. Jovanović, B. Šešelja, A. Tepavčević, *On classes of groups characterized by classes of lattices* (submitted).
- M.Z. Grulović, J. Jovanović, B. Šešelja, A. Tepavčević, *Systems of subgroups and Kurosh-Chernikov classes of groups in lattice framework* (submitted).

This research is a joint work with Andreja Tepavčević and Jelena Jovanović.

This research was supported by the Science Fund of the Republic of Serbia, Grant no 6565, Advanced Techniques of Mathematical Aggregation and Approximative Equations Solving in Digital Operational Research- AT-MATADOR.

Algebraic structures defined by the finite condensation on linear orders

Ricardo Suarez (ricardo.suarez532@myci.csuci.edu)
CSU Channel Islands

The finite condensation $\sim_F$ is an equivalence relation defined on a linear order L by $x \sim_F y$ if and only if the set of points lying between x and y is finite. We define an operation $\cdot_F$ on linear orders L and M by $L \cdot_F M = (LM)/{\sim_F}$; that is, $L \cdot_F M$ is the lexicographic product of L and M modulo the finite condensation. If $L/{\sim_F} \cong 1$ for an infinite linear order L, then L is order-isomorphic to one of $\mathbb{N}, \mathbb{N}^*$, or $\mathbb{Z}$. We show that under the operation $\cdot_F$, the set $R = \{1, \omega, \omega^*, \zeta\}$ (where ω^* is the order type of the negative integers and ζ is the order type of $\mathbb{Z}$) forms a left rectangular band. Further, each of the ordinal elements of R defines, via left or right multiplication modulo the finite condensation, a weakly order-preserving map on the class of ordinals. We describe these maps' action on the ordinals whose Cantor normal form is of finite degree.

Towers of sublocales induced by cozero elements

Joanne Walters-Wayland (jwalterswayland@gmail.com)
CECAT, Chapman University

We consider towers of sublocales of a completely regular frame induced by the cozero elements. These are constructed via Lindelofications to obtain a tower descending to a given frame; and then via the Bruns-Lakser completions (which in this setting are precisely the normal/Dedekind-MacNeille completions) to obtain a tower descending down from the given frame to its skeleton (aka booleanization). Another tower can be obtained by "hollowing out" the frame using dense elements. Some interesting topological properties may be characterized by collapsing parts of these towers.

Lattice-theoretic principles for synthetic higher category theory

Jonathan Weinberger (jweinberger@chapman.edu)
Chapman University

$(\infty, 1)$-categories generalize ordinary categories by introducing n-morphisms in each dimension n, with all of them being invertible for $n > 1$. This makes ∞-category theory much more involved than ordinary 1-category theory, due to the large amount of additional bookkeeping involved. In recent years, synthetic accounts have been developed to at least partially overcome this problem. This led to the synthetic study of ∞-categories in other foundation systems than set theory, such as homotopy type theory (HoTT). Based on Riehl–Shulman's simplicial extension of HoTT, we show how to construct the universe of ∞-groupoids in that setting, and prove that it satisfies desired properties such as being an ∞-category and directed univalence which says that its geometric arrows correspond to functors between ∞-groupoids. Our system relies on the addition of new axioms about the underlying directed interval. We will discuss these in detail and argue how they can be used in our synthetic theory to give relatively elementary combinatorial and order-theoretic proofs of theorems that in traditional set-theoretic foundations requires the development of hundreds of pages of intricate machinery.

This is joint work with Daniel Gratzer and Ulrik Buchholtz (https://arxiv.org/abs/2407.09146).

Abelian congruences in locally finite Taylor varieties

Ross Willard (ross.willard@uwaterloo.ca)
University of Waterloo

Locally finite Taylor varieties form an especially important and broad class within universal algebra. This is essentially the largest class of locally finite varieties for which anything "interesting" can be said about the structure of its members. Many characterizations are known for this class. One under-used characterization, proved originally by Hobby & McKenzie, is that a locally finite variety is Taylor iff it has a *weak difference term*. Here a 3-ary term $d(x, y, z)$ is a weak difference term for a variety $\mathcal{V}$ if it satisfies Maltsev's identities $d(x, x, y) \approx y \approx d(y, x, x)$ whenever x and y are elements of a block of an abelian congruence in a member of $\mathcal{V}$.

In this tutorial I will explain how the existence of a weak difference term shapes the structure of abelian congruences. Adapting a construction of Hagemann and Herrmann for congruence modular varieties, I will show how the blocks of an abelian congruence θ contained in a single block of its centralizer "fit together" in a single abelian group determined by θ and the relevant centralizer block. When θ is minimal and the algebra is finite, the abelian group is in fact a vector space over a finite field determined by θ. This leads to a notion of "similarity" between subdirectly irreducible algebras with abelian monolith, as well as a fruitful analysis of critical rectangular relations, generalizing work of Freese, Kearnes & Szendrei, and Zhuk.

The subpower membership problem for 2-nilpotent Mal'cev algebras

Patrick Wynne (patrick.wynne@colorado.edu)
University of Colorado Boulder

The Subpower Membership Problem for an algebra A, denoted SMP(A), is the following decision problem: given tuples $a_1, \dots, a_k, b \in A^n$, decide if b is in the subalgebra of A^n generated by $a_1, \dots, a_k$. For certain finite algebras, SMP(A) is decidable in polynomial time, while for other finite algebras, SMP(A) is EXPTIME-complete. We investigate the structure of 2-nilpotent Mal'cev algebras by decomposing the term clone using an associated clonoid between abelian Mal'cev algebras. We use this decomposition to show that a large class of finite 2-nilpotent Mal'cev algebras have Subpower Membership Problem decidable in polynomial time. In particular, if A is a 2-nilpotent Mal'cev algebra of squarefree order then SMP(A) is decidable in polynomial time.