2010 International Conference on Topology and its Applications,

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ABSTRACTS

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Contents

Abstracts

Jawad Abuhlail

Department of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
Box 5046, 31261 Dhahran, KSA
e-mail: abuhlail@kfupm.edu.sa

On Zariski-like Topologies for Modules
Mathematics Subject Classification (MSC): 16N20; 16N80; 54H13 (13C05, 13C13, 54B99)

Abstract. Given a non-zero duo left module M over an associative (not necessarily commutative) ring R, Zariski-like topologies are defined on the spectrum $\mathrm{Spec}^{\mathrm{p}}(M)$ of the R-submodules of M which are *prime in* M and the spectrum $\mathrm{Spec}^{\mathrm{c}}(M)$ of the R-submodules of M which are *coprime in* M. We study these topological spaces and, in particular, investigate the interplay between the topological properties of such spaces and the algebraic properties of the module under consideration.

References

[Abu2008] J. Y. Abuhlail, *A Zariski topology for bicomodules and corings,* Appl. Categ. Structures 16 (1-2) (2008), 13-28.

[Abu2006] J. Y. Abuhlail, *Fully coprime comodules and fully coprime corings,* Appl. Categ. Structures 14 (5-6) (2006), 379-409.

[Bou1998] N. Bourbaki, *Commutative algebra,* Springer-Verlag (1998).

[Bou1966] N. Bourbaki, *General Topology, Part I,* Addison-Wesley (1966).

[Lu1999] C-P. Lu, *The Zariski topology on the prime spectrum of a module,* Houston J. Math. 25 (3) (1999), 417-432.

[McC1984] R. McCasland, *Some Commutative Ring Results Generalized to Unitary Modules,* Ph.D. thesis, University of Texas at Arlington (1984).

[WW2009] I. Wijayanti and R. Wisbauer, *Coprime modules and comodules,* Commun. Algebra 37 (4) (2009), 1308–1333.

[Yas2001] S. Yassemi, *The dual notion of prime submodules*, Arch. Math. (Brno) 37 (2001), 273-278.

Bashir Ahmad and Sabir Hussain
Department of Mathematics,
Islamia University Bahawalpur,
Pakistan
e-mail: sabiriub@yahoo.com

On Locally γ-s-closed Spaces
Mathematics Subject Classification (MSC): 54A05, 54A10, 54D10, 54D99

Abstract. In this paper, we continue studying the applications of γ-s-closed spaces introduced and discussed in [5] and [9]. The concept of locally γ-s-closed space has been introduced. Certain important characterizations and properties of locally γ-s-closed spaces have also been established.

References

[1] B. Ahmad and S. Hussain: *Properties of γ-Operations on Topological Spaces, Aligarh Bull. Math.,* 22(1) (2003), 45-51.

[2] B. Ahmad and S. Hussain: *γ-Semi-Open Sets in Topological Spaces II, Southeast Asian Bull. Maths.,* (In Press).

[3] B. Ahmad and S. Hussain: *γ-Convergence in Topological Space , Southeast Asian Bull. Maths.,* 29(2005), 832-842.

[4] B. Ahmad and S. Hussain: *γ^*-Regular and γ-Normal Spaces , Math. Today.,* 22(1)(2006), 37-44.

[5] B. Ahmad and S. Hussain: *On γ-s-Closed Subspaces , Far East Jr. Math. Sci.,* 31(2)(2008), 279-291.

[6] B. Ahmad, S. Hussain and T. Noiri: *On Some Mappings in Topological Space , Eur. J. Pure Appl. Math.,* 1(2008), 22-29.

[7] G. Di. Maio and T. Noiri: *On s-Closed Spaces , Indian J. Pure Appl. Math.,* 18(1987), 226-233.

[8] S. Hussain and B. Ahmad: *On Minimal γ-Open Sets, Eur. J. Pure Appl. Maths.,* 2(3) (2009), 338-351.

[9] S. Hussain and B. Ahmad: *On γ-s-Closed Spaces, Sci. Magna Jr.,* 3(4)(2007), 89-93.

[10] S. Hussain and B. Ahmad: *On γ-s-Regular Spaces and Almost γ-s-Continuous Functions, Lobackevskii. J. Math.,* 30(4)(2009), 263-268. DOI: 10.1134/51995080209040039.

[11] S. Kasahara: *Operation-Compact Spaces, Math. Japonica,* 24(1979), 97-105.

[12] H. Ogata: *Operations on Topological Spaces and Associated Topology, Math. Japonica,* 36(1)(1991), 175-184.

M. R. Ahmadi Zand, S. Jafari, and T. Noiri
Department of Mathematics, Yazd University, Yazd, Iran
e-mail: mahmadi@yazduni.ac.ir

An Algebraic characterization of PS-spaces
Mathematics Subject Classification (MSC): Primary 54B05, 54C08; Secondary: 54D05

Abstract. Ring of all continuous real-valued functions on a topological space X is denoted by $C(X)$(see [9] for more information). $DO(X)$ denotes the set of all dense open subsets of X. Recall that a commutative ring R is called (von Neumann) regular if for each $r \in R$, there exists an $s \in R$ such that $r = r^2 s$. We denote the ring of all real-valued functions on a nonempty set X by $F(X, R)$ which is obviously a (von Neumann) regular ring. In [2], the first author introduced a subring $T^{'}(X)$ and a subset $T(X)$ of $F(X, R)$ for any arbitrary topological space. $T(X)$ denotes all functions f of $F(X, R)$ for which there exists a dense subset D of X such that $f \mid D \in C(D)$. But $T^{'}(X)$ denotes all functions f of $F(X, R)$ for which there exists a dense open set D of X such that $f \mid D \in C(D)$. Sometimes we use the symbol $T(X, \tau)$ (resp. $T^{'}(X, \tau)$) for $T(X)$ (resp. $T^{'}(X)$). For any topological space it is true (see [1]) that $R \subseteq C^{\star}(X) \subseteq C(X) \subseteq T^{'}(X) \subseteq T(X) \subseteq F(X, R)$, where $C^{\star}(X)$ denotes the ring of all continuous bounded real-valued functions on X and R is the set of constant real-valued functions on X.
Let X be a topological space, and let X be the union of two disjoint dense subsets, then X is called resolvable , otherwise it is called irresolvable. A subset A of a space X is called semi-open (resp. preopen, α-open [12] and semi-preopen [6]) if $A \subseteq Cl(Int(A))$ (resp. $A \subseteq Int(Cl(A))$, $A \subseteq Int(Cl(Int(A)))$ and $A \subseteq Cl(Int(Cl(A)))$), where $Int(A)$ and $Cl(A)$ denote the interior and closure of A, respectively. Their complements are called semi-closed, preclosed, α-closed and semi-preclosed, respectively. We denote the families of semi-open (resp. preopen, α-open

and semi-preopen) subsets of a topological space (X, τ) by $SO(X)$ (resp. $PO(X)$, τ^α and $SPO(X)$).

Ahmadi Zand in [1] and [2] introduced and investigated some subrings of the ring of real-valued functions which contain C(X). In this talk, we will generalized some results of [1] to spaces with isolated points and we will show that some of the results in [1] is related to PS-spaces [3] and [4]. Moreover, we answer to the question that whether in the digital n-spaces (Z^n, k^n), the equality $T^{'}(Z^n, k^n) = T(Z^n, k^n)$ is true or not in the positive.

References

[1] M. R. Ahmadi Zand, An algebraic characterization of Blumberg spaces, Quaestiones Mathematicae, 33(2010), 1-8.

[2] M. R. Ahmadi Zand, Strongly Blumberg spaces, PhD Thesis, Shahid Chamran university of Ahvaz, Ahvaz, Iran. 2006.

[3] T. Aho and T. Nieminen, Spaces in which preopen subsets are semiopen, Ricerche Mat., 43(1994), 45-59.

[4] T. Aho and T. Nieminen, On α-spaces, PS-spaces and related topics, Reports of the Department of Mathematics, Univ. of Helsinki, November 1991.

[5] B. Al-Nashef, Semi-preopen sets, Mat. Vesnik, 38(1986), 24-32.

[6] D. Andrijevic, On semipreopen sets, Q and A in General Topology, 19(2001), 203-212.

[7] M. Fujimoto, H. Maki, T. Noiri and S. Takigawa, The digital plane is quasi-submaximal, Q and A in General Topology, 22(2004), 163-168.

[8] M. Ganster, Preopen sets and resolvable spaces, Kyungpook Math. J., 27(1987), 135-143.

[9] L. Gillman and M. Jerison, Rings of continuous functions, Springer-Verlag, 1976.

[10] D. S. Janković, I. L. Reilly and M. K. Vamanamurthy, On strongly compact topological spaces, Q and A in General Topology, 6(1988), 29-39.

[11] A. S. Mashhour, M. E. Abd El-Monsef, I. A. Hasanein and T. Noiri, Strongly compact spaces, Delta J. Sci., 8(1984), 30-46.

[12] O. Njåstad, On some classes of nearly open sets, *Pacific J. Math.* 15 (1965), 961-970.

[13] I. L. Reilly and M. K. Vamanamurthy, On some questions concerning preopen sets, Kyungpook Math. J., 30(1) (1990), 87-93.

[14] J. A. Steen and J. A. Seebach, Jr., Counterexamples in Topology (Second Edition), Springer-Verlag, Berlin, Heidelberg, New York 1978.

[15] Shin-Ya Takigawa, M. Ganster, H. Maki, T. Noiri, and M. Fujimoto, The digital n-space is quasi-submaximal, Q and A in General Topology, 26(2008), 45-52.

Amna Ahmed and C. Good

School of Mathematics, University of Birmingham, B15 2TT, UK
e-mail: ahmeda@maths.bham.ac.uk

Characterizing Continuous Functions on the Rational World
Mathematics Subject Classification (MSC): 37B99, 54A10, 54B99, 54C05, 54H20

Abstract. We will consider the following problem: given a countable set X and a function $T : X \to X$, when can one endow X with a topology such that X is homeomorphic with the rational space $\mathbb{Q}$ and with respect to which T is continuous. Mekler, Nuemann and Truss characterize the situation when T is a bijection. We give characterization of the general case when T is considered to be any function.

T.M.G. Ahsanullah and Jawaher Al-Mufarrij

Department of Mathematics
King Saud University
Riyadh 11451, Saudi Arabia
e-mail: tmga1@ksu.edu.sa

On the category of L-fuzzy neighborhood groups and its connections with categories of L-convergence groups

Mathematics Subject Classification (MSC): 54A40, 54A20, 54B30, 54H11

Abstract. Motivated by the notion of L-fuzzy neighborhood system attributed to U. Höhle and A. P. Šostak [1], we introduce for a frame L, categories **S**L-**FNeighGrp**, of stratified L-fuzzy neighborhood groups, and **S**L-**FIntGrp**, of stratified L-fuzzy interior groups. We show that these two categories are isomorphic; some basic facts along with some characterization theorems are presented. Considering the notion of stratified L-pre-topological space due to H. Boustique, R. N. Mohapatra and G. Richardson [2], we show that the category **S**L-**P-TopConvGrp**, of stratified L-pre-topological convergence groups, is isomorphic to the category **S**L-**FNeighGrp**. Also, considering a full subcategory **S**L-**FNeigh**$'$, of the category of stratified L-fuzzy neighborhood spaces **S**L-**FNeigh**, we prove that the

category of stratified L-fuzzy neighborhood groups **S**L-**FNeighGrp′**, is isomorphic to a subcategory of the category **S**L-**GConvGrp**, of stratified L-generalized convergence groups [3]; the key item of this category is the notion of stratified L-generalized convergence structure initiated by G. Jäger [4]. Finally, we propose two more categories **HŠ-S**L-**FFil**, of Höhle-Šostak stratified L-fuzzy filter spaces, and **HŠ-S**L-**FFilGrp**, of stratified L-fuzzy filter groups, and discuss some of their features.

References

[1] U. Höhle and A. P. Šostak, Axiomatic foundations of fixed basis fuzzy topology, in: *Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory*, (eds.) U. Höhle and S. E. Rodabaugh, Kluwer Academic Publishers, Dordrecht, 1999, pp. 233–239.

[2] H. Boustique, R. N. Mohapatra and G. Richardson, Lattice-valued fuzzy interior operators, Fuzzy Sets and Systems **160**(2009), 2947–2955.

[3] T. M. G. Ahsanullah and J. Al-Mufarrij, Frame valued stratified generalized convergence groups, *Quaest. Math.* **31**(2008), 279–302.

[4] G. Jäger, A category of L-fuzzy convergence spaces, *Quaest. Math.* **24**(2001), 501–517.

José G. Anaya

Universidad Autónoma del Estado de México, Facultad de Ciencias, Instituto Literario 100. Col. Centro, C.P. 50000, Toluca, Estado de México, México
e-mail: jgao@uaemex.mx

Making holes in hyperspaces of subcontinua of some continua
Mathematics Subject Classification (MSC):

Abstract. Let Z be a unicoherent topological space and let z be an element of Z. We say that *z makes a hole in Z* if $Z \setminus \{z\}$ is not unicoherent. Let X be a continuum (no degenerate compact connected metric space). We are interested in working out the following problem.
Problem. Let $\mathcal{H}(X)$ be a hyperspace of X. For which elements, $A \in \mathcal{H}(X)$, does A make a hole in $\mathcal{H}(X)$?

In this talk, we are going to present the solution to such problem when X is a continuum that has an Elsa continuum (compactification of half-open interval with an arc as remainder), Y, and $\mathrm{Cl}(X \setminus Y)$ is a simple n-od and $\mathcal{H}(X)$ is the hyperspace of subcontinua of X.

Athanasios Andrikopoulos
Department of Economics
University of Ioannina
Greece
e-mail: aandriko@cc.uoi.gr

A characterization of compactness through the Schwartz set
Mathematics Subject Classification (MSC): C02, C60

Abstract. Compactness is an important topological property as it enables us to apply minimax theorems in mathematical economics. The Schwartz set is a general solution concept of choice problems when the set of best alternatives does not exist (this problem occurs when the preferences

yielded by an aggregation process are cyclic). In this paper, we show that the feasible set is compact if and only if every generalized upper tc-semicontinuous preference has non empty Schwartz set. Here “preference”means arbitrary binary relation.

References

[1] Alcantud, J. C., 2002. Characterization of the existence of maximal elements of acyclic relations. Economic Theory 19, 407-416.

[2] Andrikopoulos A., 2007. A representation of consistent binary relations. Span. Econ. Rev. 9, 299-307.

[3] Gutiérrez, M. J., 2009. A characterization of compactness through preferences. Mathematical Social Sciences 57, 131-133.

[4] Kelley, J. L., 1955. General Topology. Springer-Verlag.

[5] Schwartz, T., 1986. The logic of Collective Choice. New York: Columbia University Press.

Boris N. Apanasov
Univ. of Oklahoma, USA
e-mail: apanasov@ou.edu

The 3-sphere as a Heegaard splitting of infinite genus with ergodic 3-hyperbolic group action
Mathematics Subject Classification (MSC):

Abstract. Here we construct a Heegaard splitting of the 3-sphere S^3 into two handlebodies Ω_1 and Ω_2 of infinite genus with their common boundary Λ. This boundary “surface" Λ has a natural conformal ergodic action of a Gromov

hyperbolic group $G \subset \text{Möb}(S^3)$. The group G is a homomorphic image $\phi(\Gamma)$ of a uniform lattice Γ in the isometry group of the real hyperbolic 3-dimensional space H^3 where the homomorphism ϕ has an infinite kernel. This also gives a new view on Andreev's hyperbolic polyhedron theorem.

Gizem Gunel and Gulhan Aslim

Department of Mathematics,
Faculty of Science, Ege University,
Izmir, Turkey
e-mail: gizemgunell@gmail.com

On L-Fuzzy γ-Open Sets
Mathematics Subject Classification (MSC): 54A40

Abstract. The aim of this paper is to develop a general structure characterizing the families of sets weaker than the open sets of L-topological space. In order to establish such a generalization, we define L-fuzzy γ-open sets where γ is a monotonic function on the family of all L-fuzzy sets on a set X and investigate some fundamental properties of L-fuzzy γ-open sets.

Bogdan Atamanyuk

Department of Mathematics and Informatics
Precarpathian National State University
Ivano-Frankivsk, Shevchenka 57
Ukraine
e-mail: bogdanatamaniuk@ukr.net

The Hyperspace of Ordered Arcs of Continual Exponenta from Metric Peano Continua is Homeomorphic to the Hilbert Cube

Mathematics Subject Classification (MSC):

Abstract. In [1] I have proved that the hyperspace of ordered arcs of Continual Exponenta from connected and locally connected nowhere nonlocal compact spaces is homeomorphic to the Hilbert space l_2 (designed). In this paper we consider the compact metric case and more exactly for metric Peano continua X we conclude that the hyperspace of ordered arcs from such space is homeomorphic to the Hilbert Cube Q. To prove it we remember than the condition of belonging hyperspace of ordered arcs to the class of AR was proved by Eberhart, Nadler and Nowell in [2] and the condition of discrete approximation I have proved in [1]. We only must to verify the DACP - property (discrete approximation cell property) for the hyperspaces $\Gamma^c(X)$ from metric Peano continua X, which is formulated as: for each natural number n and for each continuous map $f : I^n \times [0,1] \to \Gamma^c(X)$ and for each $\varepsilon > 0$ exists the continuous map $g : I^n \times [0,1] \to \Gamma^c(X)$such that is satisfied following two conditions: (1) $d(f,g) < \varepsilon$ or $(f,g) < \omega$ for each beforehand given covering $\omega \in Cov(\Gamma^c(X))$, (2) the sets $g(I^n \times \{0\})$ and $g(I^n \times \{1\})$ is not intersects. To prove it we construct the map g by eight stages as it made in [1]. Stage 1. We construct the map α which will be realized the α-proximity of the maps f, g and consequently ω-proximity of the maps f and g. Stage 2. We construct the map $\gamma : \Gamma^c(X) \to (0,\infty)$ and we must to verify that it is lower-semi-continuous. Together with γ exists the special function $\beta : \Gamma^c(X) \to (0,\infty)$ which is continuous. Stage 3. Instead polyhedron K_i we triangulates each cube $I^n \times \{t\}$ for each $t \in [0,1]$ such way, that the map $f : I^n \times [0,1] \to \Gamma^c(X)$ will be satisfies the (1) - (3) conditions by [1] for each simplex Δ. Stage 4. We selects finite arbitrary small (in the sense that diameter of his ele-

ments is less than $\frac{1}{n}$ covering U of the space $exp^c(x)$ which we can take finite because it space is compact. After that we selects finite discrete subset Ω_U in each element U of selected covering U and design $Z(n) = \cup\{\Omega_U \subset U \in U_n\}$. This set will be finite and discrete subset of $exp^c(X)$. Stage 5. We defines the function $g : I^n \times [0,1] \to \Gamma^c(X)$ at the first in the each vertex p of the triangulated cube $I^n \times \{0\}$ instead polyhedron K_1 in [1]. After that we extends the map g on the segment $[pq]$ where p and q is vertexes with the help of Borsuk and Mazurkevich theorem as I have made it in [1]. Stage 6. With the help of founded conditions we proved that constructed maps f and g on the segment $[p,q]$ are α - proximity as in [1]. Stage 7. We extended the map g onto whole cube $I^n = I^n \times \{0\}$with the help of following lemma: if B^{k+1} is the $k+1$ - dimensional ball, which bounded by sphere S^k, $k \geq 1$, than follow if $f : B^{k+1} \to \Gamma^c(X)$ and $g : S^k \to \Gamma^c(X)$ such two maps, that for each $s \in S^k$ we have $f(s) \subset g(s)$ than exists such extension G of map g onto whole ball B^{k+1} that $G : B^{k+1} \to \Gamma^c(X)$ and for each $b \in B^{k+1}$we have $f(b) \subset G(b)$. To prove of this lemma were used the continuum-imaging retraction and Morse parametrization as in [1]. At such ways we may to construct any map G which is defined on the cube $I^n \times \{t\}$ for each $t \in [0,1]$. So, we choice finite division of compact segment [0,1] as $t_0 = 0 < t_1 < t_2 < \ldots < t_n = 1$ with arbitrary small diameter and we consider cubes $I^n \times \{0\}$, $I^n \times \{t_1\}$, $I^n \times \{t_2\}$,..., $I^n \times \{t_n\}$. Let $Z_p \in Z(n,p) \setminus \cup\{G(y) : y \in I^n \times \{s\}\}$ for $s \in [0, \frac{1}{2^{np}})$. Thus the images $G(I^n \times \{\frac{1}{2^{np}}\})$ will be disunited under the construction. Specifically we obtained that the sets $G(I^n \times \{0\})$ and $G(I^n \times \{1\})$ are not intersected. Stage 8. We verifyes the proximity of maps G and f. After that $\Gamma^c(X)$ is homeomorphic to the Hilbert cube

Q.

References

[1] *The Hilbert Space as Hyperspace of Ordered Arcs by Continual Exponente.* Cardinal invariants and extension of topological spaces. Collections of scientific works. Izevsk, 1989, p.57-59.

[2] C. Eberhart, S.B. Nadler, W.O. Nowell *Spaces of order arcs in hyperspaces*, Fund.Math., 112(1981), 111-120.

Atamanyuk Roman
Ivano-Frankivsk
e-mail: bogdanataman@gmail.com

The optimization of Topology and CTL of Big Integrated Circuits on the Nano-Level by Mathematical Methods
Mathematics Subject Classification (MSC):

Abstract. For optimization of topology BIC I proposed to replace the Manhetten metric onto the Euclidean metric during the proecting of topology BIC. On the nano-level we needed to calculate the distance between difference points as the length along hypotenuse of ortogonal triangle but not as the sum of katets as it was maked in Manhetten geometry. It means that the Stainer points will be obtained by another way as a half past of sum of coordinates but they will be obtained as a result of strong exactly construction of Stainer points in triangles with Euclidean metric. I proposed to optimize not only the topology of BIC, but also to optimize constructor technological limitations which leads to the nano-sizes and may be realized by using high-K-dielectrics. The constructor limitations are realized in a

system of unequalityes in the optimization tasks. We may to use the duality task. The goal of duality task is following. We can to obtained the new constructor limitations and new aim function after the replacing old aim function and old constructor limitations but we also must to change the unequal- signs onto the antipodal and to replace min onto max or reverse. The next we shall to optimize the new aim function under the new constructor limitations. That is why it is possible to minimize as the topology of BIC as the CTL- constructor technological limitations. But customary it is possible on the such level which it is permitted by the projecting norms for the given kind of BIC.

E. Athanassiadou and N. Papanastassiou

Department of Mathematics,
University of Athens,
Athens, Greece
e-mail: eathan@math.uoa.gr

On a new type of convergence of sequences of functions

Mathematics Subject Classification (MSC):

Abstract. Let X, Y be two topological spaces. We introduce a new type of convergence for a sequence $(f_n)_n$, $f_n : X \to Y$, and we obtain a characterization of a feeble T_1-continuous function which is p.w. limit of this sequence (f_n).

Konstantin Athanassopoulos

Department of Mathematics,
University of Crete,
GR-71409 Iraklion, Greece

e-mail: athanako@math.uoc.gr

One-dimensional minimal attractors
Mathematics Subject Classification (MSC): 54H20

Abstract. We are concerned with one-dimensional compact minimal sets of continuous flows on locally compact metric spaces in the spirit of Poincaré-Bendixson theory. The main result is that if A is an asymptotically stable, one-dimensional, compact minimal set of a continuous flow on a locally compact metric space X and X is locally connected at every point of A, then A is a periodic orbit. This implies that if A is an isolated, one-dimensional, compact minimal set and the intrinsic topology of its region of attraction is locally connected at the points of A, then A is a periodic orbit.

Fatemah Ayatollah Zadeh Shirazi

Faculty of Mathematics, Statistics and Computer Science
College of Science, University of Tehran
Enghelab Ave., Tehran, Iran
e-mail: fatemah@khayam.ut.ac.ir

More about indicator sequence and indicator topology of a transformation semigroup
Mathematics Subject Classification (MSC): 54H15

Abstract. In the (topological) transformation semigroup (X, S) (for more details on transformation semigroups see [2] and [4]) define, height of (X, S), $\mathrm{h}(X, S) = \sup\{n \geq 0 :$ there exists a chain $M_0 \subset M_1 \subset \cdots M_n$ of distinct closed invariant subsets of $X\}$ [3]. For any transformation semigroup (X, S) with $\mathrm{h}(X, S) = m < \infty$ the indicator sequence of (X, S), $(n_0, ..., n_m)$ of non negative integers with $n_i \leq i$ $(0 \leq i \leq m)$ and $n_0 \leq n_1 \leq \cdots \leq n_m$ has been

introduced firstly in [1]. Moreover $Z := \{\overline{xS} : x \in X\}$ can be considered under topology generated by basis $\{\{\overline{yS} : y \in \overline{xS}\} : x \in X\}$, which introduces indicator topology of (X, S) [1]. Any two transformation semigroup with homeomorph indicator topology and finite height have the same indicator sequence but not vice versa [1]. In this talk after a short review on properties of indicator sequences and indicator topologies in finite height case, we will introduce and see indicator sequences in general case, discuss on their properties and their interactions with indicator topologies.

1. Ayatollah Zadeh Shirazi, F.; Golestani, N., *On classifications of transformation semigroups*, submitted.

2. Ellis, R., *Lectures on topological dynamics*, W. A. Benjamin, New York, 1969.

3. Sabbaghan, M.; Ayatollah Zadeh Shirazi, F., *Transformation semigroups and transformed dimensions*, Journal of Sciences, Islamic Republic of Iran, 2001 (12, no. 1), 65–75.

4. Sabbaghan, M.; Tso, K.; Wu, Ta Sun, *Abelian minimal transformation semigroups*, Topology and its Applications, 1994 (60, no. 3), 201–227.

Halis Aygün and Vildan Çetkin
Department of Mathematics,
Kocaeli University
Umuttepe Campus, 41380, Kocaeli - TURKEY
e-mail: vcetkin@gmail.com

Lattice Valued Double Fuzzy Preproximity Spaces

Mathematics Subject Classification (MSC): 54A40, 54E05, 54A05

Abstract. In this study, the concept of lattice valued double fuzzy preproximity is introduced. Moreover, the relationships among the double fuzzy preproximity, double fuzzy topology and double fuzzy interior (closure) operators are studied.

References

[1] S. E. Abbas and Halis Aygün, *Intuitionistic Fuzzy Semi-regularization Spaces*, Information Sciences, 176 (2006) 745-757.

[2] S. E. Abbas, Halis Aygün and V. Çetkin, *On (L, M)-Fuzzy Closure Spaces*, under revision.

[3] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20(1) (1986) 87-96.

[4] K. Atanassov, *New Operator Defined Over the Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems, 61 (1993) 131-142.

[5] G. Birkhoff, Lattice Theory, Ams Providence, RI (1995).

[6] C.L. Chang, *Fuzzy Topological Spaces*, J. Math. Anal. Appl., 24 (1968) 182-190.

[7] D. Çoker, *An Introduction to Fuzzy Subspaces in Intuitionistic Fuzzy Topological Spaces*, J. Fuzzy Math. 4 (1996) 749-764.

[8] D.Çoker, M. Demirci, *An Introduction to Intuitionistic Fuzzy Topological Spaces in Šostak's Sense*, Busefal, 67 (1996) 67-76.

[9] V. Çetkin and H. Aygün, *On (L, M)-Fuzzy Interior Spaces*,under revision.

[10] J.A. Goguen, *The Fuzzy Tychonoff Theorem*, Math. Anal. Appl. 43 (1973) 734-742.

[11] J. G. Garcia, S.E. Rodabaugh, *Order-theoretic, topological, categorical redundancides of interval-valued sets, grey sets, vague sets, interval-valued "intuitionistic" sets, "intuitionistic" fuzzy sets and topologies*, Fuzzy Sets and Systems, 156 (2005) 445-484.

[12] U. Höhle,*Upper Semicontinuous Fuzzy Sets and Applications*, J. Math. Anall. Appl. 78 (1980) 659-673.

[13] U. Höhle and E. P. Klement, *Non-Classical Logic and Their Applications to Fuzzy Subsets*, Kluwer Academic Publisher, Dordrecht (1995).

[14] U. Höhle, A.P. Šostak, *A General Theory of Fuzzy Topological Spaces*, Fuzzy Sets and Systems, 73 (1995) 131-149.

[15] B. Hutton, *Normality in Fuzzy Topological Spaces*, J. Math. Anal. Appl., 50 (1975) 74-79.

[16] B. Hutton, *Products of Fuzzy Topological Spaces*, Topology and Appl., 11 (1980) 59-67.

[17] B. Hutton, I.Reily, *Separation Axioms in Fuzzy Topological Spaces*, Fuzzy Sets and Systems, 3 (1980) 93-104.

[18] S. Jenei, *Structure of Girard Monoids on [0,1]* Chapter 10 in: Topological and Algebraic Structures in Fuzzy Sets, S.E.Rodabaugh, E.P.Klement eds., Kluwer Acad. Publ. (2003).

[19] Y. C. Kim and K. C. Min, *L-fuzzy Preproximity and L-fuzzy Topologies*, Information Sciences, 173 (2005) 93-113.

[20] T. Kubiak, *On Fuzzy Topolgies*, Ph.D Thesis, A. Mickiewicz, Poznan (1985).

[21] Y. M. Liu, M. K. Luo, *Fuzzy Topology*, World Scientific Publishing, Singapore (1997) 323-336.

[22] A. A. Ramadan, T. M. El-Adawy and M. A. Add-Allah, *On Fuzzifying Preproximity Spaces*, The Arabian Journal for Science and Engineering, 30, IA (2005) 51-67.

[23] S. K. Samanta, T. K. Mondal, *Intuitionistic Gradation of Openness: intuitionistic fuzzy topology*, Busefal, 73 (1997)8-17.

[24] S. K. Samanta, T. K. Mondal, *On Intuitionistic Gradation of Openness*, Fuzzy Sets and Systems, 131 (2002) 323-336.

[25] A. Šostak, *Towards the Theory of M-Approximative Systems: Fundamentals and Examples*, submitted.

[26] A. P. Šostak, *On a Fuzzy Topological Structure*, Suppl. Rend. Circ. Matem. Palermo ser. II vol.11 (1985).

[27] A. M. Zahran, M. A. Abd-Allah, K. El-Saady and Abd El-Nasser G. Abd El-Rahman, *Double Fuzzy Preproximity Spaces*, International Journal of Fuzzy Logic and Intelligent Systems, vol. 7 (2007) 249-255.

[28] A. M. Zahran, M. A. Abd-Allah, K. El-Saady, A. Ghareeb, *The Category of Double Fuzzy Preproximity Spaces*, Computers and Mathematics with Applications, 58 (2009) 1558-1572.

Liljana Babinkostova
Boise State University
e-mail: liljanab@diamond.boisestate.edu

On one extension of Lebesgue covering dimension
Mathematics Subject Classification (MSC): 91A44, 16K40

Abstract. For separable metric spaces we define *game dimension* using a game motivated by selective screenability property. Then we obtain upper bounds on the game dimension using several natural classical selection principles and their associated games.

Virgil Bălan and Doina Cristina Burada

University of Craiova, Romania
e-mail: balanvirgil@yahoo.com

The topology induced by a horistology
Mathematics Subject Classification (MSC): 54A20

Abstract. In this paper we consider the concept of *horistological structures* introduced by T. Bălan in [1] as a generalization of worlds of events endowed with super-additive norms and metrics and we show how we can attach a topology at any horistology.

It is well known that in Topology, the topological structures can be defined by filters of neighbourhoods; similarly, in Horistology, the horistological structures will be defined by *ideals of perspectives* as follows:

Definition 1 The function $\chi : X \to \mathcal{P}(\mathcal{P}(X))$ is said to be a horistology on X if for each $x \in X$, $\chi(x)$ is an ideal in $\mathcal{P}(X)$ such that:

(h_1) If $P \in \chi(x)$ then $x \notin P$;

(h_2) For each $P \in \chi(x)$ there exists $L \in \chi(x)$ such that for every $y \in P$ and $Q \in \chi(y)$ we have $Q \subseteq L$.

The pair (X, χ) is called *horistological space.*

If $P \in \chi(x)$, then P is called a *perspective of x* and x is said to be a *premise* of P.

The set

$$K = \{(x, y) \in X \times X : x = y \text{ or } \{y\} \in \chi(x)\} \qquad (1)$$

is an order relation on X, called the *proper order* on X.

The function $p : \mathcal{P}(X) \to \mathcal{P}(X)$, defined by

$$p\,(A) = \{x \in X : A \in \chi(x)\}\,, \text{ for each } A \subseteq X \qquad (2)$$

is called *premise operator induced by the horistology* χ.

The simplest case of real horistological spaces is the relativist space-times (see [1]):

Example 2 If $X = \mathbb{R}^2$ is gifted with the causal order

$$K = \{((t_1, s_1), (t_2, s_2)) \in X \times X : t_2 - t_1 > |s_2 - s_1| \\ \text{or } (t_1, s_1) = (t_2, s_2)\}$$

and with the super-additive metric $\sigma : K \to \mathbb{R}_+$, defined by

$$\sigma\,[(t_1, s_1), (t_2, s_2)] = \sqrt{(t_2 - t_1)^2 - (s_2 - s_1)^2},$$

then the ideal of perspectives of the point $(t, s) \in X$ is the set

$$\chi(t, s) = \{P \in \mathcal{P}(X) : \exists r > 0 \text{ such that } P \subseteq H\,((t, s), r))\}\,,$$

where

$$H\,((t, s), r)) = \{(t', s') \in K[(t, s)] : \sigma\,[(t, s), (t', s')] > r\}$$

are the *hyperbolic (perspectives)* of center (t, s) and radius $r > 0$.

Theorem 3 Let (X, χ) be a horistological space and let p the premise operator induced by the horistology χ. Then the function $\mathcal{V} : X \to \mathcal{P}(\mathcal{P}(X))$, expressed by

$$\mathcal{V}(x) = \{V \subseteq X : \exists P \subseteq X \text{ such as } x \in p(P) \subseteq V\}\,, \\ \text{for each } x \in X \qquad (3)$$

is a neighbourhoods function for any topology on X. Moreover, if K is the proper order of χ, then $\mathcal{V}_\chi(x) \subseteq [K^{-1}[x]]$

Definition 4 We call τ_χ the topology induced by the horistology χ.

The equivalent notion in horistology of convergence of filters in topology is the emergence of ideals, defined as follows (see, [3] and [4]):

Definition 5 If (X, χ) is a horistological space and $\mathcal{J}$ is an ideal in $\mathcal{P}(X)$, we say that $\mathcal{J}$ emerge from $x \in X$ iff $\mathcal{J} \subseteq \chi(x)$.

Theorem 6 Let (X, χ) be a horistological space and let $p_\chi$the premise operator induced by the horistology χ. If $\mathcal{J}$ is an emergent ideal from $x \in X$ then $p_\chi(\mathcal{J})$ is a convergent filter to x relative to the topology induced by the horistology χ.

References

[1] Bălan T. *Generalising the minkowskian space-time (II),* Stud. Cerc. Mat. **44** Nr. 4 (1992), 267-284.

[2] Bălan V. *Topologies associées à une horistologie,* Ann. Univ. Craiova, Seria Matematica-Informatica, vol. XXIII, (1996), 60-77.

[3] Bălan V. *Émergence des idéaux dans un espace horistologique. Structures d'emergence.* Proceedings of the annual meeting of the Romanian Society of Mathematical Sciences, Cluj Napoca, (1998), 231-236.

[4] Gherman R., Predoi M. *Emergent ideals.* New Zeeland Journal of Mathematics, Vol. 28 (1999), 25-36.

Taras Banakh and Volodymyr Gavrylkiv

Department of Mathematics and Computer Sciences
Vasyl Stefanyk Precarpathian National University
Ivano-Frankivsk, Shevchenko Street 57
Ukraine
e-mail: vgavrylkiv@yahoo.com

Extending binary operations to funtor-spaces
Mathematics Subject Classification (MSC): 18B30; 18B40; 20N02; 20M50; 22A22; 54B30; 54H10

Abstract. One of powerful tools in the modern Combinatorics of Numbers is the method of ultrafilters based on the fact that each binary operation $\varphi : X \times X \to X$ defined on a discrete topological space X can be extended to a right-topological operation $\Phi : \beta X \times \beta X \to \beta X$ on the Stone-Čech compactification βX of X. The extension of φ is constructed in two step. First, for every $x \in X$ extend the left shift $\varphi_x : X \to X$, $\varphi_x : y \mapsto \varphi(x, y)$, to a continuous map $\overline{\varphi}_x : \beta X \to \beta X$. Next, for every $b \in \beta X$, extend the right shift $\bar{\varphi}^b : X \to \beta X$, $\bar{\varphi}^b : x \mapsto \bar{\varphi}_x(b)$, to a continuous map $\Phi^b : \beta X \to \beta X$ and put $\Phi(a, b) = \Phi^b(a)$ for every $a \in \beta X$. The Stone-Čech extension βX is the space of ultrafilters on X. In [6] it was observed that the binary operation φ extends not only to βX but also to the superextension λX of X and to the space GX of all inclusion hyperspaces on X. If X is a semigroup, then GX is a compact Hausdorff right-topological semigroup containing λX and βX as closed subsemigroups.

We show that an (associative) binary operation $\varphi : X \times X \to X$ on a discrete topological space X can be extended to an (associative) right-topological operation $\Phi : T\beta X \times T\beta X \to T\beta X$ for any monadic functor T in the category **Comp** of compact Hausdorff spaces. So, for the functors β, λ or G, we get the extensions of the operation φ discussed

above.

References

[1] T. Banakh, V. Gavrylkiv, O. Nykyforchyn, *Algebra in superextensions of groups, I: zeros and commutativity*, Algebra Discrete Math. **3** (2008), 1-29

[2] T. Banakh, V. Gavrylkiv, *Algebra in superextension of groups, II: cancelativity and centers*, Algebra Discrete Math. **4** (2008), 1-14

[3] T. Banakh, V. Gavrylkiv, *Algebra in the superextensions of groups, III: minimal left ideals*, Mat. Stud. **31**(2) (2009), 142-148

[4] T. Banakh, V. Gavrylkiv, *Extending binary operations to functor-spaces*, Carpathian Mathematical Publication. **1**(2) (2009), 113-126

[5] V. Gavrylkiv, *The spaces of inclusion hyperspaces over noncompact spaces*, Mat. Stud. **28**(1) (2007), 92-110

[6] V. Gavrylkiv, *Right-topological semigroup operations on inclusion hyperspaces*, Mat. Stud. **29**(1) (2008), 18-34

Iztok Banič, Matevž Črepnjak, Matej Merhar, and Uroš Milutinović

Faculty of Natural Sciences and Mathematics,
University of Maribor,
Koroška 168, Maribor 2000, Slovenia
e-mail: iztok.banic@uni-mb.si

Paths and arcs through Knaster continua
Mathematics Subject Classification (MSC): 54C60, 54B10

Abstract. Let $f_{(a,b)} : [0,1] \to [0,1]$ be a tent map with the top vertex (a, b) and the graph, which is the union of the straight line segments from $(0,0)$ to (a,b) and from (a,b) to $(1,0)$ for any $a, b \in [0,1]$. Let $K_{(a,b)}$ be the tent map inverse limit obtained from the inverse sequence with $f_{(a,b)}$ being the only bonding map. We will describe paths and arcs from a tent map inverse limit to another tent map inverse limit that go only trough tent map inverse limits in $2^{[0,1]}$.

References

[1] I. Banič, M. Črepnjak, M. Merhar, U. Milutinović, Paths through inverse limits, submitted.

Iztok Banič, Matevž Črepnjak, Matej Merhar, and Uroš Milutinović

Faculty of Natural Sciences and Mathematics,
University of Maribor,
Koroška 168, Maribor 2000, Slovenia
e-mail: matevz.crepnjak@uni-mb.si

Towards the complete classification of tent maps inverse limits

Mathematics Subject Classification (MSC): 54C60, 54B10

Abstract. Let $f_{(a,b)} : [0,1] \to [0,1]$ be a tent map with the top vertex (a,b) and the graph, which is the union of the straight line segments from $(0,0)$ to (a,b) and from (a,b) to $(1,0)$ for any $a, b \in [0,1]$. Let $K_{(a,b)}$ be the inverse limit obtained from the inverse sequence with $f_{(a,b)}$ being the only bonding map. We will present results about classification of such inverse limits $K_{(a,b)}$ depending on $(a,b) \in [0,1] \times [0,1]$.

References

[1] I. Banič, M. Črepnjak, M. Merhar, U. Milutinović, Paths through inverse limits, submitted.

[2] I. Banič, M. Črepnjak, M. Merhar, U. Milutinović, Towards the complete classification of tent maps inverse limits, submitted.

<u>Désirée Basile</u> and Angelo Bella
Università degli Studi di Catania
Dipartimento di Matematica e Informatica
Viale Andrea Doria 6
95125 Catania
Italy
e-mail: basile@dmi.unict.it

Remainders in compactifications of homogeneous spaces
Mathematics Subject Classification (MSC): 54D35, 54D40, 54E52

Abstract. Arhangel′skiĭ has recently proved two dichotomy theorems about remainders of topological groups:

- Every remainder of a topological group is either Lindelöf or pseudocompact.
- Every remainder of a topological group is either σ-compact or Baire.

We prove the following dichotomy theorem about remainders of homogeneous spaces:

- Every remainder of a homogeneous space is either realcompact and meager or Baire.

We also show that none of the Arhangel′skiĭ's theorems can be generalized to the case of homogeneous spaces.

E. Bastrykov, R. Golovastov, A. Gryzlov
Udmurt State University
Russia, Izhevsk, Universitetskaya 1
e-mail: gryzlov@uni.udm.ru

Comactifications of N as Stone spaces of some Boolean algebras
Mathematics Subject Classification (MSC): 54D35, 54D80

Abstract. We consider compactifications of a countable discrete space, which are Stone spaces of some Boolean algebras and examine their properties.

K. Belaid and L. Dridi
8 bis rue Lorca 1 Tunis 2092 Tunisia
e-mail: belaid412@yahoo.fr

Nodec spaces and compactifications
Mathematics Subject Classification (MSC): 06B30, 06F30 and 54F05

Abstract. We describe compact nodec spaces and we characterize space such that its one point compactification (resp. Wallman compactification) is nodec.

Nikola Koceić Bilan
Department of Mathematics, University of Split, Teslina 12, Split, Croatia
e-mail: koceic@pmfst.hr

The coarse shape groups
Mathematics Subject Classification (MSC): 55P55, 55Q05, 55N99

Abstract. The (pointed) coarse shape category Sh^* ($Sh^*_\star$), having (pointed) topological spaces as objects and having the (pointed) shape category as a subcategory, was recently constructed ([1]). Its isomorphisms classify (pointed) topological spaces strictly coarser than the (pointed) shape type classification. Coarse shape isomorphisms preserve some important topological invariants as connectedness, (strong) movability, shape dimension and stability ([3]). There are also several new algebraic coarse shape invariants. In this talk we introduce a new algebraic coarse shape invariant which is an invariant of shape and homotopy, as well. For every pointed space $(X,\star)$ and for every $k \in \mathbb{N}_0$, the coarse shape group $\check{\pi}^*_k(X,\star)$, having the standard shape group $\check{\pi}_k(X,\star)$ for its subgroup, is defined ([2]). Furthermore, a functor $\check{\pi}^*_k : Sh^*_\star \to Grp$ is constructed. The coarse shape and shape groups already differ on the class of polyhedra. An explicit formula for computing coarse shape groups of polyhedra is given. The coarse shape groups give us more information than the shape groups. Generally, $\check{\pi}_k(X,\star) = 0$ does not imply $\check{\pi}^*_k(X,\star) = 0$ (e.g. for solenoids), but from $pro\text{-}\pi_k(X,\star) = \mathbf{0}$ follows $\check{\pi}^*_k(X,\star) = 0$. Moreover, for pointed metric compacta $(X,\star)$, the n-shape connectedness is characterized by $\check{\pi}^*_k(X,\star) = 0$, for every $k \leq n$.

References

[1] N. Koceić Bilan and N. Uglešić, *The coarse shape*, Glasnik Mat. 42 (62) (2007) 145-187.

[2] N. Koceić Bilan, *The coarse shape groups*, Topology Appl. 157 (2010) 894-901

[3] N. Koceić Bilan, *On some coarse shape invariants*, Topology Appl., in press.

S. I. Bogataya, S. A. Bogatyy
Lomonosov MSU; Moscow
e-mail: bogatyi@inbox.ru

Embedding compacta into Eucledean space
Mathematics Subject Classification (MSC): 53A07

Abstract. Theorem. *For an infinite n-dimensional compactum X containing n-dimensional simplex Δ^n and integers $m \geq 1$, $0 \leq d \leq m$, $q \geq d+2$ the following conditions are equivalent:*

$(1)_{(n,m,d,q)}$ *The inequality*

$$qn + 1 \leq (q - d - 1)(m - d).$$

is true.

$(2)_{(X,m,d,q)}$ *The set $\mathcal{H}$ of all maps $g\colon X \to \mathbb{R}^m$ such that the preimage $g^{-1}(\Pi^d)$ of every d-dimensional plane $\Pi^d \subset \mathbb{R}^m$ has cardinality $\leq q-1$ is dense (containes dense G_δ-set) in the space of all continuous maps of X into $\mathbb{R}^m$.*

The implication (1) $\Longrightarrow$ (2) was proved by Bogatyy and Valov in a stronger form and there proof is based on a "converse of transversal Tverberg theorem". The implication (2) $\Longrightarrow$ (1) was proved by Boataya–Bogatyy–Kudryavtseva in a stronger form and there proof is based on a "topologically stable transversal Tverberg configuration".

The case $d = 0, q = 2$ corresponds to the famous Nöbeling-Pontryagin embedding theorem. The case $d = 0$ corresponds to the Hurewicz theorem. The case $d = k - 1, q = k + 1$ corresponds to the Boltyanski theorem about k-regular maps.

The importanse of k-regular maps is seen from the Haar–Kolmogorof–Rubinstein

Theorem. *For functions (continuous)* $\varphi_1, ..., \varphi_m : X \to \mathbb{R}$ *and natural* $k \le m$ *the following conditions are equivalent:*

1) *(Number of roots). Every linearly independent system of* $(m-k+1)$ *equations*

$$\begin{cases} \alpha_0^{(1)} + \alpha_1^{(1)}\varphi_1(x) + ... + \alpha_m^{(1)}\varphi_m(x) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \alpha_0^{(m-k+1)} + \alpha_1^{(m-k+1)}\varphi_1(x) + ... + \alpha_m^{(m-k+1)}\varphi_m(x) = 0 \end{cases}$$

has no more than k *roots.*

2) *(Interpolation property). For any points* $x_0, ..., x_k \in X$ *and reals* $f_0, ..., f_k \in \mathbb{R}$ *there exist a polynomial* $p_{\bar{\alpha}} = \alpha_0 + \alpha_1\varphi_1 + ... + \alpha_m\varphi_m$ *such that*

$$p_{\bar{\alpha}}(x_0) = f_0, \ ..., \ p_{\bar{\alpha}}(x_k) = f_k.$$

3) *(Chebyschev's approximation). For any function* $\varphi : X \to \mathbb{R}$ *polynomials of the best approximation constitute the set of dimension* $\le m - k$.

4) *(k-regularity). The mapping* $f : X \to \mathbb{R}^m$ *given by the formula*

$$f(x) = \big(\varphi_1(x), \varphi_2(x), \dots, \varphi_m(x)\big),$$

is k-regular, i.a. preimage of every $(k-1)$*-dimensional plane has cardinality* $\le k$.

5) *(Matrix rank). For any points* $x_0, ..., x_k \in X$

$$\operatorname{rnk} \begin{pmatrix} 1 & \varphi_1(x_0) & \varphi_2(x_0) & \dots & \varphi_m(x_0) \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \varphi_1(x_k) & \varphi_2(x_k) & \dots & \varphi_m(x_k) \end{pmatrix} = k+1.$$

Elżbieta Wagner-Bojakowska and Aleksandra Karasińska
Faculty of Mathematics and Computer Science
University of Łódź
PL-90-238 Łódź, Banacha 22, POLAND
e-mail: wagneruni.lodz.pl

Homeomorphisms of linear and planar sets of the first category into microscopic sets
Mathematics Subject Classification (MSC): 28A05, 54C50

Abstract. A set $A \subset \mathbb{R}$ is said to be microscopic if for each $\epsilon > 0$ there exists a sequence $\{I_n\}_{n \in \mathbb{N}}$ of intervals such that $A \subset \bigcup_{n \in \mathbb{N}} I_n$ and $m(I_n) < \epsilon^n$ for $n \in \mathbb{N}$.

The notion of microscopic set on the real line was introduced by J. Appell in 2001. The properties of these sets were investigated by J. Appell, E. D'Aniello and M. Väth. They proved among others that the family of all microscopic sets is a σ-ideal, situated between countable sets and sets of Lebesgue measure zero, which is essentially different from both these families.

J.C. Oxtoby and S. Ulam proved that each set of the first category in r-dimensional Euclidean space can be transformed into a set of Lebesgue measure zero by some automorphism, and the set of all such homeomorphisms constitute a residual set in the set of all automorphisms. We have improved this result changing the sets of measure zero by microscopic sets on the real line and on the plane.

We proved also that for microscopic sets the theorems analogous to Sierpiński-Erdös Duality Theorem and to Fu-

bini Theorem on the plane are valid.

References

[1] Appell J., Insiemi ed operatori "piccoli" in analisi funzionale, Rend. Istit. Mat. Univ. Trieste **33** (2001), 127-199.

[2] Appell J., D'Aniello E., Väth M., Some remarks on small sets, Ric. Mat. **50** 2001, 255-274.

[3] Erdös P., Some remarks on set theory, Ann. Math. **(2)** **44** (1943), 643-646.

[4] Oxtoby J.C., Measure and Category, Springer-Verlag, New York-Heidelberg-Berlin 1980.

[5] Oxtoby J.C., S.M. Ulam, On the equivalence of any set of first category to a set of measure zero, Fund. Math. **31** (1938), 201-206.

[6] Sierpiński W., Sur la dualité entre la première catégorie et la mesure nulle, Fund. Math. **22** (1934), 276-280.

M. R. Burke, N. D. Macheras, K. Musiał, W. Strauss

Department of Statistics and Insurance Science, University of Piraeus, 80 Karaoli and Dimitriou str., 185 34 Piraeus, Greece
e-mail: macheras@unipi.gr

Fubini type products of category densities and liftings
Mathematics Subject Classification (MSC): 54E52, 54B10, 28A51, 60B05

Abstract. Let X and Y be topological spaces such that $X \times Y$ is a Baire space. For given category densities ρ, σ on

X and Y, respectively, two 'Fubini type' products $\rho \odot \sigma$ and $\rho \boxdot \sigma$ on $X \times Y$ are introduced. A necessary and sufficient condition for $\rho \odot \sigma$ to be a category density is presented.

Under the mild condition, that the pairs (X, Y) and (Y, X) have the Kuratowski-Ulam property, we prove for given category liftings ρ and σ on the factors the existence of a category lifting π on the product, which dominates $\rho \boxdot \sigma$ and has the properties

$\pi(A \times B) = \rho(A) \times \sigma(B)$ for Baire subsets A of X and B of Y and

$\rho\big([\pi(E)]^y\big) = [\pi(E)]^y$ for all $y \in Y$ and Baire subsets E of $X \times Y$.

It is shown, that further properties of consistency with the product structure can not be expected.

References

[1] M. R. Burke, N. D. Macheras, K. Musiał and W. Strauss, *Category product densities and liftings*, Topology and its applications 153 (2006), 1164-1191.

[2] D. H. Fremlin, T. Natkaniec and I. Recław, Universally Kuratowski-Ulam spaces, Fund. Math. 165 (2000) 239-247.

[3] K. Musiał, W. Strauss and N. D. Macheras, *Product liftings and densities with lifting invariant and density invariant sections*, Fund. Math. 166 (2000), 281-303.

[4] K. Musiał, W. Strauss and N. D. Macheras, *Fubini type products for densities and liftings*, J. Theoret. Probab. 20 (2007) 545-560.

[5] J. C. Oxtoby, Measure and Category, Springer-Verlag, New York-Heidelberg-Berlin, 1980.

[6] W. Strauss, N. D. Macheras and K. Musiał, *Liftings*, in Handbook of Measure Theory, (2002), 1131-1184, Elsevier.

M. R. Burke, N. D. Macheras, W. Strauss

Universität Stuttgart, Fachbereich Mathematik, Institut für Stochastik und Anwendungen, Postfach 80 11 40, D-70511 Stuttgart, Germany
e-mail: strauss.w@arcor.de

The topological construction of densities and liftings
Mathematics Subject Classification (MSC): 54B05, 54E52, 54H10, 28A12, 28A51, 60B05

Abstract. It is well known from an example given by Erdös, that for finitely additive measures on fields densities or liftings will not exist in general. But with the help of a topology in the basic field, we can derive a subfield for which densities and liftings exist and this process works more generally for arbitrary ideals in a basic field. As compared to the classical situation these densities and liftings have unexpected good properties, i.e. they are strong and with range in the open sets. There exists even a minimal density of this sort, which is identical with the inner density of every density on this derived subfield. Perhaps most surprising, when compared with the situation for topological probability spaces, the construction of such densities can be achieved without an application of the axiom of choice. The class of derived fields comprises the most classical fields with finitely additive measures, i.e. the fields of Jordan measurable sets derived from topological probability spaces of full support.

References

[1] K. Bichteler, *On the strong lifting property*, Illinois J. Math. 16 (1972), 370-380.

[2] M.R. Burke, N.D. Macheras, K. Musial, and W. Strauss, *Category product densities and liftings*, Topology and its Applications 153(2006), 1164-1191.

[3] N.D. Macheras and W. Strauss, *Lifting topologies for primitive liftings II*, Atti Semin. Mat. Fis. Univ. Modena Regio Emilia, 56 (2008-2009), 1-30.

[4] D. Maharam, *Finitely additive measures on the integers*, Sankhya : The Indian J. of Statistics 38(1976), 41-51.

[5] S. Marcus, *La mesure de Jordan et l'integrale de Riemann dans un espace mesure topologique*, Acta Sci. Math. Szeged 20(1959), 156-163.

[6] J.C. Oxtoby, *Measure and category*, Berlin-Heidelberg-New York, 1971 (Springer-Verlag).

A.J. Calderon Martin

Universidad de Cadiz
11510 Puerto Real, Cadiz, Spain
e-mail: ajesus.calderon@uca.es

Topological Methods in Absolute Valued Structures
Mathematics Subject Classification (MSC): 17A80, 55P15

Abstract. An algebra A is called absolute valued if it is endowed with a space norm $\|\cdot\|$ such that $\|xy\| = \|x\|\|y\|$ for all $x, y \in A$. Since 1918, there was a series of results on absolute valued algebras culminating in Albert's paper [1] asserting that any finite-dimensional absolute valued real algebra is of dimension $n = 1, 2, 4$ or 8 and isotopic to one of the classical absolute valued algebras $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ or $\mathbb{O}$. Since

then, the study of absolute valued algebras has undergone an important development with a number of contributions. The absolute valued structures theory also involves the absolute valued two-graded algebras and the absolute valued triple systems, [2, 3]. In the present work we introduce the notion of homotopy of absolute valued algebras, absolute valued two-graded algebras and absolute valued triple systems. Roughly speaking two absolute valued algebras are said to be homotopic if the product of the first algebra can be continuously deformed through absolute valued products into the product of the second one. We also show how this concept let us obtain some invariants in any kind of absolute valued structure in such a way that we can give, in a sense, a unifying viewpoint of the theory and solve some problems related to absolute valued theory.

References

[1] A. A. Albert, *Absolute valued real algebras,* Ann. of Math. **48** (1947), 495-501.

[2] A. J. Calderón and C. Martín, *Absolute-valued triple systems,* Intern. Math. J. **32** (6) (2004), 2443-2455.

[3] A. J. Calderón and C. Martín, *Two-graded absolute-valued algebras,* J. Algebra **292** (2005), 492-515.

Filippo Cammaroto and Andrei Catalioto

Department of Mathematics
Faculty of Science
University of Messina
viale Ferdinando Stagno D'Alcontres 31
S. Agata 98166 - Messina (ITALY)
e-mail: acatalioto@unime.it

A note on the cardinality of the θ-closed hull

Mathematics Subject Classification (MSC): 54A25, 54D10

Abstract. In this article we studied some properties about the cardinality of the θ-closed hull $[A]_\theta$ related to cardinal functions $\chi(X)$, $bt_\theta(X)$ and $t_\theta(X)$.
In particular, considering the Urysohn cardinal function $\mathcal{U}(X)$, we prove that $|[A]_\theta| \leq |A|^{t_\theta(X)}$ with finiteness of $\mathcal{U}(X)$ and $A \subset X$.
Moreover, some known statements about Urysohn spaces can be generalized in terms of the Urysohn cardinal function.

References

[1] A. Bella and F. Cammaroto, *On the cardinality of Urysohn spaces*, **Canad. Math. Bull.**, **31** (1988) 153-158.

[2] M. Bonanzinga, F. Cammaroto and M. Matveev, *On the Urysohn number of a topological space*, **Quaestiones Mathematicae** (2010) [*in press*].

[3] F. Cammaroto and Lj. Kočinac, *On θ-tightness*, **Ser. Math. Inform.**, **8** (1993) 77-85.

[4] F. Cammaroto and Lj. Kočinac, *A note on θ-tightness*, **Rend. Circ. Mat. Palermo**, (2) **42** (1993) 129-134.

[5] R. Engelking, *General Topology*, **Heldermann Verlag**, Berlin (1989).

[6] K.P. Hart, J. Nagata and J.E. Vaughan, *Encyclopedia of General Topology*, **Elsevier North-Holland** (2004).

N.A. Carlson and G.J. Ridderbos
Department of Mathematics,
California Lutheran University,

60 W. Olsen Rd, MC 3750, Thousand Oaks, CA 91360
USA
e-mail: ncarlson@callutheran.edu

On several cardinality bounds on power homogeneous spaces
Mathematics Subject Classification (MSC): 54A25

Abstract. We show the cardinality of a homogeneous Hausdorff space X is not necessarily bounded by $2^{L(X)\pi\chi(X)}$ by providing examples of σ-compact, countably tight, homogeneous spaces of countable π-character and arbitrary cardinality. We also generalize a closing-off argument of Pytkeev to show the cardinality of any power homogeneous Hausdorff space X is at most $2^{L(X)\mathrm{pct}(X)t(X)}$. This was previously shown to hold if X is also regular by G.J. Ridderbos. Another consequence of the generalization of Pytkeev's closing-off argument is the well-known cardinality bound $2^{L(X)t(X)\psi(X)}$ for an arbitrary Hausdorff space X.

References

[1] A. V. Arhangel'skii, *On the cardinality of bicompacta satisfying the first axiom of countability*, Soviet Math. Dokl. **10** (1969), no. 4, 951–955.

[2] A. V. Arhangel'skii, *The structure and classification of topological spaces and cardinal invariants*, Uspekhi Mat. Nauk **33** (1978), no. 6 (204), 29–84, 272.

[3] A. V. Arhangel'skii, J. van Mill, and G. J. Ridderbos, *A new bound on the cardinality of power homogeneous compacta*, Houston J. Math. **33** (2007), no. 3, 781–793.

[4] A. V. Arhangel'skii, *G_δ-modification of compacta and cardinal invariants*, Comment. Math. Univ. Carolinae **47** (2006), no. 1, 95–101.

[5] N. A. Carlson, G. J. Ridderbos, *Partition relations and power homogeneity*, Top. Proc. **32** (2008), 115–124.

[6] E. K. van Douwen, *Nonhomogeneity of products of preimages and π-weight*, Proc. Amer. Math. Soc. **69** (1978), no. 1, 183–192.

[7] R. Engelking, *General Topology*, Heldermann Verlag, Berlin, second ed., 1989.

[8] A. Hajnal and I. Juhász, *Discrete subspaces of topological spaces*, Indag. Math. **29** (1967), 343–356.

[9] I. Juhász, *Cardinal functions in topology—ten years later*, second ed., Mathematical Centre Tracts, vol. 123, Mathematisch Centrum, Amsterdam, 1980.

[10] E. G. Pytkeev, *About the G_λ-topology and the power of some families of subsets on compacta*, Colloq. Math. Soc. Janos Bolyai **41** (1985), 517–522.

[11] G. J. Ridderbos, *On the cardinality of power homogeneous Hausdorff spaces*, Fund. Math. **192** (2006), 255–266.

[12] G. J. Ridderbos, *Power homogeneity in topology* (2007), Doctoral Thesis, Vrije Universiteit, Amsterdam.

[13] B. Šapirovskiĭ, *Canonical sets and character. Density and weight in bicompacta*, Dokl. Akad. Nauk SSSR **218** (1974), 58–61.

[14] R. de la Vega, *Homogeneity properties on compact spaces* (2005), Doctoral Thesis, University of Wisconsin-Madison.

[15] R. de la Vega, *A new bound on the cardinality of homogeneous compacta*, Topology Appl. **153** (2006), 2118-2123.

A. Caserta, G. Di Maio, Ljubiša D.R. Kočinac

University of Niš, Serbia
e-mail: lkocinac@gmail.com

Bornologies and function spaces
Mathematics Subject Classification (MSC): 54C35

Abstract. For a metric space (X, d), we study some closure-type properties of the space $(\mathsf{C}(X), \tau^s_{\mathfrak{B}})$ of all continuous real-valued functions on X equipped with the recently introduced topology of strong uniform convergence on a bornology $\mathfrak{B}$ on X. Characterizations of several properties (for instance, countable fan tightness, countable strong fan tightness, the (strong) Fréchet-Urysohn property, the A-space property) of this function space are obtained.

Vincent Cases and Richard S. Lemence

Ochanomizu University
2-1-1 Otsuka, Bunkyo-ku, Tokyo 112-860, Japan
e-mail: lemence.richard.santiago@ocha.ac.jp

On the Topological Properties of Residual Classes of Real Numbers
Mathematics Subject Classification (MSC): 54A05, 54C99, 08A02

Abstract. Let $\alpha > 0$ be a fixed real number and $a, b \in \mathbb{R}$. Then $a \equiv b(\text{mod}\alpha)$ if and only if $a - b = k\alpha$ for some $k \in \mathbb{Z}$. This definition is parallel to the concept of residue classes of integers Z_n for fixed $n \in \mathbb{Z}$. As a result, this also constitutes residue classes of real numbers denoted by R_α. The element $r\alpha \in R_\alpha$ is the set $\{r + k\alpha | k \in \mathbb{Z}\}$. In [2], the algebraic structure and algebraic properties of R_α were described. It is interesting to look at R_α in another

way. Since one can perceived this geometrically, its concept leads us to extend this to topology.

Consider the mapping $\gamma : \mathbb{R} \to R_\alpha$ which is defined by $\gamma(x) = r\alpha$ such that $x = r + k\alpha$ for some $k \in \mathbb{Z}$ and $0 \leq r < \alpha$. Let $\epsilon > 0$. Define the symmetric open ball in $\mathbb{R}$ center at $x \in \mathbb{R}$ of radius ϵ by

$$B(\epsilon, x) = \{y : |x - y| < \epsilon\}.$$

Through this, the basis element of R_α center at $r\alpha \in R_\alpha$ determined by $x \in r\alpha$, can be defined by the set

$$B'_x(\epsilon, r\alpha) = \{\gamma(y) : y \in B(\epsilon, x)\}.$$

The set of B'_x's generate the topology τ in R_α.

In this talk, we will present the topological properties of residue classes of real numbers – continuity, covering map, homeomorphism and connectedness.

[1] Bartle, R.G., *Introduction to Real Analysis*, John Wiley and Sons, New York, 2000.

[2] V.P. Cases, *On the Residual Classes of Real Numbers*, Graduate thesis, Mindanao State University - Iligan Institute of Technology, 2006.

[3] Fraleigh, J.B., *A First Course in Abstract Algebra*, Addison - Wesley Publishing Company Inc., 1994.

[4] Munkres, J.R., *Topology A First Course*, Prentice Hall, Engelwood Cliffs, New Jersey, 1975.

Michael G. Charalambous and Jerzy Krzempek

Department of Mathematics
University of the Aegean
83 200 Karlovassi, Samos, Greece

e-mail: mcha@aegean.gr

On Dimensionsgrad, resolutions, and chainable continua
Mathematics Subject Classification (MSC): Primary 54F45; secondary 54F15

Abstract. The necessary definitions

Let A, B be disjoint closed subsets of a space X and C a closed subset of X disjoint from $A \cup B$. C is called a (*zero*) *partition* between A and B if (C is a zero subset of X and) there are disjoint open subsets U, V of X such that $A \subset U, B \subset V$ and $X \setminus C = U \cup V$. C is called a (*zero*) *cut* between A and B if (C is a zero subset of X and) every continuum that meets both A and B meets C. Evidently, every (zero) partition is a (zero) cut.

The definitions of the topological dimension functions Ind, Ind_0, Dg, and Dg_0 are quite similar: $\mathrm{Ind}\,X$, $\mathrm{Ind}_0\,X$, $\mathrm{Dg}\,X$, or $\mathrm{Dg}_0\,X$ equals -1 iff $X = \emptyset$. For a non-empty normal space X, $\mathrm{Ind}\,X$ (respectively, $\mathrm{Ind}_0\,X$, $\mathrm{Dg}\,X$, $\mathrm{Dg}_0\,X$) is the smallest non-negative integer n for which between any pair of disjoint closed sets A and B of X, there is a partition (respectively, zero partition, cut, zero cut) C with $\mathrm{Ind}\,C$ (respectively, $\mathrm{Ind}_0\,C$, $\mathrm{Dg}\,C$, $\mathrm{Dg}_0\,C$) $\leq n-1$, if such an integer exists. If no such integer exists, we set $\mathrm{Ind}\,X$ (respectively, $\mathrm{Ind}_0\,X$, $\mathrm{Dg}\,X$, $\mathrm{Dg}_0\,X$) $= \infty$. If in the above definition of Ind, we stipulate that the set A is a singleton, we obtain the definition of ind. Dg or *Dimensionsgrad* was defined by Brouwer in 1913. The definition of ind or *small inductive dimension* was formulated by Urysohn in 1922 and, independently, by Menger in 1923. Čech defined Ind or *large inductive dimension* in 1931. In a previous paper of ours, Ind_0 proved very useful in estimating ind and Ind.

We introduce Dg_0 in this paper as a tool for computing Dimensionsgrad.

Transfinite extensions of the above dimension functions are obtained in the usual manner. Thus, for example, if X is a non-empty space, $\mathrm{trDg}_0\, X$ is the smallest ordinal α for which between any pair of disjoint closed sets A and B of X, there is a zero cut C with $\mathrm{trDg}_0\, C < \alpha$, if such an ordinal exists. If no such ordinal exists, we set $\mathrm{trDg}_0\, X = \infty$. Evidently, $\mathrm{trind} \leq \mathrm{trInd} \leq \mathrm{trInd}_0$, $\mathrm{Dg} \leq \mathrm{Ind}$ and $\mathrm{trDg} \leq \mathrm{trDg}_0 \leq \mathrm{trInd}_0$. Hence, if $\mathrm{trind}\, X = \mathrm{trInd}_0\, X$, then $\mathrm{trind}\, X = \mathrm{trInd}\, X = \mathrm{trInd}_0\, X$.

A *continuum* is a compact, Hausdorff and connected space. A continuum is *chainable* or *snake–like* if every open cover of the continuum is refined by an open chain, i.e. a finite open cover $U_1, \ldots, U_k$ such that $U_i \cap U_j \neq \emptyset$ iff $|i - j| \leq 1$.

Two questions

(1) B.A. Pasynkov, 1985: Does there exist a chainable continuum S_α with $\mathrm{trind}\, S_\alpha = \alpha$ for each ordinal number α?

(2) V.A. Chatyrko and V. V. Fedorchuk, 2005: Is the Dimensionsgrad of every non-degenerate chainable continuum equal to 1?

Results

For each natural number $n \geq 1$ and each pair of ordinals α, β with $n \leq \alpha \leq \beta \leq \omega(\mathfrak{c}^+)$, where $\omega(\mathfrak{c}^+)$ is the first ordinal of cardinality greater than $\mathfrak{c}$, there is a continuum $S_{n,\alpha,\beta}$ such that

(a) $\dim S_{n,\alpha,\beta} = n$;

(b) $\operatorname{trDg} S_{n,\alpha,\beta} = \operatorname{trDg}_0 S_{n,\alpha,\beta} = \alpha$;

(c) $\operatorname{trind} S_{n,\alpha,\beta} = \operatorname{trInd}_0 S_{n,\alpha,\beta} = \beta$;

(d) if $\beta < \omega(\mathfrak{c}^+)$, then $S_{n,\alpha,\beta}$ is separable and first countable;

(e) if $n = 1$, then $S_{n,\alpha,\beta}$ can be made chainable or hereditarily decomposable;

(f) if $\alpha = \beta < \omega(\mathfrak{c}^+)$, then $S_{n,\alpha,\beta}$ can be made hereditarily indecomposable; and

(g) if $n = 1$ and $\alpha = \beta < \omega(\mathfrak{c}^+)$, then $S_{n,\alpha,\beta}$ can be made chainable and hereditarily indecomposable.

Other results contained in the paper enable us to compute the Dimensionsgrad of a number of spaces constructed by Charalambous, Chatyrko, and Fedorchuk.

Charalampos Charitos, Ioannis Papadoperakis, and Georgios Tsapogas

Department of Mathematics,
University of the Aegean,
Samos, Greece
e-mail: get@aegean.gr

A Complex of Incompressible Surfaces for handlebodies and the Mapping Class Group
Mathematics Subject Classification (MSC): 57N10, 57N35

Abstract. For a genus g handlebody H_g a simplicial complex, with vertices being isotopy classes of incompressible surfaces in H_g, is constructed and several properties are established. As in the classical theory, the group of automorphisms of this complex is identified with the mappig class group of the handlebody.

Vitalij A.Chatyrko and Yasunao Hattori

Department of Mathematics, Shimane University
Matsue, Shimane, 690-8504 Japan
e-mail: hattori@riko.shimane-u.ac.jp

The remainders in extensions and finite unions of locally compact sets
Mathematics Subject Classification (MSC):
Primary 54D35, 54D40; Secondary 54D45

Abstract. In [1], we studied the spaces which are represented as finite unions of locally compact subspaces. The class of such spaces we denote by $\mathcal{P}_{fin}$. In this talk, we discuss the relationship between properties of spaces from the class $\mathcal{P}_{fin}$ and their remainders in extensions from the same class $\mathcal{P}_{fin}$. In particular, we show that a space $X \in \mathcal{P}_{fin}$ iff the remainder in each (some) compactification of X is in $\mathcal{P}_{fin}$. Then we study the relationship between the class of almost locally compact spaces ([3]) and $\mathcal{P}_{fin}$, present a relationship between the remainders of a space from $\mathcal{P}_{fin}$ in compact extensions and give a generalization of the theorem of Henriksen-Isbell [2].

References

[1] V. A. Chatyrko, Y. Hattori and H. Ohta, Partitions of spaces by locally compact subspaces, Houston J. Math. 32, 4 (2006) 1077–1091

[2] M. Henriksen and J. R. Isbell, Some properties of compactifications, Duke Math. J. 25 (1958) 83-106

[3] M. C. Rayburn, Compactifications with almost locally compact outgrowth, Proc. Amer. Math. Soc. 106, 1 (1989) 223–229

Vitalij A.Chatyrko and Yasunao Hattori

Linköping University, Sweden
e-mail: vitja@mai.liu.se

Around a Hurewicz's formula
Mathematics Subject Classification (MSC): 54D45, 54F45

Abstract. Some relevant questions around the Hurewicz's formula for the dimension-lowering mappings are under our discussion. In particular, we generalize the notion of a fully closed mapping introduced by V. V. Fedorchuk ([F]) and find for such new mappings an evaluation formula for the dimension Ind_0 (resp. Ind) of preimages via the dimension Ind_0 (see [C] or [I]) (resp. Ind) of images.

References

[C] M. G. Charalambous, Two new inductive dimension functions for topological spaces, Ann. Univ. Sci. Budapest. Eötvös Sect. Math. 18 (1975) 15–25 (1976)

[F] V. V. Fedorchuk, Fully closed mappings and their applications, Fundam. Prikl. Mat. 9 (2003) N4, 105–235

[I] A. V. Ivanov, The dimension of not perfectly normal spaces, Vestnik Moskov. Univ. Ser. I Mat. Meh. 31 (4) (1976) 21–27.

Lia Chechelashvili and Leonard Mdzinarishvili

Saburtalo str. Bl. 3, apt. 45, Tbilisi, 0192, Georgia
e-mail: amyrocher@yahoo.com

Čech cohomology with coefficients in a topological abelian group
Mathematics Subject Classification (MSC): 55N05

Abstract. Unlike the ordinary Čech cohomology where a group of coefficients is abelian, in this paper a continuous Čech cohomology with coefficients in a topological abelian group is defined.

It is proved that for compact spaces and any topological abelian group the continuous Čech cohomology satisfies all Eilenberg–Steenrod axioms and continuity axiom, except the excision axiom.

However, if a group of coefficients is an absolute retract or $\Omega^* G$, where G is an AR, then for compact spaces the continuous Čech cohomology satisfies the excision axiom.

E. Colebunders, B. Lowen, S. De Wachter

e-mail: sdewacht@vub.ac.be

Quantifying domains using approach spaces
Mathematics Subject Classification (MSC): 06, 54, 68

Abstract. Motivated by central problems in theoretical computer science, mathematical structures, called domains, have been created to model semantics of programming languages. A domain is a partially ordered set in which all directed sets have a supremum and which satisfies the axiom of approximation: Every element in the domain is the supremum of a directed set of base elements

which are finite in nature. Every domain has an intrinsic topology, called the Scott topology [1], which can be used to describe convergence of algorithms. However, since the Scott topology is not a metrizable topology, this theory cannot be used for a more refined quantitative reasoning. Because this quantitative information is important for applicability (One would like to know how fast an algorithm converges to a solution), domains have been endowed with a weighted quasi metric structure inducing the Scott topology. An important result, independently obtained by M. Schellekens [3] and by P. Waszkiewicz [2], states that all continuous domains having a countable basis are quantifiable. The weighted quasi metric involved is constructed by taking an infinite sum, $\sum \frac{1}{2^n}$, over some subset of the natural numbers. As observed by M. Schellekens in [3] and by H.P. Künzi in [5], the role of $(\frac{1}{2^n})_n$ could be replaced by any other sequence, so the numerical values computed through the quasi metric are not canonically determined.

We propose a canonical solution for the problem of quantifiability. Such a solution is obtained regardless of cardinality conditions on bases of the domain. We show that every domain D is quantifiable in the sense that there exists an approach structure on D [4], inducing the Scott topology. Moreover in the case of an algebraic domain, a quantifying approach space can be obtained which allows to extract the set of maximal elements of the domain, which are of great importance in applications, since they are the ideal objects which are approximated during computations.

References

[1] G. Gierz, K.H. Hofmann, K. Keimel, J.D. Lawson, M. Mislove and D.S. Scott, *A compendium of continuous lattices*, Springer Verlag, Berlin, Heidelberg, New York, 1980.

[2] P. Waszkiewicz, Quantitative continuous domains. *Applied Categorical Structures* 11 41-67, 2003.

[3] M. Schellekens, A characterization of partial metrizability: Domains are quantifiable. *Theoretical Computer Science* 305, no.1-3, 409-432, 2003.

[4] R. Lowen, *Approach Spaces. The Missing Link in the Topolgy-Uniformity-Metric Triad*, Oxford Mathematical Monographs, Oxford University Press, 1997.

[5] H.-P. Künzi, V. Vajner, Weighted quasi-metrics, *Ann. New York acad. Sci.* 728, 64-77, 1994.

Pratulananda Das and Sanjoy Kr Ghosal

Jadavpur University,
Kolkata-700032, India
e-mail: pratulananda@yahoo.co.in

On summability of nets through ideals and some topological observations
Mathematics Subject Classification (MSC): Primary 54A20; Secondary 40A05, 40A99

Abstract. The idea of convergence of a real sequence had been extended to statistical convergence by Fast [2] (see also Schoenberg [13]) as follows: If N denotes the set of natural numbers and $K \subset N$ then K_n denotes the set $\{k \in K : k \leq n\}$ and $|K_n|$ stands for the cardinality of the set K_n. The natural density of the subset K is defined by

$$d(K) = \lim_{n\to\infty} \frac{|K_n|}{n}$$

provided the limit exists.

A sequence $\{x_n\}_{n\in N}$ of points in a metric space (X, ρ) is to be statistically convergent to ℓ if for arbitrary $\epsilon > 0$,

the set $K(\epsilon) = \{k \in N : d(x_k, \ell) \geq \epsilon\}$ has natural density zero. A lot of investigations have been done on this convergence and its topological consequences after the initial works by Fridy [3] and Šalát [12]. In particular, Very recently Di maio and Kocinak [8] introduced the concept of statistical convergence in topological spaces as well as uniform spaces and established the topological nature of this convergence as also offered some applications to selection principles theory, function spaces and hyper spaces.

However if one considers the concept of nets instead of sequences (which undoubtedly play more important and natural role in topological and uniform spaces) the above approach does not seem to be appropriate because of the absence of any idea of density in arbitrary directed sets. Instead it seems more appropriate to follow the more general approach of [4].

In [4] an interesting generalization of the notion of statistical convergence was proposed. Namely it is easy to check that the family $I_d = \{A \subset N : d(A) = 0\}$ forms a non-trivial admissible ideal of N (recall that $I \subset 2^N$ is called an ideal if (i) $\phi \in I$, (ii) $A, B \in I$ implies $A \cup B \in I$ and (iii) $A \in I$, $B \subset A$ implies $B \in I$. I is called non-trivial if $I \neq \{\phi\}$ and $N \notin I$. I is admissible if it contains all singletons). Thus one may consider an arbitrary ideal I of N and define I-convergence of a sequence as follows.

A sequence $\{x_n\}_{n \in N}$ in $(X, \rho\,)$ is said to be I-convergent to $x \in X$, (in short $x = I - \lim_{n \to \infty} x_n$) if $K(\epsilon) \in I$ for each $\epsilon > 0$, where $K(\epsilon) = \{k \in N : d(x_k, x) \geq \epsilon\}$.

The aim of this presentation is to show that the idea of convergence and Cauchy condition of nets can be broadened in the same way using the concept of ideals. It is observed that two types of convergence namely, I-convergence and I^*-convergence can be considered in line of statistical and s^*-convergence of [8] as well as the corresponding Cauchy conditions. But unlike [8], these concepts are not in general equivalent even in first countable spaces (which can be shown by constructing proper examples) and only coincide if and only if the ideal satisfies a condition called condition (DP). The basic topological nature of these convergence are established and most importantly an open problem posed by Di Maio and Kocinac (Problem 2.16 [8]) is considered and we try to give some answers. This motivates us to consider the idea of completeness of an uniform space and we also consider the impact of this generalization on the notion of completeness and make certain interesting observations. Finally a kind of divergence of nets is considered in uniform spaces and its basic properties are studied.

References

[1] K.Dems, On I-Cauchy sequences, Real Anal. Exchange, 30 (1) (2004-2005), 123-128.

[2] H. Fast, Sur la convergence statistique, Colloq. Math. 2 (1951), 241-244.

[3] J.A.Fridy, On statistical convergence, Analysis, 5 (1985), 301-313.

[4] P.Kostyrko, T.Šalát, W.Wilczynki, I-convergence, Real Anal. Exchange, 26 (2) (2000/2001), 669-685.

[5] K.Kuratowski, Topologie I, PWN, Warszawa, 1961.

[6] B.K.Lahiri, Pratulananda Das, I and I^* convergence in topological spaces, Math. Bohemica, 130 (2) (2005), 153-160.

[7] B.K.Lahiri, Pratulananda Das, I and I^* convergence of nets, Real Anal. Exchange, 33 (2) (2007-2008), 431 - 442.

[8] G. Di Maio, Lj. D.R. Kocinac, Statistical convergence in topology, Top. Appl., 156 (2008), 28 - 45.

[9] A. Nabiev, S. Pehlivan, M. Gurdal, On I-Cauchy sequences, Taiwanese J. Math., 11 (2), (2007), 569 - 576.

[10] Pratulananda Das, S. Ghoshal, On I-Cauchy nets and completeness, communicated.

[11] Pratulananda Das, S Ghoshal, On some further remarks on ideal summability of nets in uniform spaces, under preparation.

[12] T. Šalát, On statistically convergent sequences of real numbers, Math. Slovaca, 30 (1980), 139-150.

[13] I.J.Schoenberg, The integrability of certain functions and related summability methods, Amer. Math. Monthly, 66 (1959), 361-375.

Pratulananda Das

Jadavpur University,
Kolkata-700032, India
e-mail: pratulananda@yahoo.co.in

Sequential approach to a problem on Dense Sets
Mathematics Subject Classification (MSC): 40A05, 54A20

Abstract. Here we consider a very fundamental topological problem as to what is the condition under which almost all subsets of a dense set in a topological space X are again dense in X. As far as our knowledge is concerned, we could not find any answer to this question in the existing literature. It is well known that in a first countable T_1 topological space X, β is a limit of $B \subset X$ if and only if there is a sequence of elements in B which is convergent to β. Since

a countable dense set can be written as a sequence , so in a first countable T_1 space, every limit point of a countable dense set B(and so of that sequence) can be thought of as a subsequential limit of that sequence and consequently $\overline{B}$ becomes the cluster set of all subsequential limit points of this sequence .

With this in mind we start with the well known correspondence between the numbers of $(0, 1]$ and the subsequences of a given sequence $a = (a_n)_{n \in N}$ in a topological space X (cf. [1], [2]). If $t \in (0, 1]$, then t has a unique non-terminating dyadic expansion

$$t = \sum_{k=1}^{\infty} c_k(t) 2^{-k}, \ldots (1)$$

$c_k(t) = 0$ or 1 $(k = 1, 2, \ldots)$ and $c_k(t) = 1$ for infinitely many k's.

If we put $\{k : c_k(t) = 1\} = \{k_1 < k_2 < k_3 < \ldots\}$ then (1) has the form $t = \sum_{n=1}^{\infty} 2^{-k_n}$. Put

$$a(t) = (a_{k_1}, a_{k_2}, \ldots, a_{k_n}, \ldots).$$

So we get a one-to-one correspondence between the numbers of $(0, 1]$ and the subsequences of a. Thus corresponding to a class of subsequences A of a, there exists a subset $A^* \subset (0.1]$ of all those t's from $(0, 1]$ that correspond to the subsequences from A. This unique correspondence enables us "to measure" the magnitude or "to find" the category position of a class A of subsequences of a by calculating the measure of the set A^* or by finding the category position of the set A^*. Using the above mentioned facts and intrinsic properties of dyadic expansion of we prove the following.

Theorem A. In a completely separable topological space X, $\Delta \subset X$ is the cluster set of a given sequence $a = (a_n)_{n \in N}$ if and only if almost all it's subsequences also have Δ as their cluster set in the sense of measure and category.

The following theorem then immediately follows which gives a partial answer to the open problem stated above.

Theorem B. In a separable, first countable, T_1 and perfect topological space X(which is infinite), almost all infinite subsets of a countable dense set are again dense in X in the sense of measure and category.

References

[1] R. C. Buck, A note on subsequences, Bull. Amer. Math. Soc., 49 (1943), 898 - 899.

[2] R. C. Buck , H. Pollard, Convergence and summability properties of subsequences, Bull. Amer. Math. Soc., 49 (1943), 924 - 931.

[3] R. Engelking, General Topology, Heldermann Verlag, Berlin, 1989.

[4] C. Kuratowski, Topologie I, PWN Warszawa, 1958.

Dikran Dikranjan and Anna Giordano Bruno

Dipartimento di Matematica e Informatica, Università di Udine, Via delle Scienze, 206 - 33100 Udine, Italy
e-mail: dikran.dikranjan@dimi.uniud.it

The Pinsker subgroup of an algebraic flow
Mathematics Subject Classification (MSC): 22D40, 28D20

Abstract. Let G be an abelian group and let $\phi : G \to G$ be an endomorphism. For a finite subset F of G and for $n \in \mathbb{N}$, let

$$T_n(\phi, F) = F + \phi(F) + \ldots + \phi^{n-1}(F).$$

Obviously, $\tau_{\phi,F}(n) = |T_n(\phi, F)| \leq |F|^n$. We show that $\tau_{\phi,F}$ has either a polynomial or an exponential growth as a function of n. Hence, the limit

$$H(\phi, F) = \lim_{n\to\infty} \frac{\log \tau_{\phi,F}(n)}{n}$$

exists. Moreover, $h(\phi) = \sup_{F\in[G]^{<\omega}} H(\phi, F)$ (the *algebraic entropy* of ϕ) coincides with the algebraic entropy of ϕ^{-1} defined in [5].

Extending a result from [3] we prove that there exists a maximum ϕ-invariant subgroup $P(G, \phi)$ of G (the *Pinsker subgroup* of ϕ), such that $h(\phi \restriction_{P(G,\phi)}) = 0$. This subgroup is also the maximum ϕ-invariant subgroup N of G such that the induced endomorphism $\bar{\phi}$ of G/N has no non-trivial quasi-periodic points. It coincides also with the maximum ϕ-invariant subgroup N of G such that $\tau_{\phi\restriction_N,F}$ has polynomial growth for every finite subsets F of N.

The counterpart of Pinsker subgroup in the case of the topological entropy (in the sense of [1]) is the Pinsker factor (see [2, 4]).

Extending a theorem of Peters [5] one can prove that if G is an abelian group and ϕ is an endomorphism of G, then the topological entropy $h_{top}(\widehat{\phi})$ of the Pontryagin dual $\widehat{\phi} : \widehat{G} \to \widehat{G}$ coincides with the algebraic entropy of ϕ.

It follows from the above results that if ψ is a continuous endomorphism a compact abelian group K, then the annihiulator $\mathcal{E}(G, \psi)$ of $P(\widehat{G}, \widehat{\psi})$ is a closed ψ-invariant subgroup of K having the following properties:

(a) the restriction $\psi \restriction_{\mathcal{E}(G,\psi)}$ is ergodic;

(b) $\mathcal{E}(G,\psi)$ is the greatest closed ψ-invariant subgroup of K with the property (a).

Item (a) justifies the term *greatest domain of ergodicity* of ψ for $\mathcal{E}(G,\psi)$. It coincides also with the greatest closed ψ-invariant subgroup N of K such that the induced endomorphism $\bar{\psi}$ of G/N has zero topological entropy (i.e., $G/\mathcal{E}(G,\psi)$ coincides with the Pinsker factor of ψ).

References

[1] R. L. Adler, A. G. Konheim and M. H. McAndrew, *Topological entropy*, Trans. Amer. Math. Soc. **114** (1965), 309–319.

[2] F. Blanchard and Y. Lacroix, *Zero entropy factors of topological flows*, Proc. Amer. Math. Soc. **119** (1993) no. 3, 985–992.

[3] D. Dikranjan, B. Goldsmith, L. Salce and P. Zanardo, *Algebraic entropy of endomorphisms of abelian groups*, Trans. Amer. Math. Soc. **361** (2009), 3401–3434.

[4] D. Kerr and H. Li, *Dynamical entropy in Banach spaces*, Invent. Math. **162** (2005), no. 3, 649–686.

[5] J. Peters, *Entropy on discrete Abelian groups*, Adv. Math. **33** (1979), 1–13.

D. Dikranjan and G. Lukács

Department of Mathematics
University of Manitoba
Winnipeg, Manitoba, R3T 2N2
Canada
e-mail: lukacs@cc.umanitoba.ca

Vedenissoff groups
Mathematics Subject Classification (MSC):

Abstract. A (Hausdorff) space X is *hereditarily disconnected* if its connected components are singletons, and X is *zero-dimensional* if its clopen sets form a base for the topology of X. Every zero-dimensional space is hereditarily disconnected, and if X is locally compact, then the reverse implication

$$\text{hereditarily disconnected} \Longrightarrow \text{zero-dimensional} \qquad (*)$$

also holds. In particular, hereditarily disconnected locally compact groups are zero-dimensional.

For a topological group G, let G_0 denote the component of the identity in G. It is well known that G/G_0 is hereditarily disconnected, but it need not be zero-dimensional. We say that the group G is *Vedenissoff* if the quotient G/G_0 is zero-dimensional. We are interested in classes of topological groups where $(*)$ holds, that is, classes of Vedenissoff groups.

A space X is *pseudocompact* if every continuous real-valued map on X is bounded. In the class of topological groups, pseudocompact groups can be characterized as the G_δ-dense subgroups of compact groups (cf. [2, 1.1]). A dense subgroup G of a locally compact group L is *locally pseudocompact* if G is G_δ-dense in L (cf. [3]), and G is *hereditarily locally pseudocompact* if every closed subgroup S of G is G_δ-dense in its closure $\mathrm{cl}_L\, S$ in L.

There are many known examples of pseudocompact groups that fail to be Vedenissoff (cf. [4, Theorem 11] and [6, 1.4.10]). Nevertheless, we show that every hereditarily locally pseudocompact group is Vedenissoff, a result that generalizes [5, 1.2], and provides a positive answer

to [1, 4.13]. We also show the existence of pseudocompact abelian groups with strong compactness-like properties that fail to be Vedenissoff.

References

[1] W. W. Comfort and G. Lukács. Locally precompact groups: (local) realcompactness and connectedness. *J. Lie Theory*, 20:347–374, 2010.

[2] W. W. Comfort and K. A. Ross. Pseudocompactness and uniform continuity in topological groups. *Pacific J. Math.*, 16:483–496, 1966.

[3] W. W. Comfort and F. J. Trigos-Arrieta. Locally pseudocompact topological groups. *Topology Appl.*, 62(3):263–280, 1995.

[4] D. Dikranjan. Dimension and connectedness in pseudocompact groups. *C. R. Acad. Sci. Paris Sér. I Math.*, 316(4):309–314, 1993.

[5] D. Dikranjan. Zero-dimensionality of some pseudocompact groups. *Proc. Amer. Math. Soc.*, 120(4):1299–1308, 1994.

[6] D. Dikranjan. Compactness and connectedness in topological groups. *Topology Appl.*, 84(1-3):227–252, 1998.

X. Dimitriou and N. Papanastassiou

Department of Mathematics,
University of Athens,
Athens, Greece
e-mail: npapanas@math.uoa.gr

Some aspects of convergence in Riesz Spaces
Mathematics Subject Classification (MSC):

Abstract. Some aspects of the theory of order and (D)-convergence in lattice groups with respect to ideals are presented. Moreover new versions of some basic Matrix theorems are given.

Giuseppe Di Maio, A. Caserta and Lj.D.R. Kočinac
Seconda Universita di Napoli, Caserta, Italy
e-mail: giuseppe.dimaio@unina2.it

Versions of properties (a) and (pp), and the Alexandroff duplicate
Mathematics Subject Classification (MSC): 54D20

Abstract. We introduce and study selective versions of properties (a) and (pp). Their applications to the Alexandroff duplicate are considered.

Dimitrova-Burlayenko S.D.
National Technical University 'KhPI', Kharkov, Ukraine
e-mail: s.dimitrova@mail.ru

On continuity of functions limit
Mathematics Subject Classification (MSC): 43A60

Abstract. In the present paper a new definition of quasi-uniform convergence of functions with values in the metric space Y that are not obligatory continuous and are given in the non-compact topological set G is introduced. Several examples proving that the quasi-uniform convergence proposed does not result in the convergence by P.S. Aleksandrov are found.

Definition. *A pointwise convergent sequence of functions* $\{f_n(t)\}_{n=1}^{\infty}$ *is called quasi-uniform convergent to the*

function $f(t) : G \to Y$ *if for any* $\varepsilon > 0$, N *and any subsequence* $\{t'_\beta\}_{\beta=1}^{\infty} \subset G$ *there exists index* $n_0 > N$ *and subsequence* $\{t_\beta\}_{\beta=1}^{\infty} \subset \{t'_\beta\}_{\beta=1}^{\infty}$ *such that* $\rho(f(t_\beta), f_{n_0}(t_\beta)) < \varepsilon$ *for* $\forall\beta = 1, 2, 3, \ldots$.

Partially continuous functions at a point are considered. The function is called partially continuous if there exists a sequence $\{t_n\}_{n=1}^{\infty}$, $\lim_{n\to\infty} t_n = t$ such that $\lim_{n\to\infty} f(t_n) = f(t)$. It is proven that a limit of the partially continuous functions is partially continuous if and only if the convergence of the functions is quasi-uniform.

The convergence mentioned above changes its properties depending on the definition domain and the type of the functions. If the definition domain is a compact set then the classical quasi-uniform convergence by P.S. Aleksandrov is stronger than the introduced one. Otherwise, the quasi-uniform convergence proposed is stronger than the convergence by P.S. Aleksandrov in the case of continuous functions. Finally, if the functions are simultaneously continuous and defined on a compact set then the both convergences are coinciding. Moreover, it is proven that the definition domain of functions is a compact set if and only if the functions are continuous and the both quasi-uniform convergences are coinciding.

D. Djurčić, Lj. D.R. Kočinac, and Mališa R. Žižović

Technical Faculty, Čačak, Serbia

e-mail: zizovic@tfc.kg.ac.rs

Rates of divergence of real sequences

Mathematics Subject Classification (MSC): 54A20, 26A12, 40A05

Abstract. We consider different classes of divergent sequences of positive real numbers defined by using the quotient speed of divergence. In particular, we are concentrated on connection between classes of sequences defined in this way and selection principles and games.

Georgi Dimov

Department of Mathematics and Informatics,
Sofia University,
Sofia, Bulgaria
e-mail: gdimov@fmi.uni-sofia.bg

Open and other kinds of map extensions over zero-dimensional local compactifications
Mathematics Subject Classification (MSC): 54C20, 54D35; secondary 54C10, 54D45, 54E05

Abstract. Generalizing a theorem of Ph. Dwinger [3], we describe the partially ordered set of all (up to equivalence) zero-dimensional locally compact Hausdorff extensions of a zero-dimensional Hausdorff space. Using this description, we find the necessary and sufficient conditions which has to satisfy a map between two zero-dimensional Hausdorff spaces in order to have some kind of extension over arbitrary, but fixed, Hausdorff zero-dimensional local compactifications of these spaces; we consider the following kinds of extensions: continuous, open, quasi-open, skeletal, perfect, injective, surjective, dense embedding. In this way we generalize some classical results of B. Banaschewski [1] about the maximal zero-dimensional Hausdorff compactification. Extending a recent theorem of G. Bezhanishvili [2], we describe the Leader's local proximities [4] corresponding to the zero-dimensional Hausdorff local compactifications.

References

[1] B. Banaschewski, Über nulldimensionale Räume, Math. Nachr. 13 (1955) 129-140.

[2] G. Bezhanishvili, Zero-dimensional proximities and zero-dimensional compactifications, Topology Appl. 156 (2009) 1496-1504.

[3] Ph. Dwinger, Introduction to Boolean Algebras, Physica Verlag, Würzburg, 1961.

[4] S. Leader, Local proximity spaces, Math. Annalen 169 (1967) 275–281.

Georgi Dimov and Elza Ivanova
Department of Mathematics and Informatics,
Sofia University,
Sofia, Bulgaria
e-mail: elza@fmi.uni-sofia.bg

A new duality theorem for locally compact spaces
Mathematics Subject Classification (MSC): 54D45, 18A40; secondary 54E05, 06E10, 06E15

Abstract. In the paper [2], de Vries' Duality Theorem [1] was extended to the category **HLC** of locally compact Hausdorff spaces and continuous maps. The composition of the morphisms of the obtained there dual category is not the usual composition of maps. The same fact holds in the case of de Vries' Duality Theorem. We now obtain a new duality theorem for the category **HLC** such that the composition of the morphisms of the dual category is a natural one; however, the morphisms of the dual category are multi-valued maps.

References

[1] H. de Vries, *Compact Spaces and Compactifications, an Algebraic Approach*, Van Gorcum, The Netherlands, 1962.

[2] G. Dimov, A de Vries-type duality theorem for the category of locally compact spaces and continuous maps - I (submitted).

Tadek Dobrowolski

e-mail: tdobrowo@pittstate.edu

On a certain Dichotomy for Metric Linear Spaces and Consequences
Mathematics Subject Classification (MSC):

Abstract. We state two mapping properties, which may or may not hold in a given metric linear space. Consequences for the topological structure of convex subsets of the space will be discussed. In particular, the negation of the second property turns out to be equivalent to the existence of a compact convex subset without the extension property in the completion of the space.

Maria Dobrynina

Chair of General Topology and Geometry, Mechanics and Mathematics Faculty,
Moscow State University,
Moscow 119899, Russia.
e-mail: mary_dobr@mail.ru

On degree spectrums of seminormal functors
Mathematics Subject Classification (MSC): 54F99

Abstract. A covariant functor $\mathcal{F} : Comp \to Comp$, acting in the category $Comp$ of compact Hausdorff spaces and

their continuous mappings is said to be seminormal if it satisfies all the normality conditions in the sense of Shchepin [1] except epimorphism and preserving weight and preimages.

In 2008 A.V. Ivanov and E.V. Kashuba proved [2] that there exists a non-metrizable compact Hausdorff space, such that for any seminormal functor $\mathcal{F}$ preserving one-to-one points and of a degree spectrum $sp\mathcal{F} = \{1, k, ...\}$ the space $\mathcal{F}_k(X)$ is hereditary normal.

In this connection we prove the following propositions.

Proposition 1. *Let $\mathcal{F}$ be seminormal functor preserving one-to-one points and of a degree spectrum $sp\mathcal{F} = \{1, k, ...\}$. Then $k \leq 3$.*

Proposition 2. *Let $\mathcal{F}$ be seminormal functor with a degree $deg\mathcal{F} \leq 2$. Then $\mathcal{F}$ preserves one-to-one points.*

Recall that seminormal functor $\mathcal{F}$ *preserves one-to-one points*, if for any mapping $f : X \to Y$ and any point $y \in Y$ such, that $|f^{-1}(y)| = 1$, the mapping $\mathcal{F}(f) : \mathcal{F}(X) \to \mathcal{F}(Y)$ also satisfies the condition: $|(\mathcal{F}(f))^{-1}(y)| = 1$. *The degree spectrum $sp(\mathcal{F})$* of a functor $\mathcal{F}$ is a set of degrees of points in spaces of the form $\mathcal{F}_n(X)$.

References

[1] E.V. Shchepin, Functors and uncountable powers of compacta, Russian Math Surveys 36 (3) (1981) 1-71

[2] A.V. Ivanov, E.V. Kashuba, On hereditary normality spaces of the form $\mathcal{F}(X)$, Sib. Math. Journal 4 (2008) 813-824.

Szymon Dolecki
Mathematical Institute of Burgundy
Burgundy University

B.P. 47870, 21078 Dijon, France
e-mail: dolecki@u-bourgogne.fr

A general theory of preservation of properties by operations
Mathematics Subject Classification (MSC): 54D99

Abstract. Let $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ be classes of topologies (more generally convergences) and let φ be a binary operation. Under what conditions

$$\alpha \in \boldsymbol{A}, \beta \in \boldsymbol{B} \Longrightarrow \varphi(\alpha, \beta) \in \boldsymbol{C}?$$

Of course, only certain combinations of $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ and only special operations φ define a problem of interest. Although there is a general pattern enabling one to cope in theory with the problem, I shall restrict myself here to the case of $\varphi(\alpha, \beta) = \alpha \times \beta$. In this case, a natural assumption on the classes $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ is that they are concretely coreflective. If $\boldsymbol{A} = \boldsymbol{B} = \boldsymbol{C}$ were reflective, then $\alpha \times \beta \in \boldsymbol{A}$ for every $\alpha, \beta \in \boldsymbol{A}$.

Consider some special instances of this general problem:

(i) When is the product of two *Fréchet* topologies *Fréchet*?

(ii) When is the product of two *strongly Fréchet* topologies *strongly Fréchet*?

(iii) When is the product of two *sequential* topologies *sequential*?

It is known that if $\alpha \times \beta$ is Fréchet and either α or β admits a non-stationary convergent sequence, then $\alpha \times \beta$ is strongly Fréchet. This shows that the second example is, in a sense, more meaningful than the second. It was proved by Jordan and Mynard that the greatest class of topologies

$\boldsymbol{B}$ such that $\alpha \times \beta$ is (strongly) Fréchet for each strongly Fréchet α and $\beta \in \boldsymbol{B}$, is the class of *productively Fréchet* topologies. Solving a problem of Tanaka, Mynard proved that if the product of a sequential topology β is sequential for each topology of countable character α is sequential, then β is *strongly sequential*. By the bye, both productively Fréchet and strongly sequential were introduced as the solutions to these problems.

To solve the Tanaka problem, Mynard used his theorem on coreflectively modified duality, while for the problem on strongly Fréchet products, Jordan and Mynard proceeded indirectly through polarities of classes of filters.

In this talk I will show that there is a common method for the both these problems (and for many others),which is based on the mentioned *Mynard theorem*. Its corollary says that the *M-modified L-bidual* convergence $\mathrm{Epi}_M^L \xi$ of ξ is the coarsest convergence θ that fulfills

$$\theta \times M\tau \geq L\,(\xi \times \tau) \text{ for each convergence } \tau,$$

where M is a concrete functor and L is a concrete reflector. On applying the theorem above for $M = JV, L = J, \xi \geq V\xi$ and $\tau \geq V\tau$ where J is an idempotent reflector and V a coreflector commuting with finite products, we obtain a variety of classical and new results for appropriately chosen J and V. A calculation of Epi_M^L for particular L and M enables one to draw some meaningful conclusions.

For instance, Fréchet, strongly Fréchet, sequential and other topologies τ can be characterized with the the aid of functorial inequalities of the type

$$\tau \geq JV\tau,$$

where J is a suitable reflector and V a suitable coreflector. By the way, the class of τ that fulfill $\tau \geq M\tau$ for an arbitrary functor is always coreflective.

Szymon Dolecki, Francis Jordan, Frédéric Mynard
Mathematical Science, Georgia Southern University, POBox 8093, Statesboro, GA 30460, USA
e-mail: fmynard@georgiasouthern.edu

Consonance versus infraconsonance
Mathematics Subject Classification (MSC):

Abstract. A family $\mathcal{A}$ of open subsets of a topological space X is *compact* if it is closed under open supersets and has the property that whenever a union of open sets belongs to $\mathcal{A}$, finitely many of these open sets have a union in $\mathcal{A}$. Compact families are exactly the open sets for the Scott topology κ on the lattice $\mathcal{O}_X$ of open subsets of X ordered by inclusion.

A topological space is *consonant* if κ coincides with the topology τ_k on $\mathcal{O}_X$ with a basis formed by sets $\mathcal{O}(K) := \{O \in \mathcal{O}_X : K \subseteq O\}$ where K ranges over compact subsets of X. The topologies τ_k and κ induce topologies on the set $C(X)$ of real-valued continuous functions on X whose subbases are formed by sets of the form

$$[\mathcal{A}, U] := \{f \in C(X) : \exists A \in \mathcal{A} : f(A) \subseteq U\},$$

where $\mathcal{A}$ ranges among τ_k-open and κ-open sets respectively, and U ranges over $\mathbb{R}$-open sets. The former is the compact-open topology, while the latter is known as *Isbell topology.* Since these two topologies coincide if X is consonant, and the compact-open topology is a vector space topology on $C(X)$, consonance of X provides an obvious sufficient condition for the Isbell topology to be a group topology. It is not known however, if this condition is necessary, even though the fact that the Isbell topology is a group topology is characterized in terms of the formally weaker following condition:

A topological space is *infraconsonant* if for every compact family $\mathcal{A}$ there is another compact family $\mathcal{B}$ such that the (non necesarily compact) family $\mathcal{B} \vee \mathcal{B}$ of pairwise intersections of elements of $\mathcal{B}$ is included in $\mathcal{A}$. Combining results of [3], [2] and [1] and more recent results we have:

Theorem 1. *Let X be a completely regular topological space. The following are equivalent:*

(i) X is infraconsonant;

(ii) Addition is jointly continuous at the zero-function of $C(X)$ for the Isbell topology;

(iii) The Isbell topology on $C(X)$ is a group topology;

(iv) The Isbell topology on $C(X)$ is a vector-space topology;

(v) $\cap : \mathcal{O}_\kappa(X) \times \mathcal{O}_\kappa(X) \to \mathcal{O}_\kappa(X)$ defined by $\cap(A, B) = A \cap B$ is jointly continuous;

(vi) The Scott topology of $\mathcal{O}_X \times \mathcal{O}_X$ coincides with the product of the Scott topologies on $\mathcal{O}_X$ at (X, X).

There is however no example to date of a completely regular space that is infraconsonant but not consonant. Avenues to obtain such an example are discussed.

References

[1] S. Dolecki, F. Jordan, and F. Mynard, *Group topologies coarser than the Isbell topology*, to appear in Top. Appl.

[2] S. Dolecki and F. Mynard, *When is the Isbell topology a group topology?*, Top. Appl. **157**(2010), no. 8, 1370–1378.

[3] F. Jordan, *Coincidence of function space topologies*, Top. Appl. **157** (2010), no. 2, 336–351.

Alan Dow and Jerry Vaughan

Department of Mathematics and Statistics,
University of North Carolina at Greensboro,
P. O. Box 26170, Greensboro, NC 27402-6170 U.S.A.
e-mail: vaughanj@uncg.edu

Ordinal Remainders of ψ-spaces
Mathematics Subject Classification (MSC): 54D35, 54D40, 54D80, 54F05, 03E05

Abstract. Let κ be an infinite cardinal and let $[\kappa]^\omega$ denote the set of all countably infinite subsets of κ. A family $\mathcal{A} \subset [\kappa]^\omega$ is called an *almost disjoint family* (ADF) provided for every $A, A' \in \mathcal{A}$ if $A \neq A'$ then $A \cap A'$ is finite. An almost disjoint family $\mathcal{A}$ is called *maximal* (MADF) if $\mathcal{A}$ is not properly contained in any other almost disjoint family. For any ADF $\mathcal{A} \subset [\kappa]^\omega$, let $\psi(\kappa, \mathcal{A})$ denote the space with underlying set $\kappa \cup \mathcal{A}$ and with the topology having as a base all singletons $\{\alpha\}$ for $\alpha < \kappa$ and all sets of the form $\{A\} \cup (A \setminus F)$ where $A \in \mathcal{A}$ and F is finite. It is known and easy to prove that ψ-spaces are locally compact, Hausdorff, and that $\mathcal{A}$ is maximal if and only if $\psi(\kappa, \mathcal{A})$ is pseudocompact. Let $\mathfrak{c}$ denote the cardinality of the continuum. In this talk we discuss our theorem that states: $\mathfrak{c}^{++}$ is the first cardinal that satisfies the following statement: if $\kappa \geq \mathfrak{c}^{++}$ then there is no $\mathcal{A} \subset [\kappa]^\omega$ MADF such that the Stone-Čech remainder $\beta\psi(\kappa, \mathcal{A}) \setminus \psi(\kappa, \mathcal{A})$ is homeomorphic to an ordinal $\delta \geq \mathfrak{c}^{++}$ with the order topology.

Othman Echi

King Fahd University of Petroleum and Minerals,
Department of Mathematics and Statistics
PO Box 5046, Dhahran 31261, Saudi Arabia

e-mail: echi@kfupm.edu.sa othechi@yahoo.com and othechi@math.com

Discrete-Time Dynamical Spaces
Mathematics Subject Classification (MSC):
Primary: 54H20, 54B30 Secondary: 18A40

Abstract. Let X be a set and $f : X \longrightarrow X$ be a mapping. The graph of f is the binary relation $\{(x, f(x)) : x \in X\}$. The transitive closure of this graph defines a principal topology on X which may be given by

$$\mathcal{P}(f) := \{O \subseteq X \mid f^{-1}(O) \subseteq O\}.$$

A topological space $(X, \mathcal{T})$ will be called a *discrete-time dynamical space* if there is some mapping $f : X \longrightarrow X$ such that $\mathcal{T} = \mathcal{P}(f)$.

The main result of this talk provides an intrinsic topological characterization of discrete-time dynamical spaces.

Some illustrative examples are also given.

Shchepin Evgeny
Steklov Mathematical Institute,
Gubkina 8, Moscow, 119991, Russia
e-mail: scepin@mail.ru

On mean value of neighbors in plane partitions
Mathematics Subject Classification (MSC):

Abstract. The main result presented solves the following problem posed by V.Serdukov: what is the maximum possible mean number of neighbors in plane partition? Under "plane partition" we mean decomposition of plane into union of connected closed sets with disjoint interiors. Two elements of the partition are called neighbors if they boundaries meets more than in one point. It is proved that mean

number of neighbors in any finite plane partition is less than 6.

Vitaly V. Fedorchuk

Moscow State University
e-mail: vvfedorchuk@gmail.com

Finite dimensions via finite simplicial complexes
Mathematics Subject Classification (MSC):

Abstract. The following two theorems give us main characterizations of the Lebesgue dimension.

THEOREM A. *A normal space X satisfies the inequality* $\dim X \leq n \geq 0$ *if and only if for every sequence*

$$\left(F_1^1, F_2^1\right), \left(F_1^2, F_2^2\right), \ldots, \left(F_1^{n+1}, F_2^{n+1}\right)$$

of $n+1$ pairs of disjoint closed subsets of X there esist partitions P_i between F_1^i and F_2^i such that $\bigcap_{i=1}^{n+1} P_i = \emptyset$.

THEOREM B. *A normal space X satisfies the inequality* $\dim X \leq n \geq 0$ *if and only if every continuous mapping $f : F \to S^n$, where F is a closed subset of X, can be extended over X.*

Pairs $\left(F_1^i, F_2^i\right)$ from Theorem A are families Φ_i of sets such that their nerves $N(\Phi_i)$ coincide with the two point set $\{0, 1\}$ which is zero-dimensional simplicial complex. Changing the two point set by a finite simplicial complex K we get a definition of a dimension function K-dim.

The sphere S^n from Theorem B is homeomerphic to the join $\overset{n+1}{*} S^0$. Changing the sphere S^0 by an arbitrary compact polyhedron L we get a definition of a dimension function L-dim.

Dimensions K-dim and L-dim were introduced in [2]. Theory of dimension L-dim is a part of extension theory

introduced by A.Dranishikov in [1]. In [3] inductive dimensions K-Ind and L-Ind where introduced and investigated. Their definition are based on general notions of partitions. We investigate also complexes K and polyhedra L with the following properties:

1) L-dim$X < \infty \Rightarrow$ dim$X < \infty$;
2) K-Ind$X < \infty \Rightarrow$ Ind$X < \infty$;
3) L-Ind$X < \infty \Rightarrow$ Ind$X < \infty$.

References

[1] A.N.Dranishnikov, *Extension of mappings into CW-complexes*, Matem. Sb. **182:9** (1991), 1300–1310.

[2] V.V.Fedorchuk, *Finite dimensions modulo simplicial complexes and ANR-compacta*, Matem. Vesnik **61:1** (2009), 25–52.

[3] V.V.Fedorchuk, *Inductive dimensions modulo simplicial complexes and ANR-compacta*, to appear in Colloq. Math.

Leobardo Fernández and Sergio Macias

Av. Galvez y Fuentes 195, Col. Educacion, Coyoacan, 04400, Mexico D.F., Mexico.
e-mail: leobardof@ciencias.unam.mx

Some properties of Professor Jones' set function $\mathcal{K}$ on irreducible continua
Mathematics Subject Classification (MSC): 54C60, 54B20

Abstract. A *continuum* is a nonempty compact connected metric space. Given a continuum X, Professor Jones' *set function* $\mathcal{K}$ is defined as follows: for each subset $A \subseteq X$,

$\mathcal{K}(A) = \bigcap\{W \mid W$ is a subcontinuum of X

such that $A \subseteq Int(W)\}$.

A continuum X is *irreducible* if there exist two points of X such that no proper subcontinuum of X contains both points. A continuum X is $\mathcal{K}$-*symmetric* if for each pair of nonempty closed subsets A and B of X, $A \cap \mathcal{K}(B) \neq \emptyset$ if and only if $\mathcal{K}(A) \cap B \neq \emptyset$.

Some properties of the set function $\mathcal{K}$ will be presented. Also, we are going to give a characterization of irreducible $\mathcal{K}$-symmetric continua.

Manuel Fernandez-Martnnez and M.A. Sanchez-Granero
Area de Geometrna y Topologna
Universidad de Almerna (UAL)
04071 Almerna (Spain)
e-mail: fmm124@ual.es

Fractal Dimension on GF-spaces: a Hausdorff Approach
Mathematics Subject Classification (MSC): Primary 28A80; Secondary 54E35

Abstract. Fractals are a kind of sets which have received attention from different fields because of its many applications. Accordingly, they have been studied from different points of view, and in particular, topology provides some interesting tools in order to model them, like fractal structures. A fractal structure is a countable family of coverings of the whole space which approaches it by a discrete sequence of levels. Likewise, one of the main tools used in the study of such sets is the fractal dimension, through the so called box-counting and Hausdorff dimensions, which can be defined over a metrizable space. Never-

theless, fractals structures constitute a perfect place where a definition of fractal dimension can be given. On [2] we gave two approaches for a definition of fractal dimension on the context of generalized-fractal spaces, following the box-counting scheme. Indeed, we generalized box-counting dimension and classified a larger volume of spaces than by using the classical definition of fractal dimension.

In this talk, we are going to present a new model based on the Hausdorff scheme, in order to get a more accurate definition of fractal dimension on a GF-space. In this way, we find some interesting properties on the elements of a fractal structure in order to be able to calculate the new fractal dimension from those gave on [2]. We show that this model generalizes also the box-counting dimension as well as the fractal dimension models studied on [2], and we realize that though the new definition is inspired on the Hausdorff dimension, it equals to the box-counting dimension under certain conditions. On the other hand, another interesting question we study consists of determining the fractal dimension of self-similar sets. Our main theorem allows to calculate this quantity from an easy equation which only involves the contraction factors on the corresponding iterated function scheme without the open set condition hypothesis used in [1, Theorem 9.3].

References

[1] K. Falconer, *Fractal Geometry,* Mathematical Foundations and Applications, John Wiley & Sons, 1990.

[2] M. Fernandez-Martnnez and M.A. Sánchez-Granero, *Fractal Dimension for Fractal Structures,* preprint.

S. García-Ferreira

Instituto de Matemáticas, Universidad Nacional Autónoma de México, Campus Morelia, Apartado Postal 61-3, Santa María, 58089, Morelia, Michoacán, México
e-mail: sgarcia@matmor.unam.mx

Some dynamical properties of certain continuous functions of the Cantor set
Mathematics Subject Classification (MSC): Primary 54G20, 54D80, 22A99: secondary 54H11

Abstract. Given a dynamical system (X, f) with X a compact metric space and a free ultrafilter p on $\mathbb{N}$, we define $f^p(x) = p\text{-}\lim_{n\to\infty} f^n(x)$ for all $x \in X$, which is called the p-iterate of f. It was proved by A. Blass (1993) that $x \in X$ is recurrent iff there is $p \in \mathbb{N}^* = \beta(\mathbb{N})\setminus\mathbb{N}$ such that $f^p(x) = x$. This suggest to consider those points $x \in X$ for which $f^p(x) = x$ for some $p \in \mathbb{N}^*$, which are called p-recurrent. We shall give an example of a recurrent point which is not p-recurrent for several $p \in \mathbb{N}^*$. Also A. Blass prove that two points $x, y \in X$ are proximal iff there is $p \in \mathbb{N}^*$ such that $f^p(x) = f^p(y)$ (we say that x and y are p-proximal). We study the properties of p-proximal points of certain continuous functions of the Cantor set.

S. García-Ferreira and H. S. Pino-Villela

Instituto de Matemáticas (UNAM)
Apartado Postal 61-3
Xangari, 58089
Morelia, Michoacán
México
e-mail: pino@matmor.unam.mx

Filter convergence on Banach Spaces

Mathematics Subject Classification (MSC): 46B25, 46B45, 54A20, 03E05

Abstract. Given a free filter $\mathcal{F}$ on $\mathbb{N}$ and a topological space X, we recall that a sequence $(x_n)_{n\in\mathbb{N}}$ in X is $\mathcal{F}$-convergent to $x \in X$ if for every neighborhood U of x, $\{n \in \mathbb{N} : x_n \in U\} \in \mathcal{F}$. Using $\mathcal{F}$-convergent sequences in Banach spaces with Schauder basis we characterize P-filters$^+$, Q-filters$^+$ and selective$^+$ filters. By using $\mathcal{F}$-convergent sequences in ℓ_1, A. Aviles-Lopez, B. Cascales-Salinas, V. Kadets and A. Leonov characterized the P-filters$^+$ and the Q-filters$^+$.

References

[1] A. Aviles-Lopez, B. Cascales-Salinas, V. Kadets and A. Leonov, *The Schur ℓ_1 theorem for filters*, J.Math.Phys.Anal. Geom. 3,4 (2007), 383-398.

[2] S. García-Ferreira and M. Sanchis, *Ultrafilter-limit points in metric dynamical systems*, Comment.Math.Univ.Carolin. 48,3 (2007), 465-485.

S. García-Ferreira, M. Hrušák, and Ángel Tamariz-Mascarúa

Universidad Nacional Autónoma de México
e-mail: atamariz@servidor.unam.mx

Independent families and resolvability of pseudocompact dense subspaces of $\{0,1\}^\gamma$ and $[0,1]^\gamma$
Mathematics Subject Classification (MSC): 54E52, 54D25, 54A10, 54A35

Abstract. By using independent families, we prove that every pseudocompact dense subspace X of $\{0,1\}^\gamma$ is $\mathfrak{c}$-resolvable, where γ is an infinite cardinal number. We

give an example of a pseudocompact dense subspace X of $\{0,1\}^\gamma$ which is not maximally resolvable for $\gamma = 2^{2^{\omega_1}}$. Moreover, we give some sufficient conditions under which a dense Baire space of $[0,1]^\gamma$ is ω-resolvable. Finally, we provide an example of a countable space which is ω-resolvable but it is not extraresolvable.

Małgorzata Filipczak and Tomasz Filipczak
Faculty of Mathematics and Computer Science
University of Łódź
PL-90-238 Łódź, Banacha 22, POLAND
e-mail: fmalfilmath.uni.lodz.pl

On Δ_2 condition for density type topologies generated by functions
Mathematics Subject Classification (MSC): 24A10, 26A10

Abstract. Let $f : (0,\infty) \to (0,\infty)$ be a nondecreasing function with $\lim_{x\to 0^+} f(x) = 0$ and $\liminf_{x\to 0^+} \frac{f(x)}{x} < \infty$. The family $\mathcal{T}_f$ of all measurable sets $E \subset \mathbb{R}$ such that for each $x \in E$

$$\lim_{h\to 0^+} \frac{m((x,x+h)\setminus E)}{f(h)} = 0$$

and

$$\lim_{h\to 0^+} \frac{m((x-h,x)\setminus E)}{f(h)} = 0$$

is a topology. If $\limsup_{x\to 0^+} \frac{f(2x)}{f(x)} < \infty$, we say that f fulfills (Δ_2) condition, similar to the condition considered in the theory of Orlicz spaces.

If f fulfills (Δ_2) then the topology $\mathcal{T}_f$ is invariant under multiplication by nonzero numbers. Such topologies are

more convenient for examination and comparising. If $\mathcal{T}_f$ is included in the density topology $\mathcal{T}_d$ and invariant under multiplication, then f fulfills (Δ_2). For topologies bigger than $\mathcal{T}_d$ connections are a bit more difficult, but (Δ_2) condition is still a useful device.

Fomenko A.T.

Moscow State University (M.V.Lomonosov),
Department of Mathematics and Mechanics,
Leninskie Gory, Moscow, 119991, Russia
e-mail: fomenko@orc.ru

Singularities and bifurcations of integrable Hamiltonian systems
Mathematics Subject Classification (MSC): 37J35

Abstract. The talk is devoted to a new results in singularity and bifurcation theory of the integrable Hamiltonian systems. In case of two degrees of freedom the topology of a system and its solutions is determined by so called “atoms” which represent the transformations of 2-dimensional Liouville tori when they cross the critical values of momentum mapping.

The first part of the talk considers “atoms” as the regular (i.e. maximally symmetric) cell decompositions of closed oriented 2-dimensional surfaces. These objects are also known as maximally symmetric oriented atoms. An atom is called reducible if it is a branched covering over another atom, with branchings at the vertices and (or) side centres of the decomposition. The following two problems arose in the theory of integrable Hamiltonian systems: 1) describe irreducible maximally symmetric atoms; 2) describe all maximally symmetric atoms which cover a given irreducible maximally symmetric atom. The work by

A.T.Fomenko, E.A.Kudryavtseva and I.M.Nikonov solves these problems in important cases. As an application, all maximally symmetric oriented atoms of the following types are listed: 1) atoms having at most 30 edges; 2) having at most 6 sides; 3) having p or 2p edges, where p is a prime.

The second part of the talk is devoted to the topology of integrable systems on Lie algebras. Two objects mainly determine the properties of such a systems: bifurcation diagram S (for momentum mapping) and discriminant D of a spectral curve, which appears for the systems of L-A-pair type, in particular, whose integrals are obtained by an arguments shift method, introduced and developed by A.S.Mischenko and A.T.Fomenko. Recently A.Yu.Konyaev discovered, that for important series of a systems on semisimple Lie algebras, the sets S and D coincide. This important result explains many topological and algebraic effects connected with such systems.

Fomenko T.N.

oscow State University (M.V.Lomonosov),
Department of Computer Sciences and Cybernetics,
Leninskie Gory, Moscow, 119991, Russia
e-mail: tn-fomenko@yandex.ru

The stability of Cascade Search Principle

Mathematics Subject Classification (MSC): 54C60

Abstract. A non-negative real set-valued functional φ defined on a metric space (X, ρ) is called (α, β)-*search* if $0 < \beta < \alpha$ and for the one-valued functional $\varphi_*(x) := \inf\limits_{\gamma \in \varphi(x)} \{\gamma\}, x \in X$, the following conditions are fulfilled.
$\forall x, x \in X, \exists x' \in X$ such that $\rho(x, x') \leq \frac{\varphi_*(x)}{\alpha}$, and $\varphi_*(x') \leq \frac{\beta}{\alpha} \cdot \varphi_*(x)$.

In recent author's papers (see, for example, [2]) the Cascade Search Principle (below CSP) was suggested. CSP states that for any search functional there exists a multicascade (=set-valued discrete dynamic system) with its limit set being equal to the nil-subspace of that functional, and gives an upper estimation for the distance between any initial point and the correspondent limit points.

New search methods for coincidence points, common fixed points, common roots, common preimages of a closed subspace *of a finite set of* $n \geq 2$ *set-valued mappings* of metric spaces are based on the CSP.

In the report several results will be discussed concerning the stability of the CSP (see also [3, 4]). Two target settings are considered: weak and strong stability problems.

The target settings and the obtained results represent an essential development of some known results concerning the stability of approximation methods (Banach fixed point principle and some results of [1]).

References

[1] Arutyunov A.V. "The stability of coincidence points and properties of covering mappings". Matematicheskie Zametki, 2009, vol.86, No 2, p.163-169.

[2] T.N.Fomenko. "Cascade search principle and its applications to the coincidence problem of n one-valued or multi-valued mappings". Topology and its Appl., 2009, 157. p.760-773.

[3] Fomenko T.N. "Cascade seach stability", Izvestiya RAN, 2010, No 5.

[4] Fomenko T.N. "Cascade search: the stability of reachable limit points". Vestnik MGU, 2010, No 5.

U.C. Gairola and A.S. Rawat
Pauri Campus of H.N.B.Garhwal University,
Pauri 246001, India
e-mail: ucgairola@rediffmail.com

Common fixed points of mappings satisfying triangle inequality of integral type
Mathematics Subject Classification (MSC): 54H25, 47H10

Abstract. The main purpose of this paper/talk is to consider a new approach for obtaining common fixed point theorems in metric spaces by subjecting the triangle inequality to a contractive condition of integral type. We use the concept of property (E.A) and R-weak commutativity there, without the assumption of completeness of the space and the continuity of the maps. Our results generalize and extend the results of Pant et.al. [1], Rhoades [2] and others.

References

[1] V.Pant, S.Padaliya and R.P.Pant, Common fixed points of mappings not satisfying contractive condition, Fasciculi Mathematici, Nr40 (2008), 71-77.

[2] B.E.Rhoades, Contractive definitions and continuity, Contemporary Math. 72 (1998), 233-245.

Alexander A. Gaifullin
Moscow State University,
Faculty of Mechanics and Mathematics,
Department of Geometry and Topology,
Leninskie Gory, Moscow, 119991 Russia
e-mail: gaifull@higeom.math.msu.su

Sets of links of vertices of simplicial and cubic manifolds
Mathematics Subject Classification (MSC): 52B70, 57R95

Abstract. To each oriented (simplicial or cubic) closed combinatorial manifold one may assign the set (with repetitions) of isomorphism classes of links of its vertices. The obtained transformation $\mathcal{L}$ is the main object of the talk. We pose a problem on the inversion of the transformation $\mathcal{L}$. Thus the problem considered is the following. For a given set $Y_1, Y_2, \ldots, Y_k$ of oriented $(n-1)$-dimensional combinatorial spheres, does there exist an oriented (simplicial or cubic) n-dimensional combinatorial manifold K whose set of links of vertices coincides up to isomorphism with the given set $Y_1, Y_2, \ldots, Y_k$. It is easy to obtain a condition of *balancing* which is a necessary condition for the existence of such manifold K, that is, a necessary condition for a set of isomorphism classes of combinatorial spheres to belong to the image of the transformation $\mathcal{L}$. We shall give an explicit construction providing that each balanced set of isomorphism classes of combinatorial spheres gets into the image of $\mathcal{L}$ after passing to a multiple set and adding several pairs of the form $(Z, -Z)$, where $-Z$ is the sphere Z with the orientation reversed. We shall also discuss the relationship of the problem considered with N. Steenrod's problem on realization of cycles and the problem of finding local combinatorial formulae for the rational Pontryagin classes of triangulated manifolds.

References

[1] *A. A. Gaifullin*, The construction of combinatorial manifolds with prescribed sets of links of vertices, *Izvestiya RAN: Ser. Mat.* **72**:5 (2008), 3–62;

English transl., *Izvestiya: Mathematics* **72**:5 (2008), 845–899; preprint, *arXiv:* 0801.4741.

Ulises Ariet Ramos-García and Michael Hrušák
Instituto de Matemáticas, UNAM, Apartado Postal 61-3, Xangari, 58089, Morelia, Michoacán, México.
e-mail: ariet@matmor.unam.mx

Precompact Fréchet topologies on countable abelian groups
Mathematics Subject Classification (MSC): 22A05

Abstract. We give a characterization of Fréchet property among precompact topologies on countable Abelian groups in terms of a γ-set type property. Using this characterization we present the results of a systematic study of these topologies.

Constancio Hernandez Garcia
e-mail: chg@xanum.uam.mx

Concepts similar to realcompactness and Dieudonne completeness in topological groups
Mathematics Subject Classification (MSC):

Abstract. We show that a topological group G is topologically isomorphic to a a closed subgroup of a topological product of metrizable groups if and only if G is ω-balanced and G_δ-closed in ρG, the Raĭkov completion of G. Then we deduce that a topological group G is topologically isomorphic to a closed subgroup of a topological product of second countable groups if and only if G is ω-narrow and G_δ-closed in ρG. This allow us to shift the concepts of

Hewitt-Nachbin and Dieudonné completeness to the realm of topological groups

Some applications of these results to PT-groups and factorizable groups are also given.

S. Sajjad. Gashti and H. Sahleh
Islamic Azad University,
Branch Rasht, Iran
e-mail: sgashti@iaurasht.ac.ir

On the proper exact sequences of locally compact groups
Mathematics Subject Classification (MSC): 54H11, 54H15

Abstract. Let $\mathcal{L}$ be the category of locally compact groups, with continuous homomorphisms as morphisms. Groups will be written additively. A morphism is called *proper* if it is open onto its image. A sequence

$$E: \quad 0 \longrightarrow A \xrightarrow{p} E \xrightarrow{q} G \longrightarrow 0$$

of objects and morphisms in $\mathcal{L}$ is said to be *proper exact* at E if and only if $Im(p) = ker(q)$ and both p and q are proper morphism.

A topological Abelian group G is Pontryagin reflexive, or P-reflexive for short, if the natural homomorphism of G to its bidual group is a topological isomorphism. An intensive study has been done to find the class of locally compact Abelian groups which are P-reflexive. Let $Hom(G, A)$ be the set of all continuous homomorphism from G to A respect to E. Then by [6] $Hom(G, A)$ is an abelian topological group with respect to the compact-open topology under pointwise addition.

All groups discussed in this paper are metrizable locally compact groups. We will show that $Hom(G, A)$ is a connected, compactly generated, locally compact, injective G-module. On the other hand, we can conclude that by Pontryagin-van Kampan theorem, $Hom(G, A)$ is Pontryagin reflexive. Furthermore, we prove that there is an open continuous homomorphism between $Hom(G, A)$ and the Pontryagin dual of $Hom(G, A)$, that is, the group of all continuous homomorphisms of $Hom(G, A)$ into the compact group $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$ equipped with the compact- open topology.

References

[1] M. J. Chasco, *Pontryagin duality for metrizable groups*, Arch. Math. 70, number. 1(1998), pp. 22-28.

[2] D. Dikranjan, *Countably compact groups satisfying the open mapping theorem*, Topology and its Applications 98 (1999), pp. 81-129.

[3] S.Hernandez and V. Uspenskij, *Pontryagin duality for spaces of continuous functions*, J. Math. Anal. Appl. 242(2000), pp. 135-144.

[4] E. Hewitt and K. A. Ross, *Abstract Harmonic Analysis*, Academic press, New York, 1963.

[5] M. Moskowitz, *Homological Algebra in Locally Compact Abelian Groups*, Trans. Am. Math. Soc.,Vol. 127, No. 3(Jun 1967),pp. 341-356.

[6] L. Pontryagin, *Topological Groups*, Princton University Press, 1946.

D.N. Georgiou, S.D. Iliadis, and K.L. Kozlov

University of Patras, Department of Mathematics, Greece
e-mail: georgiou@math.upatras.gr

On some dimension invariants of spaces
Mathematics Subject Classification (MSC): 54B99, 54C25

Abstract. Using the dimensions I and d of spaces by normal bases it is possible to introduce dimenion invariants of spaces. In [1] two such dimension invariants which are called space dimension-like functions and which are denoted by s-b^n-Ind and s-b^n-dim, were introduced. However, in [1] these dimension invariants actually are not invesigated. The invariant s-b^n-Ind (under the notation I^w an other invariant denoted by I_0^w were studied in [2]. In the present talk, the dimension invariant s-b^n-dim denoted here by $d_{\min}$, will be investigated and two new dimension invariants, denoted by $\mathrm{d}_{\min}^{\mathrm{m}}$ and $\mathrm{d}_{\min}^0$, will be introduced and studied.

Work supported by the Caratheodory Programme of the University of Patras.

References

[1] S. D. Iliadis, *Universal spaces and mappings*. North-Holland Mathematics Studies, 198. Elsevier Science B.V., Amsterdam, 2005. xvi+559 pp.

[2] S. D. Iliadis, On the base dimension I and the property of universality, Appear in Topology and its Applications.

Dimitris Georgiou, Ioannis Kougias, and Athanasios Megaritis

University of Patras, Department of Mathematics, Greece
e-mail: megariti@master.math.upatras.gr

Borel structures for the set of Borel mappings
Mathematics Subject Classification (MSC): 54C35

Abstract. In [2] and [3] the authors tryied to generalize the results of R. Arens and J. Dugundji (see [1]) for Borel spaces. Unfortunately as R. J. Aumann observed in [2] the results of [1] are not true for Borel spaces, for example for some of the simplest Borel spaces it is impossible to defined a Borel structure on the set $\mathcal{B}(Y,Z)$ of all Borel maps of a Borel space Y into a Borel space Z such that the map $e : \mathcal{B}(Y,Z) \times Y \to Z$ with $e(f,y) = f(y)$ for every $f \in \mathcal{B}(Y,Z)$ and $y \in Y$ to be Borel. Even if we consider the discrete structure on $\mathcal{B}(Y,Z)$, then e will in general not be Borel. For this reason in [2] and [3] the authors studied subsets F of $\mathcal{B}(Y,Z)$ and Borel structures on F such that the restriction of the map e on $F \times Y$ to be Borel.

In this paper we study the above problem and we try to generalize the results of [1] for Borel spaces. Specially in Section 1 we give the preliminaries. In Sections 2 and 3 we give and study Borel $\mathcal{A}$-splitting and $\mathcal{A}$-admissible structures on $\mathcal{B}(Y,Z)$, where $\mathcal{A}$ is an arbitrary family of Borel spaces, and prove that there exists at most one Borel structure on $\mathcal{B}(Y,Z)$ which is both Borel splitting and admissible. When this structure exists, it coincides with the greatest Borel splitting structure, which always exists. Also, we give and study some special Borel structures on $\mathcal{B}(Y,Z)$. In Section 4 we give some remarks for Borel structures on $\mathcal{B}(Y,Z)$. In Section 5 we define and study some relations between the Borel structures of the set $\mathcal{B}(Y,Z)$ and the Borel structures of the set $\mathcal{B}_Z(Y)$ consisting of all subsets $f^{-1}(B)$ of Y, where $f \in \mathcal{B}(Y,Z)$ and B is an element of the Borel structure of Z, concerning the notions of Borel $\mathcal{A}$-splitting and Borel $\mathcal{A}$-admissible Borel structures. Finally, we give some open questions for Borel structures on

the set of Borel mappings.

Work supported by the Caratheodory Programme of the University of Patras.

References

[1] R. Arens and J. Dugundji, *Topologies for function spaces*, Pacific J. Math., Vol. 1, 1951, 5–31.

[2] Robert J. Aumann, *Borel structures for function spaces*, Illinois J. Math. 5, 1961, 614–630.

[3] B.V. Rao, *Borel structures function spaces*, Colloquium Mathematicum, Vol. XXIII, 1971, 33–38.

P.S. Gevorgyan

The Academy of Labour and Social Relations, Russia,
Moscow Power Engineering Institute, Russia.
e-mail: pgev@yandex.ru

On equivariant movability of topological groups
Mathematics Subject Classification (MSC): 55P55, 54C56

Abstract. In the classical shape theory it is proven that for a compact connected abelian group a movability is equivalent to a local connectivity (J. Keesling, [2]). In this report we consider an equivariant shape theory and investigate a property of equivariant movability of an acting group G. The following main result is established.

Theorem 1. *A compact topological group G with a countable base is equivariant movable if and only if it is a Lie group.*

In [1] we produced an example of a movable but not equivariant movable space. Theorem 1, in particular, gives new

examples of a movable but not equivariant movable spaces. Indeed, as was shown by J. Keesling [3], there are examples of compact connected abelian topological groups which are movable but not uniformly movable, and hence not Lie group. All these groups are not equivariant movable by Theorem 1.

References

[1] P.S. Gevorgyan, Some questions of equivariant movability, Glasnik Mat., 39(59) (2004), 185-198.

[2] J. Keesling, On movability and local connectivity, Lect. Notes math., 375 (1974), 158-167.

[3] J. Keesling, The Cech homology of compact connected abelian topological groups with application to shape theory, Lect. Notes math., 438(1975), 325-331.

Chris Good and Kyriakos B. Papadopoulos

School of Mathematics,
University of Birmingham,
B15 2TT UK
e-mail: kxp878@bham.ac.uk

Topological Characterizations of Ordinals
Mathematics Subject Classification (MSC): 54

Abstract. Van Dalen and Wattel show that a space is a LOTS (linearly ordered topoogical space), if and only if it has a T_1-separating subbase consisting of two *interlocking* nested collections of open sets.

Given a collection of subsets, $\mathcal{N}$, of a set X, van Dalen and Wattel define an order $\lhd_{\mathcal{N}}$ be declaring $x \lhd_{\mathcal{N}} y$ if and only if there is some $N \in \mathcal{N}$ such that $x \in N$ but $y \notin N$.

We examine $\lhd_{\mathcal{N}}$ in light of van Dalen and Watel's theorem.

We go on to give a topological characterization of ordinal spaces, including ω_1 in these terms nested collections.

Chris Good, Sina Greenwood, Brian Raines, and Casey Sherman
University of Auckland,
Private Bag 92019,
Auckland,
New Zealand
e-mail: sina@math.auckland.ac.nz

Homeomorphisms on continua
Mathematics Subject Classification (MSC): 54A10, 54C05, 54D05, 54D30, 54F15, 54H20

Abstract. Given a set X and a bijection $T : X \to X$, we examine what restrictions on the action of T will allow X to be a continuum and T continuous.

Sina Greenwood and Judy Kennedy
Department of Mathematics
PO Box 10047
Lamar University
Beaumont TX 77710
USA
e-mail: jakennedy1@my.lamar.edu

Pseudoarcs and generalized inverse limits
Mathematics Subject Classification (MSC): 54B10, 54C60, 54D05, 54H20

Abstract. Suppose that for each $i \geq 0$, I_i is an interval, and for each $i \geq 1$, P_i is a pseudoarc contained in $I_i \times I_{i-1}$ such that $\pi_{i-1}P_i = I_{i-1}$ and $\pi_i P_i = I_i$ (π_{i-1} and π_i denote the respective projections of P_i to the intervals I_{i-1} and I_i). Then for each $i \geq 1$, there is a surjective upper semicontinuous map $f_i : I_i \to 2^{I_{i-1}}$ such that $G(f_i) = P_i$. We prove that the generalized inverse limit space $\varprojlim(I_i, f_i)$:=$\{(x_0, x_1, \ldots) \in \Pi_{i=0}^{\infty} I_i$:for each $i \geq 1$, $x_{i-1} \in f_i(x_i)\}$ is totally disconnected. It may, however, contain isolated points.

Valentín Gregori, Samuel Morillas, and Almanzor Sapena

Instituto Universitario de Matemática Pura y Aplicada, Universidad Politécnica de Valencia, Spain
e-mail: vgregori@mat.upv.es

A class of completable fuzzy metric spacess
Mathematics Subject Classification (MSC): 54A40; 54D35; 54E50

Abstract. In this talk we study some aspects of the completion of a class of fuzzy metric spaces (in the sense of George and Veeramani) called strong. Then, we give a class of completable stationary fuzzy metrics which includes the class of stationary fuzzy ultrametrics.

Valentín Gregori and Almanzor Sapena

Instituto Universitario de Matemática Pura y Aplicada, Universidad Politécnica de Valencia, Spain
e-mail: alsapie@mat.upv.es

Metrics deduced from strong fuzzy metrics

Mathematics Subject Classification (MSC): 54A40; 54D35; 54E50

Abstract. In this talk we study some properties of a class of fuzzy metric spaces, in the sense of George and Veeramani, called strong, which includes the class of stationary fuzzy metrics. If a strong fuzzy metric M is also principal then we obtain a family of metrics which are compatible with the topology induced by M.

A. Gutek, S. Moshokoa, and M. Rajagopalan

e-mail: Moshosp@unisa.ac.za

Primitive shifts, Shifts and Compact Zero-Dimensional spaces
Mathematics Subject Classification (MSC): 54C35, 47B38

Abstract. The presentation focuses on primitive and primitive shifts on metrizable and non-metrizable spaces. In particular, we present results concerning shifts on scattered compact metric spaces, real shifts on compact metric spaces and zero-dimensional spaces. Finally, primitive shifts on non-metrizable spaces are discussed.

References

[A] J. Araujo, Examples and Counterexamples of Type I Isometric Shifts, preprint

[C] R. M. Crownover, Commutants of Shifts on Banach spaces, Michigan Math. J. 19. (1972), 233-247

[FV] F. D. Farid and K. Varadarajan, Isometric Shift operators on C(X), Canad. J. Math. 46 (1994), 532-542

[GJ] L. Gillman and M. Jerison, Rings of continuous functions, D. Van Nostrand Co., inc., Princetown-Toronto-London-New York, 1960

Gonçalo Gutierres

CMUC, Department of Mathematics, University of Coimbra, 3001-454 Coimbra, Portugal
e-mail: ggutc@mat.uc.pt

Pre-Lindelöf metric spaces in ZF
Mathematics Subject Classification (MSC): 54D20, 54E35, 03E25

Abstract. A metric space is *totally bounded* (also called pre-compact sometimes) if it has a finite ϵ-net for every $\epsilon > 0$ and it is *pre-Lindelöf* it has a countable ϵ-net for every $\epsilon > 0$. Using the *Axiom of Countable Choice*, one can prove that a metric space is topologically equivalent to a totally bounded metric space if and only if it is a pre-Lindelöf space if and only if it is a Lindelöf space.

In [1] it is studied the status of the Pre-Lindelöf property in the absence of the Axiom of Choice. In this talk, we follow that study and we also discuss what should be the right definition of pre-Lindelöfness in the choice-free environment.

References

[1] K. Keremedis, *The failure of the axiom of choice implies unrest in the theory of Lindelöf metric spaces*, Math. Log. Quart. 49(2003) 179–186.

M.R. Haddadi

Department of Mathematics,
Yazd University,
Yazd, Iran
e-mail: haddadi83@math.iut.ac.ir

Topology of Continuous Best Approximations
Mathematics Subject Classification (MSC):

Abstract. The existence of a continuous best approximation of a strictly convex space by a subset is shown to imply uniqueness of the best approximation under various assumptions on the approximating subset. For more general spaces, when continuous best exist, the set of best approximants to any given element is shown to satisfy connectivity and radius constraints.

References

[1] R. Huotari and W. Li, "Continuities of metric projections and geometric consequences", J. Approx. Theory 90 (1997), 319-339.

[2] W. Li, "Continuous Selections for Metric Projections and Interpolating Subspaces", Verlag Peter Lang, Frankfurt Main, 1991.

[3] E. H. Spanier, "Algebraic Topology", McGraw Hill, New York, 1966.

Sang-Eon Han
Faculty of Liberal Education, Institute of Pure and Applied Mathematics,
Chonbuk National University, Jeonju-City Jeonbuk, 561-756, Republic of Korea
e-mail: sehan@jbnu.ac.kr

A-isomorphism and Its Applications in Digital Topology
Mathematics Subject Classification (MSC): 54A10, 54C05, 55R15, 54C08, 54F65, 68U05, 68U10

Abstract. In this talk, we expand the notion of Khalimsky homeomorphism in the study of the Khalimsky 2D space. Further, we suggest its application in digital topology.

References

[1] P. Alexandorff, Diskrete Rume, *Mat. Sb.* **2** (1937) 501-518.

[2] S. E. Han, Non-product property of the digital fundamental group, *Information Sciences* **171**(1-3) (2005) 73-91.

[3] S.E. Han, Strong k-deformation retract and its applications, *Journal of the Korean Mathematical Society* **44**(6) (2007) 1479-1503.

[4] S.E. Han, Continuities and homeomorphisms in computer topology and their applications, *Journal of the Korean Mathematical Society* **45**(4)(2008) 923-952.

[5] S.E. Han, The k-homotopic thinning and a torus-like digital image in $\mathbf{Z}^n$, *Journal of Mathematical Imaging and Vision* **31** (1)(2008) 1-16.

[6] S.E. Han, Classification of Khalimsky topological spaces in terms of an A-isomorphism *Discrete Applied Mathematics*, submitted.

[7] E.D. Khalimsky, Applications of connected ordered topological spaces in topology, *Conference of Math. Department of Provoia*, (1970).

[8] E. Khalimsky, R. Kopperman, P. R. Meyer, Computer graphics and connected topologies on finite ordered sets, *Topology Appl.*, **36(1)**(1990) 1-17.

[9] A. Rosenfeld, Digital topology, *Am. Math. Mon.* **86** (1979) 76-87.

[10] J. Šlapal, Digital Jordan curves, *Topology Appl.*, **153** (2006) 3255-3264.

Jacek Hejduk
Faculty of Mathematics and Computer Science
University of Łódź
PL-90-238 Łódź, Banacha 22, POLAND
e-mail: hejdukmath.uni.lodz.pl

On the abstract density topologies
Mathematics Subject Classification (MSC):

Abstract. Let X be a nonempty set and $\mathcal{A}$ be a family of sets in 2^X and suppose that $\oplus : \mathcal{A} \to 2^X$ is an arbitrary operator.

If the family $\mathcal{T}_\Phi = \{A \in \mathcal{A} : A \subset \Phi(A)\}$ is a topology then we say that the topology $\mathcal{T}_\Phi$ is generated by the operator Φ.

Let $(X, \mathcal{S}, \mathcal{I})$ be a space with an algebra $\mathcal{S}$ in 2^X and proper ideal $\mathcal{I} \subset \mathcal{S}$. Let us assume that an operator $\Phi : S \to 2^X$ satisfies following conditions:

1^o $\Phi(\emptyset) = \emptyset, \qquad \Phi(X) = X,$

2^o $\underset{A,B\in\mathcal{S}}{\to} \forall \Phi(A \cap B) = \Phi(A) \cap \Phi(B),$

3^o $\underset{A,B\in\mathcal{S}}{\to} \forall A \triangle B \in \mathcal{I} \implies \Phi(A) = \Phi(B),$

4^o $\underset{A\in\mathcal{S}}{\to} \forall \Phi(A) \setminus A \in \mathcal{I}.$

This operator is "almost" lower density operator $(X, \mathcal{S}, \mathcal{I})$.

Theorem. *Let $\Phi : \mathcal{S} \to 2^X$ satisfies conditions $1^o - 4^o$. If the pair $(X, \mathcal{S})$ has the hull property then the family $\mathcal{T}_\Phi = \{A \in \mathcal{S} : A \subset \Phi(A)\}$ is a topology on X.*

Some properties of such topologies are the subject of the talk.

References

[1] M. Filipczak, T. Filipczak, On f-density topology, Topology and its Applications **155** (2008), 1980-1989.

[2] M. Filipczak, J. Hejduk, On topologies associated with the Lebesgue measure, Tatra Mt. Math. Publ. **28** (2004), 187-197.

[3] J. Hejduk, On the density topologies generated by functions, Tatra Mt. Math. Publ. **40** (2008), 133-141.

[4] J. Hejduk, On the abstract density topologies in measurable space, preprint University of Lodz (2009).

[5] J. Hejduk, G. Horbaczewska, On $\mathcal{I}$-density topologies with respect to a fixed sequence, Reports on Real Analysis (Rowy), 2003, 78-85.

[6] J. Hejduk, A. Loranty, On lower and semi-lower density operators, Georgian Math. J. **14** (2007), 661-667.

[7] J. Lukás, J. Malý, L. Zajiček, Fine topology methods, in: Real Analysis and Potential Theory, Lecture Notes in Math. 1189, Springer-Verlag, Berlin 1985.

[8] J. Oxtoby, Measure and Category, Springer-Verlag, Berlin 1987.

[9] M. Terepeta, E. Wagner-Bojakowska, Ψ-density topology, Rend. Circ. Mat. Palermo **48** (1999), 451-476.

[10] R. Wiertelak, A generalization of the density topology with respect to category, RAE **32** (2006-2007), 273-285.

[11] W. Wilczyński, Density topologies, Chapter 15 in Handbook of Measure Theory, Ed. E. Pap, Elsevier 2002, 675-702.

Rodrigo Hernńdez Gutiérrez, Alejandro Illanes, and Verónica Matínez-de-la-Vega
Instituto de Matemáticas, Circuito Exterior, Cd. Universitaria, México, 04510, MEXICO
e-mail: rod@matem.unam.mx

The problem of unique hyperspaces for Peano continua
Mathematics Subject Classification (MSC):
Primary: 54B20. Secondary: 54F15, 54F50

Abstract. A Peano continuum is a locally connected metric continuum. A continuum X is said to have unique hyperspace $C_n(X)$ if every time Y is a continuum with $C_n(X)$ homeomorphic to $C_n(Y)$, then X and Y are homeomorphic. In this talk, we will be concerned with the problem of determining which Peano continua X have unique hyperspace $C_n(X)$ for $n \in \mathbb{N}$.

Previous work has been done in this direction. In general terms, almost all finite graphs and a class $\mathcal{D}$ of dendrites have been shown to have unique n-fold hyperspaces (see [2], [3], [4], [5], [6], [7], [8]). On the other hand, Acosta and Herrera-Carrasco showed in [1] that dendrites not in class $\mathcal{D}$ do not have unique hyperspace of subcontinua.

Let $\mathcal{P}(X)$ be the set of points of a continuum X that do not have neighborhoods homeomorphic to a finite graph. A continuum X will be called *almost meshed* if the set $\mathcal{P}(X)$ is nowhere dense and *meshed* if it is almost meshed and has a basis of connected neighborhoods that cannot be disconnected by $\mathcal{P}(X)$.

In this talk, we will show how meshed and not almost meshed continua are opposite classes in the study of unique hyperspaces of Peano continua. Due to these results, the problem of unique hyperspaces of Peano continua remains

open only for the class of almost meshed but not meshed continua. Significant examples of this remaining class will also be presented.

References

[1] G. Acosta and D. Herrera-Carrasco, *Dendrites without unique hyperspace.* Houston J. Math. 35 (2009), no. 2, 451–467.

[2] G. Acosta, *Continua with unique hyperspace.* Continuum theory (Denton, TX, 1999), 33–49, Lecture Notes in Pure and Appl. Math., 230, Dekker, New York, 2002.

[3] D. Herrera-Carrasco, A. Illanes, M. de J. López, F. Macías-Romero, *Dendrites with unique hyperspace* $C_2(X)$. Topology Appl. 156 (2009), no. 3, 549–557.

[4] D. Herrera-Carrasco, F. Macás-Romero,*Dendrites with unique n-fold hyperspace.* Spring Topology and Dynamics Conference. Topology Proc. 32 (2008), Spring, 321–337.

[5] D. Herrera-Carrasco, *Dendrites with unique hyperspace.* Houston J. Math. 33 (2007), no. 3, 795–805.

[6] A. Illanes, *Dendrites with unique hyperspace* $C_2(X)$. *II.* Topology Proc. 34 (2009), 77–96.

[7] A. Illanes, *Finite graphs* X *have unique hyperspaces* $C_n(X)$. Proceedings of the Spring Topology and Dynamical Systems Conference. Topology Proc. 27 (2003), no. 1, 179–188.

[8] A. Illanes, *The hyperspace* $C_2(X)$ *for a finite graph* X *is unique.* Glas. Mat. Ser. III 37(57) (2002), no. 2, 347–363.

David Herrera-Carrasco and Fernando Macías-Romero

Facultad de Ciencias Físico-Matemáticas de la Benemérita Universidad Autónoma de Puebla. Avenida San Claudio y Río Verde, Ciudad Universitaria. San Manuel, C.P. 72570, Puebla Pue., México
e-mail: dherrera@fcfm.buap.mx

Subcontinua of n-Fold hyperspace Which are Hilbert Cubes
Mathematics Subject Classification (MSC): Primary: 54B20; 54F15, 54F50

Abstract. Let X be a nonempty compact, connected, metric space (a continuum), p in X, and n be a positive integer. Let $C_n(p, X)$ the hyperspace of the nonempty closed subset of X with at most n components which contain p. We consider that $C_n(p, X)$ is metrized by the Hausdorff metric. We show that $C_n(p, X)$ is often homeomorphic with the Hilbert cube. This work "generalize" the paper: C. Eberhart, *Intervals of continua which are Hilbert cubes,* Proc. Amer. Math. Soc., 68 (1978), 220–224.

References

[1] C. Eberhart, *Intervals of continua which are Hilbert cubes,* Proc. Amer. Math. Soc., 68 (1978), 220-224.

[2] J. B. Fugate, *Small retractions of smooth dendroits onto trees,* Fund. Math. 71 (1971), 256-262.

[3] D. Herrera-Carrasco and F. Macías-Romero, *Dendrites with unique n-fold hyperspace,* Topology Proc., 32 (2008), 321-337.

[4] A. Illanes and S. B. Nadler Jr., *Hyperspaces Fundamentals and Recent Advances*, Monographs and Textbooks in Pure and Applied Math., Vol. 216, Marcel Dekker, Inc., New York, 1999.

[5] J. van Mill, *Infinite-Dimensional Topology: Prerequisites and Introduction,* Elsevier (web server: North-Holland), North-Holland Mathematical Library, 1989.

[6] S. B. Nadler, Jr., *Hyperspaces of Sets*, Monographs and Textbooks in Pure and Applied Math., Vol. 49, Marcel Dekker, Inc., New York, 1978.

[7] S. B. Nadler, Jr., *Dimension Theory: An Introduction with Exercises*, Aportaciones Matemáticas Serie Textos 18, Soc. Mat. Mexicana, 2002.

[8] H. Torumczyk, *On CE-images of the Hilbert cube and characterization of Q- manifolds,* Fund. Math. 106, (1980), 31-40.

David Herrera-Carrasco and
Fernando Macías-Romero
Facultad de Ciencias Físico-Matemáticas de la Benemérita Universidad Autónoma de Puebla. Avenida San Claudio y Río Verde, Ciudad Universitaria. San Manuel, C.P. 72570, Puebla Pue., México
e-mail: fmacias@fcfm.buap.mx

Local Dendrites With Unique n-Fold Hyperspace

Mathematics Subject Classification (MSC): Primary 54B20. Secondary 54F15, 54F50

Abstract. Let Z be a metric continuum and n be a positive integer. Let $C_n(Z)$ be the hyperspace of the nonempty closed subsets of Z with at most n components. We talk about the following result: Let X be a local dendrite such that every point of X has a neighborhood which is a dendrite whose set of end points is closed and Z is any continuum such that $C_n(X)$ is homeomorphic to $C_n(Z)$ for some $n \geq 3$, then X is homeomorphic to Z.

References

[1] G. Acosta, *On compactifications of the real line and unique hyperspace*, Topology Proc. 25 (2000), 1-25.

[2] G. Acosta, J. J. Charatonik and A. Illanes, *Irreducible continua of type λ with almost unique hyperspace*, Rocky Mountain J. of Math. 31 (2001), 745-772.

[3] G. Acosta, R. Hernández-Gutiérrez and V. Martínez-de-la-Vega, *Dendrites and symmetric products*, Glas. Math. Ser. III 44 (2009) no. 1, 195-210.

[4] G. Acosta and D. Herrera-Carrasco, *Dendrites without unique hyperspace,* Houston J. Math. 35(2) (2009), 451-467.

[5] G. Acosta, D. Herrera-Carrasco and F. Macías-Romero *Local dendrites with unique hyperspace $C(X)$*, submitted to Topology Appl.

[6] R. Duda, *On the hyperspace of subcontinua of a finite graph,* I, Fund. Math. 62 (1968), 265-286.

[7] C. Eberhart, *Intervals of continua which are Hilbert cubes,* Proc. Amer. Math. Soc., 68 (1978), 220-224.

[8] D. Herrera-Carrasco, *Dendrites with unique hyperspace,* Houston J. Math., 33(3) (2007), 795-805.

[9] D. Herrera-Carrasco, A. Illanes, M. de J. López and F. Macías-Romero *Dendrites with unique hyperspace $C_2(X)$,* Topology Appl. 156 (2009), 549-557.

[10] D. Herrera-Carrasco, M. de J. López and F. Macías-Romero *Dendrites with unique symmetric products,* Topology Proc. 34 (2009), 175-190.

[11] D. Herrera-Carrasco and F. Macías-Romero *Dendrites with unique n- fold hyperspace,* Topology Proc. 32 (2008), 321-337.

Irena Hinterleitner and Josef Mikeš
UPalacky University Olomouc, CZ
e-mail: mikes@inf.upol.cz

On certain problem of geodesic and holomorphically projective mappings
Mathematics Subject Classification (MSC): 53B05, 53B21, 53B30, 53B35, 53C05

Abstract. The lecture is devoted to certain problems of geodesic and holomorphically projective mappings.

In the paper [1] it was proved that any manifold with affine connection is locally projective equiaffine, i.e. this manifold admits a geodesic mapping onto manifold with a symmetric Ricci tensor. Moreover, it is shown that these properties hold globally.

For this reason, the solution of the problem of the projective metrizability of a manifold A_n (or equivalently of geodesic mappings of A_n onto (pseudo-) Riemannian manifolds $\bar{V}_n$) can be realized as a geodesic mapping of the equiaffine manifold $\tilde{A}_n$, which is projectively equivalent to the given manifold A_n. These results was published in [2] and [3].

The following recent results were proved (cf. [3]):

$*$ Let a manifold $V_n(B)$, $B =$ const, admit a geodesic mapping f onto a complete manifold. If $V_n(B)$ is pseudo-Riemannian then f is affine. If $B \geq 0$ then f is affine.

$*$ Let a (hyperbolic) Kähler manifold $K_n[B]$, $B =$ const, admit a holomorphically projective mapping f onto a complete manifold. If $K_n[B]$ is pseudo-Riemannian then f is affine. If $B \geq 0$ then f is affine.

$*$ Let an Einstein manifold V_n admit a geodesic (or holomorphically projective) mapping f onto a complete Einstein manifold. If V_n is pseudo-Riemannian then f is affine.

If the scalar curvature $R \leq 0$ then f is affine.

References

[1] Mikeš, J.; Hinterleitner, I.; Kiosak, V., *On geodesic mappings of spaces with affine connection,* Acta Physica Debrecina, 42, 19-28, 2008.

[2] Mikeš, J.; Hinterleitner, I., *On fundamental equations of geodesic mappings,* Itogi Nauki Tekh., Ser. Sovrem Mat. Prilozh., Temat. Obz. 2010.

[3] Mikeš, J.; Vanžurová, A.; Hinterleitner, I., *On geodesic mappings and some generalizations,* Palacky University Press, Olomouc 2009.

G. R. Hiremath
University of North Carolina Pembroke, NC 28372
e-mail: gangadhar.hiremath@uncp.edu

A note on Moore spaces and metrizable spaces
Mathematics Subject Classification (MSC):

Abstract.

Definitions:

(i) A topological space is said to satisfy (*) if every countable closed discrete subset D in the space admits a locally finite open collection $\{G_d | d \in D\}$ such that $D \cap G_d = \{d\}$ for each $d \in D$.

(ii) A topological space X is said to satisfy C_2 if for each open cover ξ, the space admits a sequence of $\{\xi_n\}_n$ open covers such that for each $x \in X$ there is n such that $\overline{St(x, \xi_n)} \subset St(x, \xi)$.

(iii) A topological space X is said to satisfy semi-C_2 if for each semi-open cover ξ, the space admits a sequence of $\{\xi_n\}_n$ semi-open covers such that for each $x \in X$ there is n such that $\overline{St(x, \xi_n)} \subset St(x, \xi)$.

Following theorems are proved in this note

Theorem 1. A Hausdorff, second countable space is metrizable iff it satisfies (*).

Corollary 1.1. A Hausdorff, second countable wM -space is metrizable.

Corollary 1.2. A Hausdorff, second countable semimetrizable space is metrizable iff it satisfies (*).

Theorem 2. A Hausdorff , Lindelof, quasi-developable β-space that satisfies (*) is second countable and metrizable.

Theorem 3. A T_1 second countable space is C-semistratifiable.

Theorem 4. A topological space X is semidevelopable iff there is a function $d : X \times X \to [0, \infty)$ such that (i)

Michael Hrusak and
Osvaldo Alfonso Téllez Nieto
Universidad Nacional Autónoma de México
Apartado Postal 61-3 (Xangari)
C.P. 58089
Morelia, Michoacán México
e-mail: otellez@matmor.unam.mx

Some properties about extremally disconnected topological groups
Mathematics Subject Classification (MSC): 22A05

Abstract. The first example of an extremally disconnected topological group was given by Sirota in 1969 under the asumption of CH.

Let (G, τ) be the group of Sirota. Then for each continuous function f from G to the Cantor set, there is an open set $A \subseteq G$ such that $f[A]$ is a nowhere dense set.

We will talk about the relevance of this property on the extremally disconnected topological groups and another properties related to this kind of topological groups.

Jalal Hatem Hussein

Department of Mathematics, University of Baghdad.
e-mail: jalalintuch@yahoo.com

Weak and strong forms of simply-continuous functions
Mathematics Subject Classification (MSC): 54A05

Abstract. In this paper we introduce three classes of functions called Simply-continuous, Strong simply-continuous and Weak simply-continuous functions as generalization of continuous function. We obtain their characterizations, their basic properties and their relationships with other forms of generalization continuous functions between topological spaces.

References

[1] Neubrunnovia, 1975, On transfinite sequences of certain types of functions, Acta Fac. Rer. Natur. Univ. Com. Math., 30, 121- 126.

[2] Ganster M., Reilly I.L. and Vamanmurthy M.K., 1992. Remarks on Locally closed sets, Mathematical Pannonica, 3, 2, 107- 112.

[3] Levine N.L., 1963 semi-open sets and semi -continuity in topological space, Amer. math. Monthly, 70, 36-41.

[4] C. Chattopadhyay and C. Bandyopadhyay, 1991, On structure of δ-sets, Bull. Cal. Math. Soc. 83, 281-290.

[5] Crossly S.G. and Hildebrand S.K., 1971, Semi-closure, Texas J. Sic., 22, 99-112.

[6] Crossley, S.G. and Hildebrand, S.K. , 1974, Semi-topological properties, Fund. Math., 74 (), 233-254.

[7] Miguel Caldas Cueva, 2000, Weak and strong forma of irresolute maps Internat. J. Math. & Math. Sci.Vol. 23, No. 4, 253-259 Hindawi Publishing Corp.

[8] G. B. Navalagi , 2002, semi-precontinuous functions and properties of generalized semi-preclosed sets in topological spaces,IJMMS Internat. J. Math. & Math. Sci. Vol. 29, no 2, 85-98.

[9] M. K. Singal and A. R. Singal, 1968, Almost-continuous mappings, Yokohama Math. J. 15, 63-73.

[10] Radhi I.M.Ali and Jalal H. H., 2008, More Results On Simply -Open Sets, J. Col. Edu. No.3, 315-320.

[11] Maxim R. Burke, Arnold W. Miller, 2005, Models in which every nonmeager set is nonmeager in a nowhere dense Cantor set, Canadian Journal of Mathematics, 57, 1129-1144.

[12] M. Ganster, S. Jafari and G. B. Navalagi, 2002, On semi-g-regular and semi-g-normal spaces, Demonstration Math., 35, 2, 414-421.

[13] Di Maio and T. Noiri, 1987, On s-closed spaces, Indian J. Pure Appl. Math., 17, 3, 226-233.

[14] D.E. Cameron, 1978, Properties of S-closed spaces, Proc. Amer. Math. Soc., 72, 581-586.

[15] D. Andrijevic, 1986, Semi-preopen sets, Mat. Vesnik 38 (), no. 1, 24-32.

[16] M. H. Stone, 1937, Applications of the theory of Boolean rings to general topology, TAMS, 41, 375-381.

[17] P. Bhattacharyya and B. K. Lahiri, 1987, Semi-generalized closed sets in topology, Indian J.of Math. 29, 3, 375-382.

S.D. Iliadis

Lomonosov Moscow State University
e-mail: iliadis@math.upatras.gr

On hierarchies of Borel type sets
Mathematics Subject Classification (MSC):

Abstract. All considered spaces are assumed to be T_0-spaces of weight $\leq \tau$, where τ is a fixed infinite cardinal. By D^τ we denote the Cantor cub, that is the product $\Pi\{X_\delta : \delta \in \tau\}$, where $X_\delta = \{0, 1\}$ for each $\delta \in \tau$.

By a *class of subsets* we mean a class $\mathbb{P}$ consisting of pairs (Q, X), where Q is a subset of a space X (of weight $\leq \tau$). The *space-component* of a class $\mathbb{P}$ of subsets is the class of all spaces X such that there exists a subsets Q of X for which $(Q, X) \in \mathbb{P}$. A class $\mathbb{P}$ of subsets is said to be *hereditary* if for every subspace Z of a space Y a pair (Q^Z, Z) belongs to $\mathbb{P}$ if and only if there exists an element (Q^Y, Y) of $\mathbb{P}$ such that $Q^Z = Q^Y \cap Z$.

In [1] the notions of a *saturated class of subsets* and a *complete saturated class of a subsets* are given.

Theorem. *Let $\mathbb{P}$ be a hereditary complete saturated class of subsets and X an element of the space-component of $\mathbb{P}$*

containing topologically D^τ. Then, there exists a subset Q of X such that $(Q, X) \in \mathbb{P}$ and $(X \setminus Q, X) \notin \mathbb{P}$.

Let X be a space and $\alpha \in \tau^+$ an ordinal. The *multiplicative class* α (respectively, the *additive class* α) of subsets of X (see [1]), denoted here by $\Pi^\tau_\alpha(X)$ (respectively, by $\Sigma^\tau_\alpha(X)$), is defined simillarly to that of the case, where X is a metrizable space and τ is the first infinite cardinal ω. Instead of the countable intersections and countable sums it is considered intersections and sums of τ many members. The elements of the set $\Pi^\tau_\alpha(X) \cup \Sigma^\tau_\alpha(X)$ are called *Borel type sets of* X *of the class* α. It is easy to verify that $Q \in \Pi^\tau_\alpha(X)$ if and only if $X \setminus Q \in \Sigma^\tau_\alpha(X)$ and $\cup\{\Pi^\tau_\beta(X) \cup \Sigma^\tau_\beta(X) : \beta < \alpha\} \subset \Pi^\tau_\alpha(X) \cap \Sigma^\tau_\alpha(X)$. Note that the class of subsets consisting of all pairs (Q, X), where $Q \in \Pi^\tau_\alpha(X)$ (respectively, $Q \in \Sigma^\tau_\alpha(X)$) is a hereditary complete saturated class of subsets.

Corollary. *For every $\alpha \in \tau^+$ there exists a subset of the Cantor cub D^τ which is a Borel type set of the class α and it is not a Borel type set of the class less than α.*

The proof of the above result, even for the case $\tau = \omega$, is different of the classical one (given for example in the book *Topology* v.I by K. Kuratowski)

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References

[1] S.D. Iliadis, *Universal Spaces and Mappings*, North-Holland Mathematics Studies 198, Elsevier 2005, xvi+559.

Alexandr O. Ivanov and Alexey A. Tuzhilin

Chair of Differential Geometry and Applications, Faculty of Mechanics and Mathematics, Moscow State University named after M. V. Lomonosov, Moscow, 119991, GSP-1, Russia
e-mail: tuz@mech.math.msu.su, alexey.tuzhilin@yahoo.com

One-dimensional branched minimal fillings of finite metric spaces
Mathematics Subject Classification (MSC): 51F99, 52C99

Abstract. The Minimal Fillings Problem appeared in M. Gromov paper [1]. Consider a manifold M endowed with a metric ρ. A manifold X with metric d is called a *filling* of metric space $\mathcal{M} = (M, \rho)$, if M is the boundary of X and for any p, $q \in X$ we have: $\rho(p, q) \le d(p, q)$. The problem is to find the least possible value of fillings volumes for a given $\mathcal{M}$ and a special class of (X, d), e.g., when X is Riemannian manifold.

We consider a generalization of a particular case of this problem, namely, we suppose that $\mathcal{M} = (M, \rho)$ is a finite metric space, and X is a branched one-dimensional manifold, i.e., a connected graph. To introduce the metric d, we endow X with a non-negative weight function ω, i.e., a non-negative function on the edges of X, and define the distance $d(p, q)$ between vertices p and q of X as the least possible weight of paths in X joining p and q. A *minimal filling* is defined as a weighted graph (X, ω) of the least possible total weight, such that $\rho(p, q) \le d(p, q)$. In the presentation we discuss various properties of minimal fillings, demonstrate the connection with Steiner problem [2], [3], suggest some possible applications to investigation of Steiner Ratio [4], in particular, to Gilbert–Pollak Conjecture [5] on the Steiner ratio of Euclidean plane.

References

[1] M. Gromov, *Filling Riemannian manifolds*, J. Diff. Geom., 1983, 18, pp. 1–147.

[2] A. O. Ivanov and A. A. Tuzhilin, *Minimal Networks: The Steiner Problem and Its Generalizations.* CRC Press, N.W., Boca Raton, Florida, 1994.

[3] A. O. Ivanov and A. A. Tuzhilin, *Branching solutions to one-dimensional variational problems*, World Scientific, Singapore-New Jersey-London-Hong Kong, 2000.

[4] D. Cieslik, *Steiner Minimal Trees.* — Kluwer Academic Publishers, 1998.

[5] E. N. Gilbert and H. O. Pollak, *Steiner minimal trees*, SIAM J. Appl. Math., 1968, 16, No 1, pp. 1–29.

A.V. Ivanov

Petrozavodsk, Russia
e-mail: ivanov@petrsu.ru

Katetov's property for seminormal functors
Mathematics Subject Classification (MSC): 54B10, 54D15

Abstract. Let $\mathcal{F}$ be a siminormal functor in the category of compact spaces and continuous mappings. We say that $\mathcal{F}$ has a Katetov's property (K-property) if for any compact space X the hereditary normality of $\mathcal{F}(X)$ implies the metrizability of X. (The classical Katetov's theorem states that for any compact spaces X if X^3 is hereditarily normal then X is metrizable, i.e. the functor $()^3$ has K-property.)

In 1989 V.V.Fedorchuk proved [1] that each normal functor of degree ≥ 3 has the K-property.

Theorem 1. If $\mathcal{F}$ - seminormal functor of finite degree ≥ 4, then $\mathcal{F}$ has the K-property.

In paper [2] a non-metrizable compact space X_0 with hereditarily normal $\lambda_3(X_0)$ was constructed under CH (λ is the superextension functor). Hence λ_3 is an example of seminormal functor of degree 3 without K-property (under CH).

It turns out that X_0 is the general testing space for K-property.

Theorem 2 (CH). Let $\mathcal{F}$ be a seminormal functor of degree 3. $\mathcal{F}$ has the K-property iff $\mathcal{F}(X_0)$ is not hereditarily normal.

Theorem 3 (CH). Each seminormal functor of degree 2 has not the K-property.

References

[1] Fedorchuk V.V. "On the Katetov's cube theorem". Moskow Univ. Math. Bull., 44, No. 4, 102-106 (1989).

[2] Ivanov A.V., Kashuba E.V. "Hereditary normality of spaces of the form $\mathcal{F}(X)$" Sibirskii Matematicheskii Zhurnal, Vol. 49, No. 4, 813-824 (2008).

A. Jabbari and A. Zohri

Payame Noor University-Iran
e-mail: zohri_a@pnu.ac.ir

Topological quasi invertible elements in topological algebras
Mathematics Subject Classification (MSC): 46H05

Abstract. Topological invertible elements of metrizible algebras is studied in [1] and [5]. Also some results on quasi square roots is given in [2].

In this article we introduced the concept of topologically quasi invertibility of elements of topological algebras and get some new results. We say $a \in \mathcal{A}$ is topologically quasi invertible if there exists a sequence (a_n) in $\mathcal{A}$, such that $a_n \circ a \to 0$ and $a \circ a_n \to 0$. Also we proved that for every LMC algebra $\mathcal{A}$ topologically quasi invertiblity gives quasi invrtibility in $[\ell^\infty(\mathcal{A})]$ and vice versa.

References

[1] Akkar, M., Beddaa, A. and Oudadess, M., *Topologically invertible elements in metrizable algebras*, Indian J. Pure. Appl. Math., 27(2) (1996), 123-127.

[2] Ansari-Piri, E. and Zohri, A., *The nth roots and the quasi square roots in compelete metrizable fundamental topological algebras*, Fareast J. of Math. Sci. 28(3) (2008), 695-699.

[3] Balachandran, V. K., *Topological Algebras*, Narosa Publishing House, India, 1999.

[4] Bonsall, F. F. and Doncan, J., *Complete Normed Algebras*, Springer, New York, 1973.

[5] Thatte, A. D. and Bhatt, S. J., *On topolizing invertibility*, Indian J. Pure. Appl. Math., 15(12) (1984), 1308-1312.

S.J. Jesmani
Education of Mathematics, Farss, Niriz
e-mail: javdjesmani@yahoo.com

Fixed point and common fixed point in topological vector spaces
Mathematics Subject Classification (MSC):

Abstract. The purpose of this paper is to introduce and discuss the concept of fixed point and common fixed point

in the topological vector space. In this note, we shall generalize some results of fixed point theorems and common fixed point theorems of normed spaces to topological vector spaces.

References

[1] L.B. C. iric, Fixed point theorems in topological spaces, Fund. Math. (1975) 1-5.

[2] Z. Liu, Some fixed point theorems in compact Hausdorff spaces, Indian J. Math. 36 (3) (1994) 235-239.

Mikael Vejdemo-Johansson, Vin de Silva, and Dmitriy Morozov
Department of Mathematics
Stanford CA 94109-2125
USA
e-mail: mik@stanford.edu

Persistent cohomology and circular coordinates
Mathematics Subject Classification (MSC): 55-04

Abstract. In recently published work [1, 2], the authors provide algorithms for the computation of persistent cohomology, as well as an approach to produce circle-valued coordinate functions from the topological structure of a point cloud sampling a manifold.

We demonstrate the persistence approach to topological data analysis, and give examples from the general approach and into the specifics of intrinsic coordinate function recovery using cohomology.

References

[1] Dmitriy Morozov, Vin de Silva, and Mikael Vejdemo-Johansson, *Persistent Cohomology and Circular coordinates*,

Proceedings of SoCG 2009. [2] Vin de Silva, Dmitriy Morozov, and Mikael Vejdemo-Johansson, *Persistent Cohomology and Circular coordinates*; to appear: Discrete and Computational Geometry, 2010

H.R. Kamali
Islamic Azad University, Ardakan Branch
e-mail: *ham − kamali*@yahoo.com

Best Approximation in Topological Spaces
Mathematics Subject Classification (MSC):

Abstract. The purpose of this paper is to introduce and discuss the concept of best approximation in topology. In this note, we shall generalize some results of best approximation of normed spaces to topological spaces.

References

[1] V. Klee, Remarks on nearest points in normed linear spaces, in "Proceedings of a Colloquium on Convexity," pp. 168-176, University of Copenhagen, Copenhagen, 1966. BEST AND NEAR BEST APPROXIMATIONS 261.

[2] W. Li, "Continuous Selections for Metric Projections and Interpolating Subspaces," Verlag Peter Lang, Frankfurt-Main, 1991.

[3] I. Singer, "Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces," Springer-Verlag, Berlin, 1970. 9. D. R. Smart, "Fixed Point Theorems," Cambridge Univ. Press, Cambridge, UK, 1974.

[4] E. H. Spanier, "Algebraic Topology," McGraw-Hill, New York, 1966.

Alex Karassev, Murat Tuncali, and Vesko Valov

Nipissing University, North Bay, Canada
e-mail: alexandk@nipissingu.ca

Finite-to-one maps into manifolds and spaces with disjoint disks properties
Mathematics Subject Classification (MSC): Primary 54F45; Secondary 55M10

Abstract. A finite-dimensional space M has the parametric regularly m-branched maps property provided for every perfect surjection $f\colon X \to Y$ between finite-dimensional metric spaces the set of all f-regularly m-branched maps $g\colon X \to M$ is dense in $C(X, M)$. We prove that the parametric regularly m-branched maps property is a local property. We use this result to show that every manifold has such property. We also obtain applications for finite-to-one maps into certain products.

D. Koelzow

Erlangen, Germany
e-mail: koelzow@mi.uni-erlangen.de

Some Correlations of Dimension Theory with Analysis and other Fields
Mathematics Subject Classification (MSC):

Abstract. 1. The term 2n+1 in topology and analysis The fact that 2n+1 appears both in the embedding theorem of Noebeling and in the superposition theorem of Kolmogorov found its explanation in the the dimension theorem of Sternfeld, for compact metric spaces, after Ostrand had extended Kolmogorov's theorem to these spaces.

Comments and questions

No constructive proof of Kolmogorov's theorem seem to exist. Based on ideas of Kurkova, Nees found an approximative, but construc- tive version of this theorem, which only workes beyond the bound 2n+1. A dualization of the superposition theorem (for continuous functions) leads to the superposition of measures. For Lebesgue measures, a cuba- ture problem occurs. For Kolmogorow's as well as for Sternfeld's theorem, a generalization to non compact spaces spaces is unknown.

2. Some results on Fourier dimension and Salem sets (i.e. sets with equal topological and Hausdorff dimension).
a. Bluhm constructed a set with prescribed topological, Hausdorff and Fourier dimension, starting out from a random recursive construction of a Salem set of topological dimension 0 and prescribed Hausdorff dimension, and using Salem's theorem on the uniqueness of symmetric Cantor sets.
b. For positive alpha, Kaufmann gave a purely deterministic construction of a Salem set S(alpha), contained in the set E(alpha) of alpha-proximable reals, having the same Hausdorff dimension as E(alpha). His proof rests on the prime number theorem. Using this theorem iteratively, Bluhm echie- ved a Cantor like representation for S(alpha).
c. A lemma, used by Kaufmann for his theorem, was applied by Bluhm, itera- tively, for the construction of a Rajchman measure on the set of Liouville numbers, so proving that this is a set of multiplicity.

Questions

ad a. A deterministic construction of a Salem set of topological dimension 0 and prescribed Hausdorff dimension is still open.
ad b. The exact dimension problem for E(alpha) and

S(alpha) is unsettled.

Menachem Kojman, David Milovich, and Santi Spadaro
Department of Mathematics
Ben Gurion University of the Negev
Be'er Sheva
84105 Israel
e-mail: santi@cs.bgu.ac.il

Order-theoretic properties of bases in products
Mathematics Subject Classification (MSC): 54A25, 03E04

Abstract. Noetherian type is an ordered variant of weight that was introduced by Peregudov in the 90s. The Noetherian type of a topological space X is defined as the least cardinal κ such that X has a base $\mathcal{B}$ with the property that every element of $\mathcal{B}$ has at most $< \kappa$ many supersets in $\mathcal{B}$. Spaces with countable Noetherian type were studied by Balogh, Bennett, Burke, Gruenhage, Lutzer, Malykhin, Mashburn, Peregudov, Shapirovskii in different contexts and under different names, since the 70s. This cardinal function has some analogies with cellularity, especially in the class of homogeneous compacta.

The Noetherian type of a product of spaces never exceeds the product of their Noetherian types. Other than that, the behavior of this cardinal function in products is quite unpredictable. For example, Malykhin proved that raising a space to the power of its weight results in a space of countable Noetherian type.

Among other things, we will show:

- An example of two spaces having uncountable Noethe-

rian type, whose product has countable Noetherian type (the construction uses a result of Todorcevic about Tukey maps between partial orders).

- Some positive evidence to the conjecture that the Noetherian type of a (compact) space cannot decrease by taking its square.

- A ZFC bound on the Noetherian type of the countably supported topology on $2^{\aleph_\omega}$ (PCF scales are essential to get this bound. The existence of certain good PCF scales considerably improves the bound).

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Danuta Kołodziejczyk
Faculty of Mathematics and Information Science,
Warsaw University of Technology,
pl. Politechniki 1, 00-661 Warsaw, Poland
e-mail: dkolodz@mimuw.edu.pl

2-polyhedra for which every homotopy domination over itself is a homotopy equivalence
Mathematics Subject Classification (MSC): 55P55, 55P15

Abstract. We distinguish some classes of 2-dimensional polyhedra for which every homotopy domination over itself is a homotopy equivalence. It is was earlier known that all the polyhedra with polycyclic-by-finite fundamental groups have this property. Now we show that (among others) for 2-dimensional polyhedra P with soluble fundamental groups

$G = \pi_1(P)$ satisfying cd$G \leq 2$. As another corollary to our main result, we get the same for 2-dimensional polyhedra whose fundamental groups are knot groups. We also consider some related questions.

Angeliki Kontolatou and John Stabakis
Department of Mathematics,
University of Patras,
Athens, Greece
e-mail: kontolat@math.upatras.gr

Valuation Rings and Topology on a Skew Field by a Generalized Valuation
Mathematics Subject Classification (MSC):

Abstract. Given a non-commutative ring R which fulfils the *Ore conditions, a skew field* K is defined (*the field of fractions of* R). On R we define as *G-valuation*, a homomorphism w which fulfils the properties:

If $w(x) > \gamma$ and $w(y) > \gamma$, then $w(x+y) > \gamma$, $\gamma \in G_+^*$.

$w(x) = \infty$ iff $x = 0$ and $w(-1) = 0$, if the order of $w(-1)$ is not 2.

We remark that from the classical definition of a valuation the symbol $>$ has substituted the symbol $\geq$. We also remark that the last property gives that $w(x) = w(-x)$. On the other hand, we, always, have that the *G-valuation* has a unique extension on the field K.

This *G-valuation* gives another form to the field, permits to be defined the *open* sets rather than the *closed ones* for a topology and its *value group* is a partially ordered group which may have a *torsion.*

Theorem 1. *Let K be a skew field G-valuated by a G-valuation w, whose the value group G is an abelian po-group*

which is splitting and

$$G = G_0 \oplus \Gamma,$$

where G_0 is the torsion subgroup of G and its elements are parallel one to another and Γ is a group of representatives of the quotient group G/G_0. Then, the family

$$\mathbf{V}_\beta = (V_\beta(0))_{\beta \in G_+^*},$$

where

$$V_\beta(0) = \{x \in K | w(x) > \beta\}, \ \beta \in G_+^*$$

constitutes a neighborhood system of 0 for a topology. Moreover the same family is a neighborhood system for every point e of G_0, the set

$$\mathbf{V} = \cup \{\mathbf{V}_\beta | \beta \in G_+^*\}$$

is a topological ring and K is a topological field. Finally the set

$$C_0 = \{x \in K | w(x) \in G_0\}$$

is a topological group and the set

$$P = \{x \in K | w(x) \in G_+^*\}$$

is a maximal ideal of $\mathbf{V}$.

The afore G-valuation permits for a field K to be expressed under the form:

$$K = C_0 \oplus \Delta^* \oplus \overline{T}, \qquad (*)$$

where C_0 is the set such that $w(C_0) = \{0\}$, Δ^* is the family of sets C_e with $w(C_e) = \{e_i\}$, where the elements e_i, $i \in I$ are the elements of G_0 and the set $\overline{T}$ is isomorphic to the set Γ.

After *Szpilrajn* every po-set may be extended to a totally ordered set. In the present case we extend the set G/G_0 which leads to the analysis of K according to the type $(*)$.

We prove that in any step of the extension and for any $\alpha \in K^* = K \setminus \{0\}$, either $\alpha \in R$ or $\alpha^{-1} \in R$. Moreover for any $c \in K^*$ there holds $cRc^{-1} \subseteq R$, that is the ring R is an *invariant* and *total ring*, which means that it is a *valuation ring*.

Eva Kopecká

Institut of Analysis, Kepler University, A-4040 Linz, Austria
e-mail: eva@bayou.uni-linz.ac.at

Extending Lipschitz mappings continuously

Mathematics Subject Classification (MSC): Primary: 54C20; Secondary: 46C05

Abstract. Let X be a Euclidean space and B its closed unit ball. For a closed subset A of B let $L(A)$ be all the contractions, that is 1-Lipschitz mappings from A to X. We consider them as a subset of the continuous mappings from A to X endowed with the supremum norm.

According to Kirszbraun's theorem, every $f \in L(A)$ admits an extension to B which has the same Lipschitz constant as f. The extension is, however, in general not unique.

We construct a *continuous* mapping from $L(A)$ to $L(B)$ which assigns to each $f \in L(A)$ an extension having the same Lipschitz constant as f. We also show that the mapping assigning each $f \in L(B)$ its restriction to A is open. As a corollary, the Lipschitz isometries which preserve the

length of the intersection of any curve with A are residual in $L(A)$.

K. L. Kozlov

Faculty of Mechanics and Mathematics,
Moscow State University,
Moscow 119991, Russia
e-mail: kkozlov@mech.math.msu.su

Rectangularity of products and extensions of actions

Mathematics Subject Classification (MSC):

Abstract. The extension of action $\alpha : G \times X \to X$ from a space X onto a space Y in which X is dense is one of the central questions in the equivariant topology. Each Dieudonné complete extension $\tilde{X}^{\mathcal{U}}$ of X can be obtained as the completion of X with respect to some uniformity $\mathcal{U}$. One of the sufficient conditions in case we want to extend action on $\tilde{X}^{\mathcal{U}}$ may be formulated in the following way.

The action $\alpha : G \times X \to X$ can be extended to the action $\tilde{\alpha} : G \times \tilde{X}^{\mathcal{U}_X} \to \tilde{X}^{\mathcal{U}_X}$ if there is a uniformity $\mathcal{U}_G$ on G such that the action α is uniformly continuous with respect to uniformities $\mathcal{U}_G \times \mathcal{U}_X$ on $G \times X$ and $\mathcal{U}_X$ on X.

For the completions which correspond to the functors of the Stone-Čech compactification β, the Hewitt realcompactification ν or the Dieudonné completion μ the action can always be extended to the mapping from $\beta(G \times X)$ to βX, from $\nu(G \times X)$ to νX or from $\mu(G \times X)$ to μX. Thus, the distribution of the functors β, ν or μ with the operation of taking product is sufficient to extend action on βX, νX or μX respectively.

Definition. An open subset $U \times V$ of $X \times Y$ is a *cozero-set rectangle* if U and V are cozero-sets in X and Y respectively.

(Z. Frolík) A mapping $f \in C^*(X \times Y)$ satisfies the *rectangle condition* if for any $\epsilon > 0$ there is a finite cover w of $X \times Y$ by cozero-set rectangles such that $\mathrm{osc}_W f < \epsilon$ for any $W \in w$. The product is said to satisfy *rectangle condition* if every mapping $f \in C^*(X \times Y)$ satisfies the rectangle condition.

(A. Chigogidze) The product $X \times Y$ is *strongly rectangular* if every countable normal open cover of $X \times Y$ is refined by a countable cover consisting of cozero-set rectangles.

(B. Pasynkov) The product $X \times Y$ is *rectangular* if every normal cover of $X \times Y$ is refined by a σ-locally finite cover consisting of cozero-set rectangles;

The following theorem is due to T. Hoshina and K. Morita in case of μ (T.Proselkova proved it for countable products), to T.Proselkova in case of ν (A. Chigogidze proved the sufficiency of the strong rectangularity for the fulfillment of the equality $\nu(X \times Y) = \nu X \times \nu Y$) and to Z. Frolík in case of β.

Theorem. *A product space $X \times Y$ is rectangular (strongly rectangular, satisfies the rectangle condition) iff $\mu(X \times Y) = \mu X \times \mu Y$ and $\mu X \times \mu Y$ is rectangular ($\nu(X \times Y) = \nu X \times \nu Y$ and $\nu X \times \nu Y$ is strongly rectangular, $\beta(X \times Y) = \beta X \times \beta Y$ and $\beta X \times \beta Y$ satisfies the rectangle condition).*

Theorem. If the product $G \times X$ is rectangular (strongly rectangular, satisfies the rectangle condition) then any action $\alpha : G \times X \to X$ can be extended on μX (νX, βX).

The situation when the product $G \times X$ is rectangular (strongly rectangular, satisfies the rectangle condition) will be discussed.

Boris Kruglikov
Department of Mathematics and Statistics,
University of Tromsø, Tromsø, 9037, Norway
e-mail: boris.kruglikov@uit.no

Topology of attractors of dynamical systems

Mathematics Subject Classification (MSC): 37J35, 37B40, 37C45

Abstract. In this talk I refer to the known dichotomy of integrability vs. chaos in deterministic dynamics [7, 2] and briefly discuss how it influences topology/dimension of the phase space.

I start with the question how integrability of Hamiltonian systems can restrict topology of the manifold [1, 5, 8]. For instance, our joint result with V.Matveev [3] implies that geodesic flows with "good" quadratic integrals can exist only on rationally elliptic manifolds.

On the other end I indicate that the chaotic properties can be influenced by dimensional characteristics of dynamics [6]. Namely presence of negative Lyapunov exponents can force breakdown of the topological Ruelle-Margulis inequality. This is manifested by examples, from our joint work with M.Rypdal [4], of contracting maps with simple attractor but positive entropy. I will discuss dimensional restrictions on attractors provided some information on expansion and singularities.

References

[1] A. V. Bolsinov, A. T. Fomenko, *Integrable Hamiltonian systems. Geometry, topology, classification*, Chapman & Hall, 2004.

[2] A. V. Bolsinov, I. A. Taimanov, *Integrable geodesic flows with positive topological entropy*, Invent. Math. **140**, no. 3 (2000), 639–650.

[3] B. S. Kruglikov, V. S. Matveev, *Strictly non-proportional geodesically equivalent metrics have* $h_{top}(g) = 0$, Ergod. Th. & Dynam. Sys. **26**, no. 1 (2006).

[4] B. Kruglikov, M. Rypdal, *"A piece-wise affine contracting map with positive entropy"*, Discrete Contin. Dyn. Syst. **16**, no.2, 393-394 (2006).

[5] G. Paternain, *On the topology of manifolds with completely integrable geodesic flows - II*, Journ. Geom. Phys. **123**(1994), 289–298.

[6] Y. Pesin, *"Dimension theory in dynamical systems"*, Chicago Lect. in Math. Ser., The University of Chicago Press (1997).

[7] I. Prigogine, I. Stengers, *Order out of Chaos: Man's new dialogue with nature*, Flamingo (1984).

[8] I. A. Taimanov, *Topology of Riemannian manifolds with integrable geodesic flows*, Proc. Steklov Inst. Math. **205** (1995), 139–150.

P. Krupski

Mathematical Institute, University of Wrocław
Pl. Grunwaldzki 2/4, 50–384 Wrocław, Poland
e-mail: Pawel.Krupski@math.uni.wroc.pl

Hyperspaces of separators of Euclidean cubes

Mathematics Subject Classification (MSC): 57N20

Abstract. Let $\mathcal{M}_1^2$ denote the absolute small Borel class of all subsets of the Hilbert cube I^ω which are intersections of G_δ-sets with F_σ-sets. It turns out that, for $n \geq 3$, the hyperspace $\mathcal{S}(I^n)$ of all closed $(n-1)$-dimensional separators of the cube I^n is an $\mathcal{M}_1^2$-absorber in the hyperspace 2^{I^n} of all nonempty closed subsets of I^n. Similarly, $\mathcal{S}(I^n) \cap C(I^n)$

is an $\mathcal{M}_1^2$-absorber in the hyperspace $C(I^n)$ of all subcontinua of I^n.

Jan Kubarski

Institute of Mathematics,
Technical University of Łódź, Poland
e-mail: jan.kubarski@p.lodz.pl

Linear direct connections in Lie groupoids, their underlying linear connections and characteristic classes

Mathematics Subject Classification (MSC): 57R20, 58H05, 53C05

Abstract.

1. Teleman's results

N. Teleman in the papers [T1], [T2], shows how the Chern character of the tangent bundle of a smooth manifold can be obtained from the geodesic distance function $r : M \times M \to [0, \infty)$ by means of cyclic homology. For this purpose, we take a cut-off smooth monotone decreasing real valued function χ, equal identically to 1 on a neighbourhood of 0, and of sufficiently small support so that $\chi \circ r^2$ is well defined and smooth. For sufficiently close points $x, y \in M$ the linear mapping $A\,(x, y) : T_y M \to T_x M$ given by the formula $A\,(x, y) \left(\sum_i \xi^i \frac{\partial}{\partial y^i} \right) = \sum_{i,j,k} \xi^i \frac{\partial^2 \left(\chi \circ r^2 \right)(x,y)}{\partial x^i \partial y^j} g^{jk}\,(x) \frac{\partial}{\partial x^k}$ is independent of the local coordinates and has the following properties: (a) $A\,(y, x)$ is an isomorphism, i.e. $A\,(y, x)$ is an element of the Lie groupoid $GL\,(TM)$ of all linear isomorphisms between fibres of the tangent bundle TM, (b) $A\,(x, x)$ is the identity.

Then the function $\Phi_k : U_{k+1} \to \mathbb{R}$, where U_{k+1} is a neighbourhood of the diagonal in M^{k+1}, is defined by

$$\Phi_k(x_0, x_1, ..., x_k) := Trace\ A(x_0, x_1) \circ A(x_1, x_2) \circ ...$$
$$... \circ A(x_{k-1}, x_k) \circ A(x_k, x_0).$$

Teleman shows that the function Φ_k is a cyclic cycle over the algebra $\mathcal{A} = C^\infty(M)$ and Φ_k determines (by the Connes' isomorphism) a closed differential form $\Omega(\Phi_k)$. The main results is: the top degree component of the cyclic homology class of Φ_k is equal to $[\Omega(\Phi_{2k})] = c \cdot Ch_k(M)$ where c is a constant and $Ch_k(M)$ is the k-component of the Chern character of the tangent bundle of M.

The mapping $A : U \to GL(TM)$, $(x, y) \longmapsto A(x, y)$, defined above [for (x, y) from some neighborhood U of the diagonal $\Delta = \{(x, x);\ x \in M\}$ is a so-called linear direct connection (=linear quasi-connection), it is a function defined in an open neighborhood of the diagonal satisfying two conditions (a) and (b) above.

2. Kubarski-Teleman's results

In the paper [K-T] we construct the "infinitesimal part" ∇ of any direct connection A and show that in this way we obtain a usual linear connection. We next determine the curvature tensor R of this linear connection and show that $\Omega(\Phi_{2k}) = c \cdot Tr\ R^k$.

As an application of these results, we present a direct proof of N. Teleman's theorem, which had shown that it was possible to represent the Chern character of smooth vector bundles as the periodic cyclic homology *class* of a specific periodic cyclic cycle, manufactured from a direct connection, rather than from a smooth linear connection as the Chern - Weil construction does. In addition, we show that the image of the cyclic cycle into the de Rham cohomology (through the A. Connes' isomorphism) coincides with the *cycle* provided by the Chern - Weil construction applied to the underlining linear connection.

3. Groupoids' point of view

N.Teleman in [T2] said: "*The arguments discussed here may be extended to the language of groupoids*".

We give here the Lie groupoid's point of view on linear direct connections and characteristic classes. We will show much more, namely that for each principal fibre bundle (equivalently for a Lie groupoid), the primary characteristic classes can be recovered from any direct connection in the Lie groupoid.

Let Φ be an arbitrary transitive Lie groupoid with the anchor α and the target β. We denote by u_y the unit of Φ at y. By a *linear direct connection* in Φ we mean a mapping $A : (M \times M)_{|U} \to \Phi$, where $U \subset M \times M$ is an open neighborhood of the diagonal $\Delta = \{(x,x)\,;\ x \in M\}$, such that $\alpha\left(A(x,y)\right) = y$, $\beta\left(A(x,y)\right) = x$, and $A(x,x) = u_x$. We will show how you can get the Chern homomorphism of the Lie groupoid Φ with the direct connection A. Let us fix a point y, and let us take the submanifold $\Phi_y = \alpha^{-1}(y) \subset \Phi$ and the mapping $A(\cdot, y) : M \to \Phi_y$, $x \longmapsto A(x,y)$. It is a smooth mapping such that $\beta \circ A(\cdot, y) = \mathrm{id}$. Taking the differential $A(\cdot, y)_{*y} : T_yM \to T_{u_y}(\Phi_y)$ we define the splitting of the Atiyah sequence of Φ, i.e. a usual connection in the Lie algebroid $A(\Phi) = u^*(T^\alpha\Phi)$, $\nabla^A : TM \to u^*(T^\alpha\Phi) = A(\Phi)$, $\nabla^A(v_y) = A(\cdot, y)_{*y}(v_y)$,

$$0 \to \mathbf{g} \to u^*(T^\alpha\Phi) \underset{\underset{\nabla^A}{\longleftarrow}}{\to} TM \to 0.$$

The connection ∇^A will be called the **underlying linear connection of the linear direct connection** A. Consider the curvature tensor $\Omega^A \in \Omega^2(M;\boldsymbol{g})$ of ∇^A,

$$\Omega^A(X,Y) = \left[\left[\nabla^A_X, \nabla^A_Y\right]\right] - \nabla^A_{[X,Y]}.$$

The linear direct connection A determines the mapping

$$\Psi^A_k : \Big(\underbrace{M \times ... \times M}_{k+1}\Big)_{|U} \to \Phi,$$

$$\Psi^A_k(x_0, x_1, ..., x_k) = A(x_0, x_1) \cdot A(x_1, x_2) \cdot ... \\ ... \cdot A(x_{k-1}, x_k) \cdot A(x_k, x_0)$$

having the values in the associated **Lie group bundle,**

$$\Psi^A_k(x_0, x_1, ..., x_k) \in \Phi^{x_0}_{x_0}.$$

For $k = 2$, the function

$$\Psi^A_2 : (M \times M \times M)_{|U} \to \Phi,$$
$$\Psi^A_2(x_0, x_1, x_2) = A(x_0, x_1) \cdot A(x_1, x_2) \cdot A(x_2, x_0),$$

is called the *curvature* of A. Analogously to the previous cases we can associate some **differential form** $\Omega\left(\Psi^A_k\right) \in \Omega^k(M;\boldsymbol{g})$ to the function Ψ_k by

$$\Omega\left(\Psi^A_k\right)(x) = \frac{1}{k!}\sum_{i_1,i_2,..,i_k} \frac{\partial}{\partial x_1^{i_1}}\frac{\partial}{\partial x_2^{i_2}}\frac{\partial}{\partial x_k^{i_k}} \\ \Psi^A_k(x_0, x_1, ..., x_k)_{x_0=x_1=...=x_k=x}\, dx^{i_1} \wedge ... \wedge dx^{i_k}.$$

The fundamental role is played by the following

Theorem 1 *For an arbitrary linear direct connection* $A : (M \times M)_{|U} \to \Phi$ *in the Lie groupoid* Φ*, the curvature form of* A *and the curvature form of the underlying connection in* $A(\Phi)$ *differ by a constant*

$$\Omega\left(\Psi^A_2\right) = \frac{1}{4} \cdot \Omega^A.$$

This theorem is the basis for solving the problem of describing the characteristic classes of the Lie groupoid Φ (equivalently, of the principal fibre bundle) via any direct connection A on the level of differential forms. We use a mechanism to designate the characteristic classes of a Lie groupoid using the Chern-Weil homomorphism of a Lie algebroid, which is presented in [K].

References

[K] J. Kubarski, *The Chern-Weil homomorphism of regular Lie algebroids*, UNIVERSITE CLAUDE BERNARD – LYON 1, Publications du Départment de Mathématiques, nouvelle série, 1991, 1-70.

[K-T] J.Kubarski, N.Teleman, *Linear direct connections,* Banach Center Publications, Vol. 76, Warszawa 2007, pp. 425-436.

[T1] N.Teleman, *Distance Function, Linear quasi-Connections and Chern Character,* June 2004, IHES/M/04/27.

[T2] N.Teleman, *Direct Connections and Chern Character,* Proceedings of the International Conference in Honor of Jean-Paul Brasselet, Luminy, May 2005.

Hans-Peter A. Künzi and
Charly Makitu Kivuvu
University of Cape Town, 7701 Rondebosch, South Africa
e-mail: hans-peter.kunzi@uct.ac.za

Filter pairs on quasi-uniform spaces
Mathematics Subject Classification (MSC): 54E15

Abstract. In the following the set of all filter pairs on a quasi-uniform space $(X, \mathcal{U})$ will be equipped with its stan-

dard partial order: A filter pair $\langle \mathcal{F}_1, \mathcal{G}_1 \rangle$ on X is called *coarser* than the filter pair $\langle \mathcal{F}_2, \mathcal{G}_2 \rangle$ provided that $\mathcal{F}_1 \subseteq \mathcal{F}_2$ and $\mathcal{G}_1 \subseteq \mathcal{G}_2$. Over the years many different kinds of filter pairs on quasi-uniform spaces $(X, \mathcal{U})$ have been studied by Deák, Doitchinov, Kopperman, Romaguera and others. We next recall some of the more important properties: A pair of filters $\langle \mathcal{F}, \mathcal{G} \rangle$ on the set X is called a *Cauchy filter pair* on $(X, \mathcal{U})$ provided that for each $U \in \mathcal{U}$ there are $F \in \mathcal{F}$ and $G \in \mathcal{G}$ such that $F \times G \subseteq U$. A Cauchy filter pair is said to *converge to* $x \in X$ provided that the neighborhood filter pair $\langle \mathcal{U}^{-1}(x), \mathcal{U}(x) \rangle$ is coarser than $\langle \mathcal{F}, \mathcal{G} \rangle$. A filter pair $\langle \mathcal{F}, \mathcal{G} \rangle$ on X is called *linked* provided that $F \cap G \neq \emptyset$ whenever $F \in \mathcal{F}$ and $G \in \mathcal{G}$. A Cauchy filter pair on X is called *minimal* provided that there is no Cauchy filter pair coarser than it. A Cauchy filter pair on X is called *stable* provided that for each $U \in \mathcal{U}$, $\bigcap_{G \in \mathcal{G}} U(G) \in \mathcal{G}$ and $\bigcap_{F \in \mathcal{F}} U^{-1}(F) \in \mathcal{F}$. A Cauchy filter pair $\langle \mathcal{F}, \mathcal{G} \rangle$ on X is called *symmetric* provided that $\langle \mathcal{F}, \mathcal{G} \rangle$ is a Cauchy filter pair, too. A Cauchy filter pair $\langle \mathcal{F}, \mathcal{G} \rangle$ on X is called *weakly concentrated* provided that for each $U \in \mathcal{U}$ there is $V \in \mathcal{U}$ such that $V(x) \in \mathcal{G}$ and $V^{-1}(y) \in \mathcal{F}$ imply that $(x, y) \in U$. Numerous properties of quasi-uniform spaces can be defined in terms of these conditions: For instance Deák called a quasi-uniform space $(X, \mathcal{U})$ *Cauchy* provided that whenever $\langle \mathcal{F}_1, \mathcal{G}_1 \rangle$ and $\langle \mathcal{F}_2, \mathcal{G}_2 \rangle$ are Cauchy filter pairs on X such that the pairs $\langle \mathcal{F}_1, \mathcal{F}_2 \rangle$ and $\langle \mathcal{G}_1, \mathcal{G}_2 \rangle$ are linked, then $\langle \mathcal{F}_1 \cap \mathcal{F}_2, \mathcal{G}_1 \cap \mathcal{G}_2 \rangle$ is a Cauchy filter pair on X, too. Among other things, he proved that each totally bounded Cauchy quasi-uniformity is a uniformity. Our talk surveys the literature on filter pairs and complements work done by various authors.

Krystyna Kuperberg and Andrzej Trybulec
Department of Mathematics and Statistics,
Auburn University.
Auburn AL 36830, USA
e-mail: kuperkm@auburn.edu

Computer-assisted proofs: Mizar
Mathematics Subject Classification (MSC): 68T15

Abstract. Mizar is a computer system verifying mathematical proofs translated to or written in the Mizar language. Andrzej Trybulec, the founder of Mizar, leads the project http://mizar.uwb.edu.pl/ .

The Mizar Mathematical Library (MML) is the world's largest repository of formalized and computer-checked mathematics. Over two hundred authors contributed to the MML, which at the present includes almost ten thousand definitions and over fifty thousand theorems with complete proofs. The Nagata-Smirnow metrization theorem (Mizar proof by Karol Pąk, 2004) and the Brouwer fixed point theorem (Mizar proof by Artur Kornilowicz and Yasunari Shidama, 2005) serve as examples.

The goal of this talk is to describe Mizar possibilities and advances towards formalization of dynamical systems.

Raja M. Latif, M. Caldas , S. Jafari, and T. Noiri
Department of Mathematics and Statistics
King Fahd University of Petroleum and Minerals
Dhahran 31261 Saudi Arabia
e-mail: raja@kfupm.edu.sa

On G. Λ_π^{gp}-Sets, Λ_π^{gp}-Closure Operator and the Associated Topology $\tau^{\Lambda_\pi^{gp}}$
Mathematics Subject Classification (MSC):
Primary: 54D30, 54A05; Secondary: 54H05, 54G99

Abstract. In this talk we introduce the concept of Λ_π^{gp}-sets (resp. V_π^{gp}-sets) which is the intersection of πgp-open (resp. union of πgp-closed) sets and investigate the notions of generalized Λ_π^{gp}-sets and generalized V_π^{gp}-sets in a topological space (X, τ). Also we define a new closure operator and thus a new topology $\tau^{\Lambda_\pi^{gp}}$ on (X, τ) by using generalized Λ_π^{gp}-sets and generalized V_π^{gp}-sets and shall examine some of the properties of this new topology.

References

[1] H. Maki, Generalized Λ-sets and the associated closure operator, *The special Issue in commemoration of Prof. Kazuada IKEDA's Retirement*, (1986), 139–146.

[2] Noiri T., Maki H, Umehara J. Generalized preclosed functions, *Mem. Fac. Sci. Kochi Univ. Ser. A (Math)*, 19 (1998),13–20.

[3] Park, On $\pi gp-$closed sets in topological spaces, Indian J. Pure Appl. Math., (In press)

[4] Park J.H, Park J.K, On $\pi gp-$continuous functions in topological spaces, *Chaos, Solitons and Fractals*, 20 (2007), 467-477.

[5] Stone M.H., Applications of the theory of Boolean rings to general topology, *Trans. Amer. Math. Soc.*, 41 (1937), 375-481.

[6] Zaitsev V., On certain classes of topological spaces and their bicompactifications, *Dokl Akad Nauk SSSR*, 178 (1968), 778-9.

Sami Lazaar and Othman Echi
Faculty of Sciences of Tunis. University Tunis-El Manar.
"Campus Universitaire" 2092 Tunis, TUNISIA.
e-mail: salazaar72@yahoo.fr

F-Door Spaces And F-Submaximal Spaces
Mathematics Subject Classification (MSC): 54B30, 54D10, 54F65, 46M15

Abstract. Let X be a topological space and F a covariant functor from **TOP** to itself. X is said to be F-door (resp., F-submaximal) if its F-reflection is door (resp., submaximal).

In this paper T_0-door (resp., T_0-submaximal) and ρ-door (resp., ρ-submaximal) are characterized.

References

[1] A.V. Arhangel'skii and P.J. collins, *On Submaximal Spaces*, Topology and its applications **64** (1995), 219 – 41.

[2] G. Bezhanishvili, L. Esakia and D. Gabelaia, *Some results on modal axiomatization and definability for topological spaces*, Studia Logica **81** (2005), 325 – 55.

[3] K. Belaid, O. Echi, S. Lazaar, *$T_{(\alpha,\beta)}$-spaces and the Wallman Compactification*, Int. J. Math. Math. Sc. (2004) **68**, 3717 – 3735.

[4] N. Bourbaki, *Eléments de Mathématiques, Topologie Générale Chapitres* 1 à 4 , 1990.

Jimmie Lawson
Louisiana State University
Baton Rouge, LA 70803
e-mail: lawson@math.lsu.edu

Multidimensional Barycenters in NPC-metric Spaces
Mathematics Subject Classification (MSC): 54E50, 53C70

Abstract. The purpose of this presentation is to consider a weak metric notion of nonpositive curvature (NPC) in a complete metric space X, namely

$$d(x\#y, z) \leq (1/2)(d(x, z) + d(y, z))$$

for all x, y, z, where $x\#y$ denotes some distinguished metric midpoint of x and y, and to derive in this setting a natural method of defining barycenters of finite subsets. This leads in turn to the definition of barycenters of probability measures on X and to a strong law of large numbers for an i.d.d. sequence of random variables into X.

Anna Loranty and Ryszard J. Pawlak

Faculty of Mathematics and Computer Science,
Łódź University,
Banacha 22, 90-238 Łódź, Poland
e-mail: rpawlak@math.uni.lodz.pl

On the entropy of discrete dynamical systems in (generalized) topological spaces
Mathematics Subject Classification (MSC): 54C70; 54C60; 54A05; 54C08; 54H25; 54C05; 54B20

Abstract. In the 1990s Á. Császár generalized the notion open set and therefore it became possible to consider generalized topology: A family $\mathcal{G} \subset \exp X$ is called generalized topology (GT) on X if $\emptyset \in \mathcal{G}$ and any union of its elements belongs to $\mathcal{G}$. In 2002 it was shown that every GT in X can be generated by monotonic map $g : \exp X \to \exp X$. In 2007 the Chinese mathematician J. Li pointed out that the

above facts are useful in research connected with approximation spaces widely considered in the theory of computer science (in the 1980s). Simultaneously, recently a lot of papers are connected with mutual correspondence between basic properties of discrete dynamical systems of functions, suitable multivalued functions and some maps (entropy, orbits, fixed points, chaos, transitivity, etc.). In our lecture we will connect both directions of investigations mentioned above.

The starting point of our talk will be Li observation, which will lead us to the considerations connected with discrete dynamics of some operators in very poor topological structure. Consequently, the basic tools for our considerations will be notions connected with suitably defined matrices. Supported by this base we will consider problems connected with the entropy of maps, multifunctions and functions both in generalized topological spaces, as also compact metric spaces and even unit interval. The consideration in this lecture will be concentrated on the properties of (topological) entropy. At this opportunity we will refer to the (generalization of) Vietoris topology, fixed points, orbits, properties of conjugate functions (via suitable homeomorphism) and variation of functions mapping unit interval into itself.

References.

[1] J. Andres, P. Šnyrychowá, P. Szuca *Sharkovskii's Thorem for connectivity G_δ - realtions,* Int. J. Bifurkation Chaos 16 (2006), 2377-2393.

[2] J. Andres, L. Jüttner, *Perid three plays a negative role in a multivalued version of Sharkovskii's theorem*, Nonlinear Analysis 51 (2002), 1101-1104.

[3] J. Banks, *Chaos for induced hyperspace maps*, Chaos,

Solitions & Fractals 25 (2005), 681-685.

[4] Á. Császár, *Generalized open set*, Acta Math. Hung. 75(1-2), (1997), 65-87.

[5] Á. Császár, *Generalized topology, generalized continuity*, Acta Math. Hung. 96 (4) (2002), 351-357.

[6] Á. Császár, *Separation axioms for generalized topologies*, Acta Math. Hung. 104 (1-2) (2004), 63-69.

[7] Á. Császár, *On generalized neighbourhood systems*, Acta Math. Hung. 121 (4) (2008), 395-400.

[8] Á. Császár, *Mixed constructions for generalized topologies*, Acta Math. Hung. 122 (1-2) (2009), 153-159-357.

[9] A. R. Khan, *Properties of fixed point set of a multivalued map*, J. Appl. Math. Stoch. Anal. 3 (2005), 323-331.

[10] J. Li, *Generalized topologies generated by subbases*, Acta Math. Hung. 114 (1-2) (2007), 1-12.

[11] L. Górniewicz, *Topological fixed point theoryfor multivalued mappings*, Kluwe Publ. Dordrecht (1999).

[12] R. Gu, *Kato's chaos in set valued discrete systems*, Chaos, Solitions & Fractals 31 (2007), 765-771.

[13] X. Ma, B. Hou, G. Liao, *Chaos in hyperspace system*, Chaos, Solitions & Fractals 40 (2009), 653-660.

[14] S. B. Nadler Jr., *Periodic points of multivalued ε-contractive maps*, Topol. Methods in Nonlinear Analysis, Journal of the Juliusz Schauder Center 22 (2003), 399-409.

[15] J. S. C. Pena, G. S. López, *Topological entropy for induced hyperspace maps*, Chaos, Solitions & Fractals 28 (2006), 979-982.

[16] R. J. Pawlak, *On the entropy of Darboux functions*, Coll. Math. 116.2 (2009), 227-241.

[17] R. J. Pawlak, A. Loranty, A. Bąkowska, *On the topological entropy of continuous and almost continuous function*, to appear.

[18] Z. Pawlak, *Rough sets*, International Journal of Computer and Information Science 11 (5) (1982), 341-356.

[19] Z. Pawlak, *Rough classification*, International Journal of Man-Machine Studies 20 (1984), 469-483.

[20] J. A. Pomykała, *Approximation operations in approximation space*, Bull. of the Polish Academy of Sci. Mathematics, Theoretical Computer Science 15 (9-10) (1987), 653-662.

S. Louvros and I. Kougias

Department of Telecommunication Systems and Networks
Technological Educational Institute of Messolonghi
Greece
e-mail: splouvros@gmail.com; kougias@teimes.gr

LTE Planning Optimization based on Queueing Modelling and Network Topology Principles
Mathematics Subject Classification (MSC):

Abstract. In LTE networks the allocation of appropriate resources to handset users is essential for interference reduction. Interference is produced by the number of handset users in the same cell and also in surrounding cells. Admission algorithm is responsible for the teletraffic allocation, looking forward the blocking probability schemes and also the capacity reservation per user and service. Using queueing models, several metrices could be calculated, such as the probability that a call could be blocked, average number of subcarriers allocated and bandwidth - capacity

considerations and, a decision could be provided by the Admission control algorithm. Network topology is also a critical parameter for the network planners and affects the Admission decisions and the overall QoS. In this paper a mathematical analysis is performed by exctracting an analytical model for the decision of Admission control algorithm, in order to be used as an efficient tool for network planners.

R. Lowen and W. Rosiers
Department of mathematics and Computer Science,
University of Antwerp,
Middelheimlaan 1, 2020 Antwerpen, Belgium
e-mail: wannes.rosiers@ua.ac.be

A convergence-wise look at Lipschitz spaces
Mathematics Subject Classification (MSC): 18B10, 18C15, 18C20, 54A20, 54E99

Abstract. The characterization of the category Ap via approach systems [3] is well known. Recently this notion is linked to the convergence of so called (prime) functional ideals [5].
A Lipschitz space is a set X with a Lipschitz system $(\mathcal{L}(x))_{x\in X}$ [2]. In this talk we will give another characterization by introducing (prime) Lipschitz ideals and describing their convergence.
It is a well-known fact that the category Ap is the category of lax algebras for the prime functional ideal monad [4]
In [6] Pisani gave a proof that the category Top is the category of lax algebras for the ultrafilter monad using the characterization of the interior of subsets. The category Ap can be characterized by an interior of functions. Then we are able to adapt the proof of Pisani and give another

proof that the category Ap is the category of lax algebras for the prime functional ideal monad.
In this talk I will give some preliminary results on describing the category Lip as the category of lax algebras for the (prime) Lipschitz ideal monad. I will do this by characterizing a Lipschitz space by an interior of functions as well. We will finish with adapting the proof of Pisani and state where the last unsolved problem in this setting occurs.

References

[1] E. Colebunders, R. Lowen, W. Rosiers: *Lax Monads and Algebras via Initial Monad Morphisms*

[2] R.B. Fraser, Jr.: *Axiom Systems for Lipschitz Structures*, Fundamenta Mathematicae LXVI, 1969

[3] R. Lowen: *Approach Spaces: The Missing Link in the Topology-Uniformity-Metric Triad*, Oxford Mathematical Monographs, 1997

[4] R. Lowen, T. Vroegrijk: *A New Lax Algebraic Chararcterization of Approach Spaces*

[5] R. Lowen, C. Van Olmen, T. Vroegrijk: *Functional Ideals and Topological Theories*

[6] C. Pisani: *Convergence in Exponentiable Spaces*, Theory and Applications of Categories, Vol. 5, No. 6, pp. 148-162

Sergio Macias and Sam B. Nadler, Jr.

Insituto de Matematicas, UNAM Circuito Exterior, Ciudad Universitaria, Mexico D. F., C. P. 04510 Mexico.
e-mail: sergiom@matem.unam.mx

On hereditarily decomposable continuum
Mathematics Subject Classification (MSC): 54C60

Abstract. A continuum is a nonempty, compact, connected, metric space. A continuum X is decomposable provided that X can be written as the union of tow of its nondegenerate subcontinua. The continuum X is hereditarily decomposable if each of its nondegenerate subcontinua is decomposable.

A continuum X is homogeneous provided that for each pair of points x and y of X, there exists a homeomorphism h of X onto X such that h(x)=y.

Professors J. Krasinkiewicz and P. Minc asked, independently: Is the simple closed curve the only nondegenerate hereditarily decomposable homogeneous continuum? We present some partial answers to this question.

Vladimir V. Marchenko

Russian University of Peoples' Friendship,
Moscow, Russia
e-mail: wmarchenko@rambler.ru

Rational shape type of 1-shape-connected spaces
Mathematics Subject Classification (MSC):

Abstract. Consider a connected pointed topological space $(X,*)$ with a polyhedral resolution $\underline{p} = (p_\lambda)\colon X \to \underline{X} = ((X_\lambda,*), p_{\lambda\lambda'}, \Lambda)$ in the sense of S. Mardešić. We also suppose that $(X,*)$ is 1-shape-connected, i. e. $pro\text{-}(\pi_1(X_\lambda,*)) = 0$.

Using results of Š. Ungar we may assume that all X_λ are connected and 1-connected CW-complexes.

We consider the functor $E_2 \circ Sing$ from the category $\mathcal{T}$ of topological spaces and continuous maps to the category $\mathcal{S}_2$ of 2-reduced simplicial sets and simplicial maps. $SingX$ is a singular complex of X and E_2S is the Eilenberg subcomplex of S consisting of those simplices of S

whose 1-skeleton is at the basepoint. We obtain the inverse system $\underline{\tilde{X}} = \left(\tilde{X}_\lambda, \tilde{p}_{\lambda\lambda'}, \Lambda\right)$ of 2-reduced simplicial sets $\tilde{X}_\lambda = E_2 Sing\,(X_\lambda)$ and corresponding simplicial maps.

The category *pro-*$\mathcal{S}_2$ is a closed model category (the closed model category structure of $\mathcal{C}$ can be extended to *pro-*$\mathcal{C}$ using D. Edwards' and H. Hastings' construction).

Using D. Quillen's results we show that the category $Ho_{\mathbb{Q}}(pro\text{-}\mathcal{S}_2)$ is equivalent to both of the categories $Ho_{\mathbb{Q}}(pro\text{-}DGL_1)$ and $Ho_{\mathbb{Q}}(pro\text{-}DGC_2)$ where DGL_1 and DGC_2 are the categories of reduced differential graded Lie algebras over $\mathbb{Q}$ and 2-reduced differential graded (cocommutative coassociative) coalgebras over $\mathbb{Q}$ respectively. $Ho_{\mathbb{Q}}\mathcal{C}$ is obtained from $\mathcal{C}$ by formally inverting rational homotopy equivalences. Hence each of the categories $Ho_{\mathbb{Q}}(pro\text{-}DGL_1)$ and $Ho_{\mathbb{Q}}(pro\text{-}DGC_2)$ determine the *rational homotopy type* $Ho_{\mathbb{Q}}(\underline{S})$ of any $\underline{S} \in pro\text{-}\mathcal{S}_2$ which is defined to be the class of objects isomorphic to $\underline{S}$ in $Ho_{\mathbb{Q}}(pro\text{-}\mathcal{S}_2)$.

We define *a rational shape type of* X to be a rational homotopy type of $\underline{\tilde{X}}$: $Sh_{\mathbb{Q}}(X) = Ho_{\mathbb{Q}}(\underline{\tilde{X}})$.

Vlasta Matijević

Department of Mathematics, University of Split, N. Tesle 12, 21 000 Split, Croatia
e-mail: vlasta@pmfst.hr

A covering map over a topological group which is not a covering homomorphism

Mathematics Subject Classification (MSC): 14H30, 22C05, 57M10

Abstract. Let Y be a connected group and let $f : X \to Y$ be a covering map with the connected total space X. We

consider the following question: Is it possible to define a topological group structure on X in such a way that f becomes a homomorphism of topological groups (i.e. a covering homomorphism). The answer is positive in some particular cases: if Y is a pathwise connected and locally pathwise connected group or if f is a finite-sheeted covering map over a compact connected group Y. However, using shape-theoretic techniques and Fox's notion of an overlay map, we answer the question in the negative. First we show that an infinite-sheeted covering map $f : X \to \Sigma_{\boldsymbol{P}}$ with the connected total space X over a solenoid $\Sigma_{\boldsymbol{P}}$ does not admit a topological group structure on X such that f becomes a homomorphism of topological groups. Then we construct a connected space X and an infinite-sheeted covering map $f : X \to \Sigma_2$ over the dyadic solenoid Σ_2.

H. Mazaheri

Yazd University ,Yazd, Iran
e-mail: hmazaheri@yazduni.ac.ir

The orthogonality in the topological vector spaces
Mathematics Subject Classification (MSC): 41A65, 46B50, 46B20, 41A50

Abstract. The purpose of this paper is to introduce and discuss the concept of orthogonality in the topological vector space. In this note, we shall consider the relation between this concept and best approximation, and obtain some results on orthogonality the subsets of normed spaces.

References

[1] G. Brikhoff, Orthogonality in linear metric space, Duke Math. J. 1 (1935) 169-172.

[2] F. Mazaheri, F. M. Maalek Ghaini, Quasi-orthogonality of the best approximant sets, Nonlinear Anal. 65 (2006), no. 3, 534-537.

[3] H. Mazaheri, S. M. Vaezpour, Orthogonality and ϵ-Orthogonality in Banach spaces, Aust. J. Math. Anal. Appl., 2 (2005) no. 1, Art. 10 , 5pp.

Andrea Medini

University of Wisconsin-Madison Mathematics Dept.
e-mail: medini@math.wisc.edu

Products and h-homogeneity
Mathematics Subject Classification (MSC): 54B10

Abstract. A topological space X is h-homogeneous if all non-empty clopen subsets of X are homeomorphic. The Cantor set, the rationals, the irrationals or any connected space are examples of h-homogeneous spaces.

Building on work of Terada, we will show that h-homogeneity is productive in the class of zero-dimensional spaces. Our main tool will be Glicksberg's classical theorem on the Stone-Čech compactification of products.

If time permits, we will also discuss the h-homogeneity of infinite powers of zero-dimensional first-countable spaces.

References

A. Medini. Products and h-homogeneity. (To appear in the special issue of Topology and its Applications in honor of Ken Kunen.)
Preprint available at http://arxiv.org/abs/0911.1023

V.Meshkov, A.Mishunin, A.Omelchenko

194021, Khlopina st. 8 build. 3, Saint-Petersburg, Russia
e-mail: avo-travel@yandex.ru

Classification of textile links and enumeration of alternating k-tangles
Mathematics Subject Classification (MSC): 57M99, 57M27

Abstract. By a textile link we mean a double periodic link (interlacing). Such a link can be considered as a virtual link of genus 1 (i.e. a link in a thickened torus). But because of their obvious practical importance textile links are worth of special studying. Some issues around the classification problem for textile links are discussed including the auxiliary problem of k-tangles enumeration.

Jan van Mill
Faculty of Sciences
Department of Mathematics
VU University Amsterdam
De Boelelaan 1081[a]
1081 HV Amsterdam
The Netherlands
e-mail: vanmill@few.vu.nl

On countable dense and strong n-homogeneity
Mathematics Subject Classification (MSC): 54C35, 47B38

Abstract. We prove that if a space X is countable dense homogeneous and no set of size $n-1$ separates it, then X is strongly n-homogeneous. Our main result is the construction of an example of a Polish space X that is strongly n-homogeneous for every n, but not countable dense homogeneous.

A.S.Mishchenko
Moscow State University
e-mail: asmish@mech.math.msu.su

Signature of manifolds with proper action of a discrete group and the Hirzebruch type formula
Mathematics Subject Classification (MSC): 55N22, 55N15, 57R19, 57R22

Abstract. V.A.Roklin was the first ([1]) who has written the formula for the signature of 4-dimensional manifolds in the terms of the Pontryagin classes. For manifolds of arbitrary dimension this formula is known as the Hirzebruch formula. The formula was generalized during throughout more than 50 years in various directions.

Here we consider a case of manifolds with proper action of a discrete group G, that is if for any point its isotropy subgroup is finite and the quotient space is compact. It is a natural generalization of the category of non simply connected compact manifolds where a variety of geometric and topological constructions can be extended.

In particular on the category of manifolds with proper action one can canonically construct a bordism relation. For that category in the paper by P.Baum, A.Connes and N.Higson ([2]) a universal space was constructed to which any manifold with proper action of discrete group can be mapped equivariantly up to equivariant homotopy. Due to papers by S.Illman ([3]) and T.Korppi ([4]) we know that any smooth proper action is simplicial with respect to a simplicial structure on the manifold M. It allows to extend for proper actions many combinatorial constructions and to construct correspondent invariants.

Simplicial structure on the manifold with proper action of a discrete group G allows to construct so called algebraic

Poincare complex (APC). In particular the APC has noncommutative (symmetric) signature as an element of Hermitian K–theory of the group G, **sign** $(M) \in \mathbf{K}^*(\mathbf{Q}[\mathbf{G}])$. **sign** (M) is both homotopy invariant of the manifold M and invariant of bordisms.

Hence the problem of search of the Hirzebruch type formula for the signature **sign** (M) arises in the terms of the feasible characteristic classes of the quotient space M/G. The trouble is that the quotient space is manifold with singularities. But one can show that the space M/G is the Poincare space for rational homology and the Pontryagin classes has representations as invariant differential forms relative to proper action. It allows to express usual signature of the quotient space M/G by means of the Hirzebruch type formula.

For noncommutative signature **sign** $(M) \in \mathbf{K}^*(\mathbf{Q}[\mathbf{G}])$ one need to restore a bundle on the quotient space M/G with structural group $GL(n, C^*[G])$, the analog of canonical bundle $\xi_{C^*[G]} \in K_{C^*[G]}(BG)$, that is defined by a natural representation of the group G into the group C^*-algebra $C^*[G]$.

To clarify the bordism concept for proper action one can apply so called the Conner-Floyd construction for fixed points. Calculation of equivariant bordisms for manifolds with proper action is reduced to description of the classifying space for equivariant vector bundles for the case of quasi-free action of the group G on the base ([5]).

References.

[1] V.A.Rokhlin, *New results in the theory of 4-dimensioanl manifolds,(in Russian)* Dokl. AN SSSR,**84,**(1952), p. 221–224.

[2] P.Baum, A.Connes, and N.Higson. Classifying space

for proper actions and k-theory of group c^*-algebras. *Contemp. Math.*, 167:241–291, 1994.

[3] S. Illman. Existence and uniqueness of equivariant triangulations of smooth proper gmanifolds with some applications to equivariant whitehead torsion. *J. Reine Angew. Math.*, 524:129–183, 2000.

[4] T. Korppi. Equivariant triangulations of differentiable and real-analytic manifolds with a properly discontinuous action. In *Annales Academiæ acientiarum fennicæ matematica dissertationes,*, number 141. Suomalainen Tiedeakatemia, XVC05/4352. b20453085., Helsinki, 2005.

[5] A. S. Mishchenko and Quitzeh Morales Melendes. Description of the vector G-bundles over G-spaces with quasi-free proper action of discrete group G. *arXiv:0901.3308v1[math.KT]*, page 15, 2009.

Christopher G. Mouron

Department of Mathematics and Computer Science
Rhodes College
Memphis, TN 38112
e-mail: mouronc@rhodes.edu

Hierarchies of Chaotic Maps on Continua
Mathematics Subject Classification (MSC):
Primary 54H20; Secondary 54F15

Abstract. Let $f : X \longrightarrow X$ be a onto map of a continuum X, $C(X)$ be the hyperspace of continua of X and

$$E_F(f) = \{Y \in C(X)| \text{ for every } \epsilon > 0 \text{ there exists } N \in \mathbb{N} \text{ such that } \mathrm{d}_H(f^n(Y), X) < \epsilon \text{ for every } n \geq N\}.$$

If $E_F(f)$ is the set of all non-degenerate subcontinua of X, then f is *continuum-wise fully expansive.* If $E_F(f)$ is dense in $C(X)$, then f is *weakly continuum-wise fully expansive.*

A map $f : X \longrightarrow X$ is *transitive* if for any two open sets, $U, V \subset X$, there exists $n \in \mathbb{N}_0$ such that $f^n(U) \cap V \neq \emptyset$. Let $X^n = \Pi_{i=1}^n X$ and for $f : X \longrightarrow X$ let $\widehat{f_n} : X^n \longrightarrow X^n$ be the *induced map* of f on X^n defined by $\widehat{f_n}(x_1, ..., x_n) = (f(x_1), ..., f(x_n))$. We say that f is *topologically n-transitive* if $\widehat{f_n}$ is transitive. f is *ω-transitive* if it is n-transitive for all n. f is *topological mixing* if for every open sets U, V of X^n, there exists an M such that $(\widehat{f_n})^m(U) \cap V \neq \emptyset$ for all $m \geq M$. 2-transitive is also known as *weak topological mixing.*

We say that a function $f : X \longrightarrow X$ has *sensitive dependence on initial conditions* if there exists $c > 0$ such that for every $x \in X$ and open set U that contains x, there exists $n \in \mathbb{N}_0$ and $y \in U$ such that $\mathrm{d}(f^n(x), f^n(y)) \geq c$.

In this talk, we will discuss the following result:

Theorem 1. *Suppose $f : X \longrightarrow X$ is a map of any continuum X. Then*

f is continuum-wise fully expansive $\Rightarrow$
$\Rightarrow$ *f is weakly continuum-wise fully expansive*
$\Rightarrow$ *f topological mixing*
$\Rightarrow$ *f is ω-transitive*
$\Rightarrow$ *f is n-transitive*
$\Rightarrow$ *f is k-transitive$(k \leq n)$*
$\Rightarrow$ *f has sensitive dependence on initial conditions.*

Additionally, we will discuss topological conditions on X that guarantee when reverse implications hold and examples when reverse implications do not hold.

P.P. Murthy
Department of Pure and Applied Mathematics
Guru Ghasidas University
(A Central University)
Koni, Bilaspur, C.G., 495 009, India
e-mail: ppmurthy@gmail.com

Recent trends of metric fixed point theory and applications
Mathematics Subject Classification (MSC): Primary: 54H25, Secondary 47H10

Abstract. A remarkable Theorem in the field of Fixed Point Theory and Applications introduced by **Banach** (Surles operations dans les ensembles abstraits et leur applications anx equations integrables, Fund. Math. 3(1922), 131 - 181) in the year 1922 and **Kannan**(Some results on fixed points, Bull. Cal. Mth. Soc. 60(1968), 71 - 76) introduced the concept of non-contraction maps which is different from Banach, **Boyd and Wong**(On non-linear contractins, Proc. Amer. Math. Soc. 20(1969), 458 - 464) introduced a control. In this context we shall discuss different kind of contraction, non-contraction and weak-contraction conditions under the arena of Metric Fixed Point Theorems and its Applications. Also we shall discuss different kinds of non-commuting and compatible pair of maps in Metric and their related spaces. I would like to discuss some ***TOOLS*** and their importance for obtaining fixed points and common fixed points quickly. A few applications in the field of Dynamic Programming, Integral Equations and Variational Inequalities, etc. Very recently the concept of Cone Metric Space introduced by **Haung and Zhang**(Cone metric spaces and fixed point theorems of contractive mappings, J. Math.Anal. Appl. 332(2007),

1468 - 1476) and proved some common fixed point theorems in this space. We shall discuss in detail about this space and few results in this line by generalizing some results of ***Metric Fixed Point Theorems***

Oleg R. Musin
Department of Mathematics,
University of Texas at Brownsville,
80 Fort Brown, Brownsville, TX, 78520
e-mail: oleg.musin@utb.edu

On Borsuk-Ulam type theorems for G-manifolds
Mathematics Subject Classification (MSC): 55M30, 55M35

Abstract. We are considered a geometric proof of the Borsuk-Ulam theorem (see [1]). It can be extended for a wide class of G-manifolds. Using the equivariant cobordism theory can be defined obstructions for G-maps in cobordisms (see details in [2, Sec. 5]). For the case $m = n$ we have:

Theorem 1. *Let a finite group G acts free on a closed connected PL-manifold M^n. Let G acts linearly on $\mathbb{R}^n$. Then for any continuous equivariant map $f : M^n \to \mathbb{R}^n$ the zeros set Z_f is not empty if and only if there is a continuous equivariant transversal to zeros $h : M^n \to \mathbb{R}^n$ with $|Z_h| = (2k+1)|G|$, $k \in \mathbb{Z}$.*

In particular, for the classical case $G = \mathbb{Z}_2$ we have:

Theorem 2. *Let M^n be a closed connected PL-manifold with a free involution T. Then the following statements are equivalent:*

(a) For any continuous map $f : M^n \to \mathbb{R}^n$ *there is a point* $x \in M$ *such that* $f(T(x)) = f(x)$.

(b) M *admits an antipodal continuous transversal to zeros map* $h : M^n \to \mathbb{R}^n$ *with* $|Z_h| = 4k+2$, *where* $k \geq 0$ *is integer.*

(c) There exists an equivariant triangulation Λ *of* M *and a Tucker's labeling of vertices of* Λ *such that the number of Tucker's edges is* $4k+2$ *with integer* k.

(d) $[M^n, T] = [\mathbb{S}^n, A] + [V^1][\mathbb{S}^{n-1}, A] + \ldots + [V^n][\mathbb{S}^0, A]$ *in* $\mathfrak{N}_n(\mathbb{Z}_2)$.

(e) $\operatorname{cat}(M/T) = \operatorname{cat}(\mathbb{RP}^n) = n$.

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References

[1] J. Matoušek, Using the Borsuk-Ulam theorem, Springer-Verlag, Berlin, 2003.

[2] O. R. Musin, Borsuk-Ulam type theorems for manifolds, preprint, arXiv:0912.0787.

Vadoud Najjari

Islamic AZAD University of Maragheh, Maragheh, Iran
e-mail: fnajjary@yahoo.com

Computation of the centralizers of tori
Mathematics Subject Classification (MSC): 11F23, 53D22, 11F99

Abstract. In this paper we will find the centralizers of all tori Φ_M such that M consist of only one PM-block. in general, matrices in the centralizer of Φ_M won't be in $GL(n, \mathbb{C})$ however, we will find conjugates that are.

References

[1] T. Brocker and T. tom Dieck, epresentation of compact Groups, Springer-Verlag,1985

[2] E. Lerman and S. Tolman, Hamiltonian torus actions on symplectic orbifolds and toric varieties, Trans. Amer. Math.Soc.349(1997), 4201-4230

[3] J. E. Marsden and T. S. Ratiu., Introduction to Mechanics and Symmetry, 2nd ed,Springer-Verlag,(1999).

[4] D. McDuff and D. Salamon, Introduction to Symplectic Topology, Oxford University Press, December 1995.

[5] S.F Singer,J. Talvacchina and N. Watson, Nontoric hamiltonian circle actions on four dimensional symplectic orbifolds,Proc. Amer. Math. Soc. 127, 1999.

[6] N. Watson, Symplectic vector orbi-spaces with torus actions, senior paper, Haverford College, 1991.

Van Nall

University of Richmond
Richmond, VA 23173
e-mail: vnall@richmond.edu

Connected Inverse Limits with Set Valued Functions
Mathematics Subject Classification (MSC): 54C60, 54B10, 54D80

Abstract. An inverse limit with n-dimensional connected factor spaces and continuous single valued bonding maps is an n-dimensional continuum. On the other hand, there are very simple set valued interval maps with connected graphs whose inverse limit is not connected. We present

several results concerning the connectedness of inverse limits with set valued functions. In particular we concentrate on the case with one bonding map and where that bonding map has a graph that is the union of bonding maps with connected inverse limits.

Nikolay M. Nikolov and Svetoslav Zahariev
Department of Mathematics, Engineering and Computer Science
LaGuardia Community College, C.U.N.Y.
31-10 Thomson Avenue
Long Island City, NY 11101
USA
e-mail: szahariev@lagcc.cuny.edu

Transferring Curved A_∞-Structures and Simplicial Chern-Weil Theory
Mathematics Subject Classification (MSC): 18G55, 57R20

Abstract. A curved A_∞-algebra is a non-associative generalization of the notion of a curved differential graded algebra. I will discuss how curved A_∞-algebras arise as deformations of A_∞-algebras and how the former structures can be transferred along chain contractions using homological perturbation theory. As an example, given a vector bundle on a Lipschitz manifold M, I shall exhibit a natural curved A_∞-structure on the complex of matrix-valued cochains of any fine enough triangulation of M. (Recall that every topological manifold of dimension different than 4 admits a Lipschitz structure.) I will use this curved A_∞-structure to develop a simplicial version of Chern-Weil theory on triangulated, not necessarily smooth, topological manifolds.

Atamanyuk Oksana
Department of Mathematics and Informatics
Precarpathian National State University
Ivano-Frankivsk, Shevchenka 57
Ukraine
e-mail: if260887aob@email.ua

The preservation of geometric properties by diskrete nonautonomic inverse dynamical systems under the topological conjugations

Mathematics Subject Classification (MSC):

Abstract. S.F. Kolyada in [1] put out some questions about preservation of geometric properties by dynamical systems under the topological conjugation. We have investigated the preservation different geometric properties of projections by dynamical systems. The topological conjugation is such homeomorphism which is realized by commutative diagrams for projections as $\pi \circ p_n = q_n \circ \pi$. When π is not homeomorphism but it is surjective map and when it is realized by the commutative diagrams for the projections $\pi \circ p_n = q_n \circ \pi$ therefore it named as semiconjugation.

Theorem 1. *Such geometrical properties are preserved under the topological conjugation as (1) thin homotopic equalence, (2) Z_n – set property, (3) the property of projection to be almost homeomorphism, (4) semicontinuous from below and semicontinuous from above, (5) the diskrete cell - approximation property, (6) the property of uniformly local K - connected - (ULC^k), (7) soft of projections, (8) the approximative soft of projections , (9) the approximative stratification by Gurevich, (10) Z - approximation soft property, (11) strong C - universality of projections.*

Theorem 2. *Such geometrical properties as $Helder(\alpha)$ [2]*

for projections is preserved under the nonexpansive semi-conjugation by factor-map.

We remember the definitions which used in theorem 1. Soft map T means that if for each closed $B \subset A$ and for each $g,\ h$ such that diagram $T \circ g = h \circ i$ is commutative than exists $\varphi : A \to X$ such that will be commutative following two diagrams 1) $g = \varphi \circ i$, 2) $h = T \circ \varphi$. So T is defined as approximative soft if for each covering $\omega \in Cov(X)$ and for each closed B belong to arbitrary A and for each maps $g : B \to X_{i+1}$ and $h : A \to X_i$ the condition $T \circ g = h \circ i$ follow that exists map $\psi : A \to X_{i+1}$ such that will be commutative two diagrams (1) $\psi \circ i = g$, (2) $(T \circ \psi, h) < \omega$. We prove for instance that strong C - universality for map is invariant under the topological conjugation between dynamical systems. The map $\varphi : X \to Y$ is defined as SCU-map (strong C-universality) if for each closed $B \subset A \in C$ for each enclosing $f : B \to X$ and for each map $g : A \to Y$ such that diagram $\varphi \circ f = g|_B$ is commutative or $\varphi \circ f = g \circ i$ follow that exists the enclosing $F : A \to X$ which breaks big diagram onto two commutative triangle diagrams (1) $F \circ i = f$, (2) $\varphi \circ F = g$. We note, that if we change the word "enclosing" onto word "map" we have obtained the definition of C-soft map. We take each enclosing $\varphi : B \to Y$ and we take any map $G : A \to W$. Let such diagram $S \circ \varphi = G \circ i$ is commutative. We construct the map $f : B \to X$ by formula $f = \pi^{-1}(\varphi)$ and we construct $g = \pi^{-1}(G)$. We will to prove commutativity $T(f) = g(i)$. Really $T(f) = T \circ \pi^{-1}(\varphi)$. On the other hand $g(i) = \pi^{-1} \circ G(i)$ and we have equality $T \circ \pi^{-1}(\varphi) = \pi^{-1} \circ G(i)$. And so we use of the condition topological conjugation $\pi^{-1} \circ S = T \circ \pi^{-1}$. Put it under the beforehand equality $T \circ \pi^{-1}(\varphi) = \pi^{-1}(S) \circ \varphi$ or $T(f) = \pi^{-1} \circ S(\varphi)$. But $S(\varphi) = G(i)$ and put it under the beforehand equality

$T(f) = \pi^{-1} \circ G(i) = g(i)$.

We have obtained following equality $T(f) = g(i)$. So we used the definition that map T have property strong C-universality t.i. exists enclosing $F : A \to X$ which breaks the diagram $T(f) = g(i)$ onto two commutative triangle diagrams (1) $F \circ i = f$, (2) $T \circ F = g$. So, we construct the map $\Phi : A \to X$ by the formula $\Phi = \pi(F)$. So we will to prove that map Φ breaks the diagram $S(\varphi) = G(i)$ onto two commutative triangles (1) $\Phi(i) = \varphi$ and (2) $S \circ \Phi = G$. Really $\Phi(i) = \pi \circ F(i) = \pi(f) = \varphi$, therefore $\Phi(i) = \varphi$. So $S \circ \Phi = S \circ \pi(F)$ under the condition of topological conjugation it equal $\pi \circ T(F) = \pi(g) = G$. It finish the proof.

References

[1] S.F. Kolyada *Topological dynamic.* Doctor thesis 2004. Inst. Math. NAN Ukraine, 2004.

[2] Yoav Benyamini and Yoram Lindenstrauss *Geometric Nonlinear Functional Analysis.* AMS, Colloquium Publications, Volume 48

Piotr Oprocha

Departamento de Matemáticas, Universidad de Murcia, Campus de Espinardo, 30100 Murcia, Spain
e-mail: oprocha@agh.edu.pl

Topological weak mixing and disjointness
Mathematics Subject Classification (MSC): 37B

Abstract. The notion of disjointness was introduced in the context of topological dynamics by H. Furstenberg in 1967. He proved that weakly mixing system with dense periodic points is disjoint from any minimal system. Since

then, the full characterization of systems disjoint from all minimal systems is an open question.

In this talk we will present some partial answers to the above question, and relate them to another open problem of weak product recurrence.

Sergio Alejandro Orjuela Vargas, Filip Rooms, and Wilfried Philips
Ghent University, Department of Telecommunicationsand Information Processing (TELIN-IPI-IBBT), Belgium
e-mail: seraleov@telin.ugent.be

Geometric Local Binary Pattern, a new approach to analyse texture in images
Mathematics Subject Classification (MSC): 20B30

Abstract. Texture plays an important role in image processing analysis. It can be defined as the pattern describing a surface due to variations of data at scales smaller than the scale of interest [1]. The study of texture permits to characterize and discriminate regions in images. In some cases objects can be masked by texture and removing the texture facilitates the detection of the object.

One of the most important topics of image analysis based on texture is to automate the visual inspection of surface in materials. This type of analysis is commonly performed by human experts offering subjective results and being unhealthy for the inspectors. Therefore an automated inspection system converts traditional subjective inspections into objective ones. Several techniques for two-dimensional texture analysis have been developed, increasing the role of texture analysis in practical industrial applications [2]. Specifically Local Binary Pattern (LBP) tech-

niques have proved to be successful for texture classification and texture characterization in related domains [3].

The basic implementation of the LBP technique consist in describing with a code the relation of a fixed pixel and its neighbours using a predefined window size [3]. A rotational variant represents relationships between points on a circle around a central pixel [3]. Equally spaced points are considered on the circle and a corresponding pixel value in each point is interpolated from the four closest pixels to the point. Each point gets a code bit 0 or 1 assigned depending on whether it has a higher or lower gray value than the central pixel. These bits are read out clock wise and placed in a diadic code word named pattern. Rotated versions of patterns are grouped using a look up table. Other pattern versions such as mirrored and complemented can also be grouped making the representation more compact and invariant to noise [4].

To analyse texture a multiresolution analysis is commonly required. This can be achieved by changing the number of circular neighbours and their distance from the center pixel or by evaluating images at different scales factors. However, these methods do not evaluate connections of the features among the scales scales. To overcome this, we extend the LBP technique by describing structures around the given pixel. For this, we propose to evaluate the relationship between the points on the circles with different radii around a central pixel. The connections are binarized and placed into a matrix, which can be evaluated using symmetry to group similar versions of patterns.

As an initial approach we have grouped GLPB patterns with specific structures. We have compared this method to an extension of the LBP technique grouping rotational, mirrored and complemented variants of patterns computed from eight transitional textures using intensity and depth images. These textures have been obtained by submitting new carpet samples of the same type to different revolutions in a test device which simulates traffic exposure. The comparison has been performed evaluating the statistical discrimination of consecutive degrees of wear offered by the methods. The results have shown that the GLBP technique performs better than the LBP technique.

We believe that this approach can be extended by using the theory of symmetric groups to perform a better grouping of similar versions in patterns. Thus, specific geometries can be studied. Research using topologies in image analysis has been conducted for description, compression and classifications of images by investigating connection between components, contour extraction and curves of levels among others [5,6,7].

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References

[1] M. Petrou and P. G. Sevilla. *Image Processing Dealing with Texture*. Wiley, January 2006.

[2] M. K. Pietikainen, Editor. *Texture Analysis in Machine Vision*, Vol. 40 of Series in Machine perception and artificial intelligence. World Scientific Publishing Company, January 2000.

[3] T. Ojala, M. K. Pietikainen, and T. Menp. *Multiresolution Gray Scale and Rotation Invariant Texture Classification with Local Binary Patterns*. IEEE Transactions on

Pattern Analysis and Machine Intelligence, Vol. 24, Issue 7, 2002, 971–987.

[4] S. A. Orjuela, E. Vansteenkiste, F. Rooms, S. De Meulemeester, R. De Keyser, and W. Philips. *Evaluation of the Wear Label Description in Carpets by Using Local Binary Pattern Techniques*. Textile Research Jounal, Submission ID: TRJ-10-0080, 2010.

[5] C. Maire and M. Datcu. *Object and Topology Extraction from Remote Sensing Images*. In Proceedings: ICIP, 2005.

[6] G. Eichmann and T. Kasparis. *Topologically Invariant Texture Descriptors*. Computer Vision, Graphics, and Image Processing, Vol. 41, Issue 3, 1988, 267–281.

[7] J. Gu, J. Zhou and D. Zhang. *Singular Points Analysis in Fingerprints Based on Topological Structure and Orientation Field*, Advances in Biometrics, Springer Berlin / Heidelberg, Vol. 4642, 2007, 261–270.

Alexander V. Osipov
INSTITUTE FOR MATHEMATICS AND MECHANICS
S.Kovalevskoi 16 GSP-384 Ekaterinburg 620219 RUSSIA
e-mail: OAB@list.ru

Weakly set-open topology
Mathematics Subject Classification (MSC): 54C35

Abstract. The properties of the weakly set-open topology on the set $C(X)$ of all real-valued functions defined on a Tikhonov space X are studied. The relation between the $\mathbb{R}$-compact-open topology and the well-known set-open and uniform topology on the set $C(X)$ is investigated.

Alexey Ostrovsky
Helmut Kaeutner Str. 25, 81739 Munich, Germany
e-mail: Alexey.Ostrovskiy@UniBw.de

An alternative approach to decomposition of functions and spaces
Mathematics Subject Classification (MSC): Primary 54H05, 54C10. Secondary 54C08, 28A20

Abstract. During his research the author has repeatedly encountered situations where mappings $f : X \to Y$ possessed the following two properties at the same time:

1. f takes open sets U to G_δ-sets B.
2. $X = \bigcup_{i=1}^{\infty} X_i$, where $f|X_i$ is a closed or open mapping.

This brought up a hypothesis that 1. often implies 2. It appears natural to verify the hypothesis for the case of mappings f where B is a union of clopen and closed sets. In particular, the author raised a question about the preservation of completeness by the above mentioned class of mappings. Recently the question was solved affirmatively by the author and then translated to other cases of B by other authors.

We will point out some cases of the affirmative answers to the above hypothesis for any metric spaces X. This would allow us to strengthen a number of classic results.

For instance, we will point out some natural cases of the affirmative solution to the old question of Lusin whether there exists a Borel measurable function which cannot be decomposed into countably many continuous functions.
(This question was answered negatively by Keldysh, Adjan and Novikov).

Ch. Papachristodoulos and N. Papanastassiou
Department of Mathematics,
University of Athens,
Athens, Greece
e-mail: papach@math.uoc.gr

On q-convergence in measure and q-density topologies
Mathematics Subject Classification (MSC):

Abstract. We define for each positive real number q, a convergence for sequences of real valued measurable functions, stronger than the convergence in measure. As a result in the space $\mathcal{M}$ of all sequences of measurable functions converging in measure to zero we introduced a quasi-norm under which $\mathcal{M}/\sim$ turns to be a complete metric space. Also we apply this type of convergence to define density topologies.

Ch. Papachristodoulos and N. Papanastassiou
Department of Mathematics,
University of Athens,
Athens, Greece
e-mail: npapanas@math.uoa.gr

On ℓ_p statistical convergence in measure of a sequence of real-valued measurable functions
Mathematics Subject Classification (MSC):

Abstract. We define a new type of convergence (the $\mu\alpha-$statistical convergence) of a sequence of measurable functions which is strictly between convergence in measure and asymptotically convergence.

Also for each positive real number p the ℓ_p statistical convergence is introduced and corresponding density topologies are studied.

R. Parvinianzadeh
Department of Mathematics
Faculty of Basic Sciences
University of Mazandaran
Babolsar, 47416 –1468
Iran
e-mail: r.parvinian@umz.ac.ir

Spectrum preserving maps between von Neumann algebras
Mathematics Subject Classification (MSC): Primary 46H05, 47B48; Secondary 47A10

Abstract. Let A be a unital von Neumann algebra and B a unital semi-simple Banach algebra. Let ϕ be a surjective spectrum preserving additive map from A onto B. Then ϕ is continuous and Jordan isomorphism.

Acknowledgements: This research is partially supported by the Research Center in Algebraic Hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran.

References

[1] W.B. Arveson, *An Invitation to C^*-algebras*, Springer GTM, **39**, 1976.

[2] B. Aupetit, *Spectrum-preserving linear mappings between Banach algebras or Jordan-Banach algebras*, J. London. Math. Soc. **62**(2000), 917–924.

[3] B. Aupetit, *A primer on spectral theory*, Springer, New York, 1991.

[4] B. Aupetit and H. du Toit Mouton, *Spectrum preserving linear mappings in Banach algebras*, Studia Math. **109**(1994), 91–100.

[5] M. Bresar and P. Semrl, *Linear maps preserving the spectral radius*, J. Funct. Anal. **142**(1996), 360–368.

[6] L. B. Brown and G.K. Pedersen, *C^*-algebras of real rank zero*, J. Funct. Anal. **99**(1991), 131 149.

[7] H. G. Dales, *Banach Algebras and Automatic Continuity*, London Mathematical Society Monographs 24, Clarendon Press, Oxford, 2000.

[8] H. G. Dales, P. Aiena, J. Eschmeier, K. Laursen, G. Willis, *Introduction to Banach Algebras, Operators, and Harmonic Analysis*, London Mathematical Society Monographs 57, Cambridge university Press 2003.

[9] J. Dixmier, *C^*-Algebras*, North-Holland, Amsterdam, 1977.

[10] B. E. Johnson, *The uniqueness of the (complete) norm topology*, Bull. Amer. Math. Soc. **73** (1967), 537-539.

[11] G. Frobenius, *Über die Darstellung der endlichen Gruppen durch lineare Substitutionen I*, Sitzungberichte Königlich Preussischen Akademie Wissenschaften Berlin (1897), 994–1015.

[12] L.A. Harris, *Invertibility preserving linear maps of Banach algebras*, Israel Mathematical conference proceedings. **Vol. 15**, (2001).

[13] I.N. Herstein, *Topics in ring theory*, Springer, Berlin, 1991.

[14] A.A. Jafarian, *A survey of invertibility and spectrum preserving linear maps*, Bull. Iran. Math. Soc. **35**(2009), 1–10.

[15] A.A. Jafarian and A.R. Sourour, *Spectrum-preserving linear maps*, J. Funct. Anal. **66**(1986), 255–266.

[16] J.P. Kahane and W. Zelazko, *A characterization of maximal ideals in commutative Banach algebras*, Studia Math. **29**(1968), 339–343.

[17] I. Kaplansky, *Algebraic and analytic aspects of operator algebras*, CBMS Regional conference series in Math. 1, Amer. Math. Soc. Porvidence, 1970.

[18] S. Kowalski and Z. Slodkowski, *A characterization of maximal ideals in commutative Banach algebras*, Studia Math., **67**(1980, 215–223.

[19] A.M. Gleason, *A characterization of maximal ideals*, J. Analyse Math. **19**(1967), 171–172.

[20] M. Mbekhta, *Linear maps preserving the generalized spectrum*, Extracta Mathematicae. **22**(2007), 45–54.

[21] N. Boudi and M. Mbekhta, *additive maps preserving strongly generalized inverses*, J. Operator Theory, in press.

[22] N. Boudi and M. Mbekhta, *On linear maps preserving generalized invertibility and related properties*, J. Math. Anal. Appl. **345**(2008), 20–25.

[23] Gerald Murphy,, *C*-algebras and operator theory*, Hardbound, 1990.

[24] T. J. Ransford, *A short proof of Johnson"s uniqueness-of-norm theorem*, Bull. London Math. Soc. **21** (1989), 487-488.

[25] S. SAKAI, *C^*-algebras and W^*-algebras*, Springer, New York, 1971.

[26] P. SEMRL, *Two chractreizations of automorphisims on $B(X)$*, Studia Math. **105**(1993), 143–149.

[27] A.R. SOUROUR, *Invertibility preserving linear maps on $L(X)$* , Trans. Amer. Math. Soc. **348**(1996), 13–30.

[28] W. ZELAZKO, *A characterization of multiplicative linear functionals in complex Banach algebras*, Studia Math. **30**(1968), 83–85.

Mikhail A. Patrakeev

Institute of mathematics and mechanics of Ural branch of Russian academy of sciences, Ekaterinburg, Russia
e-mail: patrakeev@mail.ru

Spaces with Lusin π-bases
Mathematics Subject Classification (MSC): 54E99

Abstract. Recall that a *Lusin scheme on a set X* is a family $(V_s)_{s\in\mathbb{N}^{<\mathbb{N}}}$ of subsets of X such that

$V_s \supset \bigsqcup_{n\in\mathbb{N}} V_{s\hat{}n}$, if $s \in \mathbb{N}^{<\mathbb{N}}$

(where $\mathbb{N}^{<\mathbb{N}}$ is the set of all finite sequences of natural numbers, $\hat{}$ denotes a concatenation and $\bigsqcup$ denotes a disjoint union). Consider the special case of Lusin schemes:

Definition. A *strict Lusin scheme on a set X* is a family $(V_s)_{s\in\mathbb{N}^{<\mathbb{N}}}$ of subsets of X such that

(i) $V_\varnothing = X$;
(ii) $V_s = \bigsqcup_{n\in\mathbb{N}} V_{s\hat{}n}$, if $s \in \mathbb{N}^{<\mathbb{N}}$;
(iii) $|\bigcap_{n\in\mathbb{N}} V_{x|n}| = 1$, if $x \in \mathbb{N}^{\mathbb{N}}$

(where $\varnothing$ is the empty sequence and $x|n$ denotes the restriction of infinite sequence x to its first n members).

Definition. A *Lusin π-base for a space X* is a strict Lusin scheme $(V_s)_{s\in\mathbb{N}^{<\mathbb{N}}}$ on X which consists of open sets and satisfies the condition:

for any $x \in X$ and any neighborhood U of x there exist $s \in \mathbb{N}^{<\mathbb{N}}$ and $n_0 \in \mathbb{N}$ such that $x \in V_s$ and $\bigcup_{n\geq n_0} V_{s\hat{}n} \subset U$.

The examples of spaces with Lusin π-bases are the space of irrational numbers and the Sorgenfrey line. The talk is devoted to the class of spaces with Lusin π-bases.

Christina P. Podara

Department of Mathematics, University of Athens, Greece
e-mail: chpodara@gmail.com

On the bidual of quasinormable Fréchet algebras
Mathematics Subject Classification (MSC): Prim.: 46A20; Sec.: 46A04, 46A13, 46H20, 46K05, 46M18, 46M40

Abstract. We know (due to work of Dierolf, Meise, Vogt and others) that the bidual of a quasinormable Fréchet space is, up to a topological isomorphism, the inverse limit of the biduals of the Banach space steps. By use of the Arens product and homological methods, we extend this result to the quasinormable Fréchet m-convex algebra setting. As a consequence, we obtain that the bidual of a σ-C*-algebra A with the Arens product is a unital σ-C*-algebra, which is commutative if A is commutative.

Elżbieta Pol

Institute of Mathematics, University of Warsaw, Banacha 2, 02-097 Warszawa, Poland e-mail: E.Pol@mimuw.edu.pl

On a construction of universal hereditarily indecomposable continua based on the Baire category
Mathematics Subject Classification (MSC):
Primary 54F15, 54F45, 54E52

Abstract. We give a new proof of a theorem of T.Maćkowiak on the existence of universal n-dimensional hereditarily indecomposable continua, based on the Baire-category method.

Madeti Prabhakar
Department of Mathematics, Indian Institute of Technology Ropar, Rupnagar-140001, Punjab, India
e-mail: prabhakar@iitg.ernet.in

A note on Unknotting Numbers
Mathematics Subject Classification (MSC):
Primary 57M25

Abstract. In this paper our focus is on finding a bound on the unknotting number of any given knot. In many cases the given bound is exactly equal to the unknotting number. We have utilized quasitoric braid representation for a given knot in finding the bound.

References

[1] V. O. Manturov, *A combinatorial representation of links by quasitoric braids*, European J. Combin. **23** (2002), 207–212.

[2] T. Kawamura, *The unknotting numbers of* 10_{139} *and* 10_{152} *are* 4, Osaka J. Math. **35** (1998), 539–546.

[3] Y. Nakanishi, *Unknotting numbers and knot diagrams with the minimum crossings*, Math. Seminar notes Kobe Univ. **11** (1983), 257–258.

[4] Y. Nakanishi, *A note on unknotting number. II.*, J. Knot Theory Ramifications **14** (2005), no. 1, 3–8.

Yury V. Sadovnichy
Moscow State University
e-mail:

Infinite-dimensional spaces of probability measures
Mathematics Subject Classification (MSC):

Abstract. In [1] there were investigated properties of infinite-dimensionality of spaces of type $\mathcal{F}(X)$, where $\mathcal{F}$: Tych $\to$ Tych is a covariant normal functor and X is a paracompact space. In particular, it was proved that a spase $P_R(X)$ of all Radon probability measures on an infinite paracompact p-space X is strongly infinite-dimensional. Here we prove stronger versions of this theorem.

Given a Tychonoff space X let βX be its Čech-Stone compactification and

$$P_R(X) \subset P_\tau(X) \subset P_\sigma(X) \subset P(\beta X)$$

be the following subspaces of $P(\beta X)$:

$P_R(X) = \{\mu \in P(\beta X) : \mu(K) = 1$ for some σ-compact subset $K \subset X \subset \beta X\}$;

$P_\tau(X) = \{\mu \in P(\beta X) : \mu(K) = 0$ for every compact subset $K \subset \beta X \setminus X\}$;

$P_\sigma(X) = \{\mu \in P(\beta X) : \mu(K) = 0$ for any closed G_δ-*set*$K \subset \beta X$ with $K \cap X = \emptyset\}$.

Theorem. *Let X be an infinite Tychonoff space. Then spaces $P_R(X)$, P_τ, and $P_\sigma(X)$ are strongly infinite-dimensional.*

REFERENCES

[1] V.V.Fedorchkuk and Yu.v.Sadovnichy, *Weakly infinite-dimensional spaces and probability measures*, Modern problems of math. and mech., MSU, volume III, issue 2, pp. 139–150.

M.A. Sánchez-Granero, H.P.A. Künzi, and S. Romaguera
Area of Geometry and Topology
Faculty of Science
Universidad de Almería
04071 Almería
Spain
e-mail: misanche@ual.es

T_0 *-Compactification in the hyperspace

Mathematics Subject Classification (MSC): Primary 54B20; Secondary 54E15, 54D35

Abstract. A *-compactification of a T_0 quasi-uniform space $(X,\mathcal{U})$ is a compact T_0 quasi-uniform space $(Y,\mathcal{V})$ that has a $\mathcal{T}(\mathcal{V} \vee \mathcal{V}^{-1})$-dense subspace quasi-isomorphic to $(X,\mathcal{U})$.

In this paper we study when the hyperspace with the Hausdorff-Bourbaki quasi-uniformity is *-compactifiable and describe some of its *-compactifications. In particular, we study when the hyperspace of the bicompletion is a *-compactification of the hyperspace.

Also, we show that *-compactifiableness of the hyperspace is equilavent to compactifiableness of the stability space, where the stability space is the bicompletion of the hyperspace. As a result, we study when the stability space is compact.

Denis I. Saveliev
127206 Russia, Moscow, Staroe shosse, 9-1-5
e-mail: denissaveliev@mail.ru

Ultrafilters without Choice
Mathematics Subject Classification (MSC): 03E55, 03E60, 22A22, 54D35, 54D80

Abstract. We study spaces of ultrafilters in the Zermelo–Fraenkel set theory, ZF, without the Axiom of Choice, AC, or with alternative axioms (weaker than, or inconsistent with AC). Working in ZF alone, we restate a few simple (well-known under AC) facts, and obtain some new results concerning algebra of ultrafilters. We consider natural extensions of given groupoids to the groupoids of ultrafilters and prove that κ-complete ultrafilters form subgroupoids.

Any investigation of ultrafilters in ZF alone has a conditional character since the existence of non-principal ultrafilters is unprovable there. Under the assumption that there exists at least one non-principal ultrafilter, we evaluate cardinalities of some spaces of ultrafilters. Furthermore, we show that a stronger assumption that any filter can be extended to an ultrafilter, known as the Prime Ideal Theorem, PI, is equivalent to compactness of all spaces of ultrafilters. Moreover, many spaces of ultrafilters under PI have cardinalities close to those under AC.

Finally, we consider generalizations of PI to higher cardinalities (with extensions preserving κ-completeness). With AC the generalizations specify large cardinals, while without AC the cardinals can be rather small (though still are large in inner models). A particular emphasis is on the situation under the Axiom of Determinacy, AD. We show that under AD, the space of ultrafilters over a well-ordered cardinal λ is Lindelöf whenever the real line can be parti-

tioned into λ parts (the least λ for which this fails is a large cardinal under AD).

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Pavel V. Semenov

Department of Mathematics,
Moscow City Pedagogical University,
2-nd Selskokhozyastvennyi pr. 4, Moscow, Russia 129226
e-mail: pavelsorc.ru

Paraconvexity as a generalized convexity
Mathematics Subject Classification (MSC):
primary - 54C60, 54C65, 41A65;
secondary: 54C55, 54C20

Abstract. Typically, a creation of "generalized convexities", is usually related to an extraction of several principal properties of the classical convexity which are used in one of the key mathematical theorems or theories and, consequently deals with analysis and generalization of these properties in maximally possible general settings. Based on the ingenious idea of Michael who proposed the notion of a paraconvex set, to each closed subset PB of a Banach space we have associated a numerical function, say $\alpha_P : (0, +\infty) \to [0, 2)$, the so-called function of nonconvexity of P. The identity $\alpha_P \equiv 0$ is equivalent to the convexity of P and the more α_P differs from zero the "less convex" is the set P. For a function $\alpha : (0, +\infty) \to [0, 1)$ with all right upper limits less than 1 the nonempty closed set P is said to be *α-paraconvex*, whenever α majorates the function $\alpha_P(\cdot)$ of nonconvexity of the set P.

Such classical results about multivalued mappings as

the Michael selection theorem, the Cellina approximation theorem, the Kakutani-Glicksberg fixed point theorem, the von Neumann - Sion minimax theorem, etc. are valid with the replacement of the convexity assumption for values $F(x)$, $x \in X$ of a mapping F by some appropriate control of their functions of nonconvexity.

So the natural question arises immediately: *Does paraconvexity of a set with respect to the classical convexity structure coincide with convexity under some generalized convexity structure?* Corollary 2, based on continuous choice of a retraction (see Theorem 1), in particular provide an affirmative answer. Below, $bexp_\alpha(B)$ denotes the hyperspace of all bounded $\alpha-$paraconvex subsets of B endowed with the Hausdorff distance and $C_b(B,B)$ is the Banach space of all bounded continuous selfmappings.

Theorem 1 Let $0 \le \alpha < \frac{1}{2}$ and $F : X \to bexp_\alpha(B)$ be a continuous multivalued mapping of a paracompact space X into a Banach space B. Then there exists a continuous singlevalued mapping $\mathcal{F} : X \to C_b(B,B)$ such that for every $x \in X$ the mapping $\mathcal{F}_x : B \to B$ is a continuous retraction of B onto the value $F(x)$ of F.

Corollary 2 Under the assumptions of Theorem 1 if in addition all values $F(x)$, are pairwise disjoint then the metric subspace $Y = \bigcup_{x\in X} F(x)B$ admits a convex metric structure σ (in the sense of Michael) such that each value $F(x)$ is convex with respect to σ.

Theorem 3 For a Hilbert space H Theorem 1 holds with the replacement of $\alpha < \frac{1}{2}$ by $\alpha + \alpha^2 + \alpha^3 < 1$.

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Z.N. Silayeva

Belarus, Brest, Mayakovsky str., 20,16
e-mail: szn2006@yandex.ru

Characterizing filtered absolute extensors
Mathematics Subject Classification (MSC): 54C55, 54E35

Abstract. We investigate the problem of topological characterization of filtered absolute extensors. In [1] such a characterization for finite-dimentional spaces was established: the metric filtered space $X = \bigcup_{i=1}^{\infty} X_i, X_i = \mathrm{Cl} X_i \subseteq X_{i+1}$, with $\dim X < \infty$ is a filtered absolute neighbourhood extensor for the class of metric filtered spaces ($X \in \mathcal{N}$-ANE) if and only if the family $\{X_i\}_{i\in\mathbb{N}}$ of filtration elements is equi-LC (possesses the property of equilocal contractibility).

The purpose of this talk is to show that the finite-dimensional assumption is essential. It turns out, the locally contractible compactum of K.Borsuk that fails to be ANE admits a special filtration consisting of ANE-spaces and having the property of equi-LC. Because this compactum is not ANE, the corresponding filtered space is not $\mathcal{N}$-ANE.

References

[1] Z.N.Silayeva, Kuratowski-Dugundji's theorem for filtered spaces, Vestnik Belorusskogo Universiteta, №3, 2009. P. 89–94.

Mikhail Skopenkov

Institute for information transmission problems of the Russian Academy of Sciences, Bolshoy Karetny 19 build. 1, Moscow 127994, Russia e-mail: skopenkov@rambler.ru

Rational classification of embeddings
Mathematics Subject Classification (MSC):
Primary 57R52, 57R40; Secondary: 57R65

Abstract. This talk is on classification of embeddings of manifolds. Given a manifold N and a number m, we study the following question: *is the set of isotopy classes of embeddings* $N \to S^m$ *finite*? In case when the manifold N is a sphere the answer was given by A. Haefliger in 1966 [1]. The case when N is a disjoint union of spheres was treated by D. Crowley, S. Ferry and independently by the author in 2008. In this talk we consider the next natural case when N is a product of two spheres.

Theorem. Assume that $m > 2p + q + 2$ and $m < p + 3q/2 + 2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \to S^m$ is infinite if and only if either $q + 1$ or $p + q + 1$ is divisible by 4, or there exists a point (x, y) in the set $U(m - p - q, m - q)$ such that $(m - p - q - 2)x + (m - q - 2)y = m - 3$.

Here $U(i, j) \subset \mathbb{Z}^2$ is a concrete subset defined in the talk, which depends only on the parity of i and j.

Our approach is based on a group structure on the set of embeddings [2] and a new exact sequence, which in some sense reduces the classification of embeddings $S^p \times S^q \to S^m$ to the classification of embeddings $S^p \sqcup S^q \to S^m$ and $D^p \times S^q \to S^m$.

References

[1] A. Haefliger, *Differentiable embeddings of* S^n *in* S^{n+q} *for* $q > 2$, Ann. Math., Ser.3 **83** (1966) 402–436.

[2] A. Skopenkov, *A new invariant and parametric connected sum of embeddings*, Fund. Math. **197** (2007), 253–269.

Josef Šlapal

Department of Mathematics, Brno University of Technology, 616 69 Brno, Czech Republic
e-mail: slapal@fme.vutbr.cz

On a generative topology on the digital plane
Mathematics Subject Classification (MSC): 54D05, 54B15, 68U05

Abstract. We introduce a special topology on $\mathbb{Z}^2$ and discuss four of its quotients including the Marcus and Khalimsky topologies. In particular, for each of the four quotients, we prove a digital Jordan curve theorem by the help of the digital Jordan curve theorem proved for the topology introduced in [1].

References

[1] J. Šlapal, Digital Jordan curves, *Top. Appl.* **153** (2006), 3255–3264.

Sławomir Solecki

University of Illinois
e-mail: ssolecki@illinois.edu

Groups generated by generic measure preserving transformations
Mathematics Subject Classification (MSC):

Abstract. Consider the group of all measure preserving transformations of Lebesgue measure with the canonical weak topology. We show that for a generic transformation T in this group, the closed group generated by T is isomorphic to a subgroup of $L_0(\text{measure}, S^1)$ that is the image of a closed linear subspace of $L_0(\text{measure}, \mathbb{R})$ via the exponential map. This sharpens and generalizes several

older results. Using certain factorization theorems, we analyze unitary representations of the group of continuous functions with values in S^1. This analysis together with a result of del Junco and Lemańczyk allows us to show that the closed group generated by a generic transformation T is not isomorphic to the whole group $L_0(\text{measure}, S^1)$.

Fernando Orozco-Zitli, Włodzimierz J. Charatonik, José G. Anaya, Enrique Castañeda, Félix Capulín

Universidad Autónoma del Estado de México, Facultad de Ciencias.
Instituto Literario 100. Col. Centro, C.P. 50000, Toluca, Estado de México, México
e-mail: forozcozitli@gmail.com

On representation spaces
Mathematics Subject Classification (MSC): Primary 54B20; Secondary 54F55

Abstract. Consider $\mathcal{C}$ of all continua (up to homeomorphism). Let $\mathcal{P}$ be a subset of $\mathcal{C}$, and let $X \in \mathcal{C}$. Then we write $X \in \text{Cl}(\mathcal{P})$ to mean that for each $\varepsilon > 0$, there exist $Y_\varepsilon \in \mathcal{P}$ and an ε-map, f_ε, from X onto Y_ε. The operator Cl is a topological operator closure. In this talk we will present some results about the topological espace $\mathcal{C}$, for example: the class of locally connected continua is dense in $\mathcal{C}$, $\text{Cl}(\{[0,1]\}) = \{X \in \mathcal{C} : X \text{ is chainable}\}$, $\text{Cl}(\{Z \in \mathcal{C} : Z \text{ arcwise connected}\})=$
$\{X \in \mathcal{C} : X \text{ is continuum chainable}\}$.

Neset Ozkan Tan and Eftal Tan

Department of Mathematics,
Usak University,
Usak, TURKEY
e-mail: n.ozkan.tan@gmail.com

Disaster in Topology without choice, Countable Sums

Abstract. The formation of countable sums is one of the simplest constructions in topology. In ZFC it preserves most familiar properties of topological spaces, in particular:

1. metrizability,
2. normalitiy,
3. seperability,
4. second countability,
5. the Lindelöf property,
6. dimension zero.

In this study we show that If CC(Z) fails there exists a sequence of separable, metrizable,compact spaces (Yn) with dim$Yn = 0$, such that $\sum_n Yn$ is neither metrizable, nor normal, nor separable, nor second countable, nor Lindelöf, nor with dimension 0.

References

[1] E. K. van Douwen. Horrors of topology without AC: A nonnormal orderable pce. Proc. AMS, 95:101-105, 1985.

[2] H. Herrlich. Products of Lindelöf T2-spaces are Lindelöf - in some models of ZF. Comment. Math. Univ. Carolinae, 43:319-333, 2002.

[3] K. Keremedis and E. Tachtsis. Topology in the absence of the axiom of choice. Mathematica Japonica, 59:357-406, 2004.

M. Lellis Thivagar
Arul Anandar College,
Karumathur Madurai-625514. INDIA
e-mail: mlthivagar@yahoo.co.in

Another form of weakly continuous functions
Mathematics Subject Classification (MSC): 54C08

Abstract. Only in twentieth century, mathematicians defined the concepts of sets and functions to represent problems. This way of representing problems is more rigid. This difficulty was overcome by the topological concepts. Kelly, initiated a systematic study of such topological spaces and Njasted, Noiri have contributed to the development of the Modern topological concepts. In this paper, continuous functions have been introduced. Using these new types of functions, several characterizations and its properties have been obtained. Also relationships between continuous functions and other existing continuous functions have been obtained and some results have been established. Although these concepts classified as pure mathematics, when converted into Fuzzy and Digital topologies it becomes applications oriented around. Hence these results turned out to be highly interesting to go deeper into these power methods and try to make some further contributions in making these methods much more applicable.

References

[1] Abd El-Monsef, M.E.; Rose Mary, S.; Lellis Thivagar, M.: closed sets in topological spaces, Assuit Journal of Mathematics and Computer Sciences, Vol.36(1), 2007, 43-51.

[2] Balachandran, K.; Sundaram, P.; Maki, H.: On generalized continuous functions in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 12 (1991) 5-13.

[3] Maki, H.; Devi, R.; Balachandran, K.: Generalized -closed sets in topology, Bull. Fukuoka Univ. Ed. Part III, 42 (1993), 13-21.

[4] Veera Kumar, M.K.R.S.: closed sets in topological spaces, Bull. Allah. Math. Soc., 18 (2003), 99-112.

Vladimir Todorov

University of Architecture, Civil Engineering and Geodesy, Dept. of Mathematics, 1, Hristo Smirnenswki Blvd., 1046 Sofia, Bulgaria
e-mail: tt.vladimir@gmail.com

On minimal (n, ε) continua
Mathematics Subject Classification (MSC): 57N99

Abstract. The theory of Cantor Manifolds developed from an initial effort to give a rigorous description of a degree of connectedness of some basic objects. A typical example in this attitude is the n-dimensional cube I^n (I=[0,1]). By 1925, Urysohn established that the n - dimensional cube cannot be separated by any $(n-2)$-dimensional closed subset. In other words, I^n is not a sum of two proper closed sets whose intersection is no more than $(n-2)$ - dimensional. In 1957, Alexadroff proved that I^n is even so-called continuum (V^n) . Later various ways in establishing properties of connectedness of I^n appeared, namely, in 1969 Wilkinson and in 1970 Hadziivanov. They proved that In is not a union of countable many proper closed sets whose pair-wise intersections are no more than (n-2) - dimensional. At the end of this short survey we should note that there are various different results in this direction. For example, using the classical theorem of Sierpinski, Urysohn have proved that I^n is not cut by $(n-2)$ - dimensional

G_δ subsets. However, it is worthy of mentioning that at present, it seems that the best description of a connectedness of I^n appears in the concept of V^n -continua. In this note, we are developing a simplified approach for the description of Alexandroff's manifolds, which allows precision in the above mentioned results or reduces some of them to simple terms.

M. Tuncali
Nipissing University, North Bay, Ontario, Canada
e-mail: muratt@nipissingu.ca

Hereditary separability in Hausdorff continua
Mathematics Subject Classification (MSC):
Primary: 54F15; Secondary: 54C05, 54F05, 54F50

Abstract. Recent studies on Suslinian continua and perfecty normal continua led to the study of hereditarily separablity in Hausdorff continua. In this talk, we consider a Hausdorff continuum X such that each separable subspace is hereditarily separable, and we present some results on the structure of such continua which are also rim-metrizable or rim-separable.

A.V. Uglanov
Russia
e-mail: auglanov@yandex.ru

A Space of Vector Measures, Weak Topology and Prokhorov Theorem

Abstract. The report's topic is related to analysis on non-normed and/or nonmetrizable locally convex spaces topological vector space (LCS). So-called vector integral has

been defined in [1] (or see [2, 3]). A vector integral is the integral of the function which takes values in an abstract LCS with respect the measure which takes values in (generally speaking, vector) dual LCS, the domain of definition of the function and measure is an abstract measurable space. It should be noted the following. 1. The integral said has all natural properties of integral. 2. The property of continuty presents in our constraction. Namely, if the measure considered is real-valued measure then integral introduced is an ordinary Pettis integral; if in addition a LCS of values of integrable function is a Banach space then integral introduced is an ordinary Bochner integral. 3. The integral constructed has found many applications in various mathematical domains: functional and stochastic analysis, partial differential equations, optimal control (including control of stochastic processes), other. In particular integral has allowed to to receive a criterion of weak compactness for vector measures (an analog of famous Prokhorov theorem).
Notations: T – a complete separable metric space; Σ_T – the Borel σ-algebra of the space's T subsets; X – an abstract LCS, having the B-property (see [4] for definition; we do note here that B property is very weak one and practically all spaces, meeting in analysis - metric, dual-metric, nuclear, dual-nuclear, other – have this property); $\mathbb{U}(X)$ – the fundamental system of closed absolutely convex neighborhoods of point $0 \in X$; X^* – the space, conjugate to X considered with strong topology; $M = M(T, X)$ – the space of all X-valued bounded measures Σ_T (the definition is given below); $C = C(T, X^*)$ – the space of continuous bounded functions from T to X^*.

All linear spaces under consideration are assumed to be real.

Let π_T be the collection of all countable measurable partitions of the space T.

Definition. A countably additive functions $\mu : \Sigma \to X$ is called bounded measure if for any set $U \in \mathbb{U}(X)$ the inequality

$$|\mu|_U \stackrel{\text{def}}{=} \left\{ \sup_{\{Q_n\}\in\pi_T} \sum_n p_U(\mu(Q_n)) \right\} < \infty.$$

For $f \in C$ and $\mu \in M$ the vector integral $I(f,\mu) = \operatorname*{int}_T f\, d\mu$ is defined. The function $I : C \times M :\to \mathbb{R}^1$ is bilinear one and therefore the pair M, C may be considered as dual one with this duality.

Theorem 1. *Let X be reflexive space, and there exists a sequence of finite-dimensional projectors $X^* \to X^*$, convergenting to identical map pointwisely. Then a subset $M_0 \subset M$ is relatively compact in the weak topology $\sigma(M, C)$ if and only if the following conditions are fulfilled:*

$$\forall\, U \in \mathbb{U}(X) \quad \sup_{\mu\in M_0} |\mu|_U(T) < \infty;$$

for any bounded set $F \subset C$ and for any $\varepsilon > 0$ there exists a compact set $K \subset T$, such that

$$\sup_{\mu\in M_0,\, f\in F} \left| \operatorname*{int}_{T\setminus K} f\, d\mu \right| < \varepsilon.$$

The work has been supported by Russian Foundation for Basic Research (project 09-01-00677).

References.

[1] Uglanov A.V. Vector integrals. *Dokl. Acad. Nauk Russia*, 2000, Vol. 373, No 6, p. 737–740; English transl.: *Russian Acad. of Sci. Dokl.*, 2000.

[2] Uglanov A.V. On measures in locally convex spaces. *Progress in Probability*, Birkhauser Verlag Basel/Switzerland, 2003, Vol. 55, p. 43 - 54.

[3] Uglanov A.V. Vector integrals and its applications. *Dokl. Acad. Nauk Russia*, 2004, Vol. 394, no. 3, p. 1 – 4; English transl.: *Russian Acad. of Sci. Dokl.*, 2004.

[4] Pietsch A. *Nukleare lokalkonvexe räume.* Academie-Verlag, Berlin, 1965.

N. Uglesic
e-mail: nuglesic@unizd.hr

A duality between ultrametrics and monotone families of equivalence relations
Mathematics Subject Classification (MSC): 03E02; 54E35

Abstract. Monotone families (decreasing or increasing) of equivalence relations on a set and the (pseudo)ultrametrics on it are the two sides of the same coin.

Vesko Valov
Nipissing University, North Bay, Canada, ON, P1B 5C6
e-mail: veskov@nipissingu.ca

Parametric Bing and Krasinkiewicz maps
Mathematics Subject Classification (MSC): 54F15, 54F45, 54E40

Abstract. We prove the existence of a residual set of parametric Bing (resp., Krasinkiewicz) maps, and apply this result to establish some theorems for extensional dimension.

S. Varsaie
Institute for Advanced Studies in Basic Sciences,
P.O.Box 45195- 1159, Zanjan, Iran.
e-mail: mlthivagar@yahoo.co.in

ν- Manifolds
Mathematics Subject Classification (MSC): 58A50, 58C50

Abstract. ν- manifolds are superspaces which have naturally arisen in course of generalizing the concept of Chern class in supergeometry. Deducing from common geometry and classifying spaces one see's that the generalization relates to Manin's question[1]. Indeed his question is read as " In what homology theory there are classes of projective suerspaces say $P^{m|n}$?". In [2], it is shown that in a sensible homology theory all projective superspaces of equal even dimensions are homologous. This shows that in case of existing such homology theory this is not satisfactory. Indeed it does not carry any information on superstructures. Thus the lack of suitable extension for Chern classes in supergeometry may be rooted in the lack of a suitable generalization for projective spaces. In [3] through an analysis of projective superspaces, it is shown that these constructions are not suitable for studying superstructures. then ν-projective spaces and their canonical line bundles are introduced and their Chern classes are studied. In this talk ν- manifolds are introduced and some basic examples are discussed.

References.

[1] Yu. I. Manin and I. B. Penkov, "The formalism of left and right connections on supermanifolds", *Lectures on Supermanifolds, Geometrical Methods of Conformal Groups* (World Sci. 1989), p.3.

[2] S. Varsaie, "On Manin's question", *in prepration.*

[3] S. Varsaie, "ν- projective spaces, ν- line bundles and Chern classes", *Submitted to International Journal of Mathematics.*

Jan Harm van der Walt

Department of Mathematics and Applied Mathematics,
University of Pretoria,
Pretoria 0002 South Africa
e-mail: janharm.vanderwalt@up.ac.za

Sheaves of Uniform Convergence Spaces: Completions
Mathematics Subject Classification (MSC): 54A20, 54B40, 54E15, 18F20, 46F05, 46F99

Abstract. In this paper we introduce sheaves of uniform convergence spaces over a topological space, and the behavior of such sheaves with respect to completions of uniform convergence spaces. We consider the Weil completion of a Hausdorff uniform space, as well as the Wyler completion of a Hausdorff uniform convergence space. It is shown that structural properties of the sheaf remain invariant under these completions. Applications of these results are made to spaces of generalized functions that appear in the analysis of partial differential equations.

Władysław Wilczyński

Faculty of Mathematics and Computer Science
University of Łódź
PL-90-238 Łódź, Banacha 22, POLAND
e-mail: wwiluni.lodz.pl

Generalized continuous convergence
Mathematics Subject Classification (MSC): 40A05, 26A15

Abstract. Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of functions transforming a metric space (X,d) into a metric space (Y,ρ) and let $f : X \to Y$. A sequence $\{f_n\}_{n\in\mathbb{N}}$ is continuously convergent to f if and only if $f_n(x_n) \underset{n\to\infty}{\longrightarrow} f(x_0)$ whenever $x_n \underset{n\to\infty}{\longrightarrow} x_0$. Let $\mathcal{I} \subset \mathbb{N}$ be an ideal of sets. We say that $\{f_n\}_{n\in\mathbb{N}}$ is continuously $\mathcal{I}$-convergent to f if and only if $f_n(x_n)$ $\mathcal{I}$-converges to $f(x_0)$ whenever x_n $\mathcal{I}$-converges to x_0. Recall that x_n $\mathcal{I}$-converges to x_0 if and only if $\{n \in \mathbb{N} : d(x_n, x_0) \geq \epsilon\} \in \mathcal{I}$ for each $\epsilon > 0$. It is shown that this kind of convergence is more general than continuous convergence and that the limit function is always continuous.

Władysław Wilczyński and <u>Wojciech Wojdowski</u>

FACULTY OF MATHEMATICS, UNIVERSITY OF ŁÓDŻ,
90 - 238 ŁÓDŻ, Banacha 22, POLAND
e-mail: Wojwoj@gmail.com

A generalization of the Lebesgue density topology
Mathematics Subject Classification (MSC): 28A05, 54A10

Abstract. Wilczynski's reformulation of the Lebesgue density point given in [W] opened the possibility of studying more subtle properties of the notion of the density point and density topology, their various modifications and most of all category analogues.

In [W1] we introduced a notion of an $\mathcal{A}_d$−density point of measurable set on the real line as a generalization of the

Lebesgue density based on the definition given by Wilczyński. In [W1] and [W2] we proved that the $\mathcal{A}_d-$density topology generated by this notion is strictly finer then the Lebesgue density topology and is completely regular but not normal.

We construct an ascending sequence $\left\{\mathcal{T}_{\mathcal{A}_{d(n)}}\right\}$ of density topologies which leads to the $\left\{\mathcal{T}_{\mathcal{A}_{d(\omega)}}\right\}-$density topology including all previous topologies. We discuss also the notion in more general settings of $\sigma-$algebra S and $\sigma-$ideal I, with appropriate assumptions. We examine the continuity with respect to the topology.

[W] W. Wilczyński, *A generalization of the density topology*, Real. Anal. Exchange **8(1)** (1982-83), 16-20.

[W1] W. Wojdowski, *A generalization of density topology*, Real. Anal. Exchange,Vol. **32(2)**, (2006/2007), 1-10.

[W2] W. Wojdowski, *On a generalisation of the density topology*, Real. Anal. Exchange,Vol. **33(1)**, (2007/2008), 201-216.

Sophia Zafiridou

Departmant of Mathematics, University of Patras, Greece
e-mail: zafeirid@math.upatras.gr

Classification of dendrite with a countable set of end points

Mathematics Subject Classification (MSC): 54C25, 54F50

Abstract. Given a dendrite X we denote by $E(X)$ the set of end points of X and by $R(X)$ the set of ramification points of X. For every countable ordinal α the subcontinuum $X_{(\alpha)}$ of X is defined by induction as follows

$X_{(0)} = X$;

$$X_{(\alpha+1)} = \begin{cases} irr(\overline{R(X_{(\alpha)})}), & \text{if } R(X_{(\alpha)}) \neq \emptyset \\ \emptyset, & \text{if } R(X_{(\alpha)}) = \emptyset; \end{cases}$$

$X_{(\alpha)} = \bigcap_{\beta<\alpha} X_{(\beta)}$ for a limit ordinal α.

Let X be a dendrite with a countable set of end points.

The ramification degree of X is the smallest ordinal α such that $X_{(\alpha)} = \emptyset$.

The type of the set $E(X)$ is the smallest ordinal α such that the α-derivative of $E(X)$ is empty.

We discuss the relationship between the ramification degree of X and the type of the set $E(X)$ and prove that:

(i) For each countable ordinal α in the family of all dendrites such that the α-derivative of the set of end points is empty there is no universal element.

(ii) For each natural number $n > 0$ in the family of all dendrites with ramification degree $\leq n$ there exists a universal element.

A. V. Zarelua

Department of Mechanics and Mathematics,
Moscow State University,
Moscow 119991, Russia
e-mail: zareluaav@gmail.com

On p-Adic group actions and rings of continuous functions

Mathematics Subject Classification (MSC): 57S10

Abstract. If p-Adic group G acts on a compact space X and $Y = X/G$ is the orbit space then general theory of zero-dimensional mappings shows that the algebraic closure $Z[C(Y), C(X)]$ of the ring of continuous functions

$C(Y)$ in the ring $C(X)$ is dense in $C(X)$. The group G acts on $Z[C(Y), C(X)]$ and the ring $C(Y)$ is fixed under this action. The standard filtration of the group G generates a filtration both in the rings $C(X)$ and $Z[C(Y), C(X)]$ that correspond to the representation of the space X as an inverse system limit $X = \varprojlim X_i$ in the category of spaces over Y.

We study homological and homotopical structures of these spaces using algebraic and analytical structures of the ring $Z[C(X), C(X)]$. An important role plays here the integration along pre-images of points with respect, mainly, Haar mesure and the resulting symmetrization of functions and mappings. The convolution with characters gives rise to series of nice approximations of functions and mappings and lead to many relations between sheaf cohomology of the space X and its orbit space Y. Technically, these relations are obtained by manipulations with some exact sequences and spectral sequences of zero-dimensional mappings introduced previously by the author. In this way we obtain new results about connections between cohomological dimensions of the space X and its orbit space Y, and new versions of the famous theorems by C. T. Yang about the raising of the cohomology dimension when p-Adic group G acts effectively on a manifold.

V. Zinchenko and E. Yakovlev

Nizhniy Novgorod State University,
23 Prospekt Gagarina, 603950, Nizhniy Novgorod, Russia
e-mail: vera.zinchenko@gmail.com

Classification of the fiber bundles over polyhedron with a finite group of multivalued automorphisms
Mathematics Subject Classification (MSC): 55R15

Abstract. According to [1], principal fiber bundles $\xi = (E, p, B, T^k)$ with the projection map $p : E \to B$ and structure group T^k could be useful when studying the corresponding gyroscopic system Γ with the configurational manifold B and a multivalued action functional S. The existence of finite symmetry group Δ for Γ is equivalent to ξ having the structure of almost Δ-bundle [2]. Systems of such type can be also considered on manifolds with singularities. Since such spaces are usually triangulable, we expanded some results of [2] to the case when the base space B of the fiber bundle ξ is a polyhedron. We define almost Δ-bundles over polyhedron and build their classification in terms of the cohomology of the base B (Theorem 4).

Let $K(B)$ be a simplicial complex of a polyhedron B. Given an open $U \subset B$ and $q \in Z$ we define a differential q-form ω_U on U as follows: $\omega_U = \{\omega_{U\cap\sigma} | \sigma \in K(B)\}$, where $\omega_{U\cap\sigma}$ is a smooth q-form on non-empty intersection $U \cap \sigma$, $\sigma \in K(B)$, and if $\tau \in K(B)$ is a face of a simplex $\sigma \in K(B)$ then $\omega_{U\cap\sigma}|_{U\cap\tau} = \omega_{U\cap\tau}$. For the first time such construction for $U = B$ was considered by Thom and Whitney [3]. In particular, it allows to define smoothnesses on B and E that turn them into smooth premanifolds and ξ into a smooth fiber bundle.

Consider a regular simplicial Δ group action $R : B \times \Delta \to B$ on B. Let $\mathcal{U}$ be an open covering of B with R-invariant elements. $\mathcal{A}(\mathcal{U})$ is an atlas of ξ associated with $\mathcal{U}$, θ - the canonical 1-form on group T^k with values in it's Lie algebra $\mathfrak{t}$. If all the transition functions ξ_{VU} of the atlas $\mathcal{A}(\mathcal{U})$ are smooth and the transition forms $\theta d\xi_{VU}$ are R-invariant, we call $\mathcal{A}(\mathcal{U})$ almost Δ-atlas and ξ together with the equivalence class of the atlas $\mathcal{A}(\mathcal{U})$ - smooth almost Δ-fiber bundle. Together with the corresponding morphisms over B they form category $\mathcal{KP}(B, T^k, \Delta, R)$. The equivalence classes of the objects of this category $\mathcal{KP}(B, T^k, \Delta, R)$ form group $\mathcal{BP}(B, T^k, \Delta, R)$.

Proposition 1. For every $U \in \mathcal{U}$ it is possible to define a smooth $\mathfrak{t}$-valued 1-form ω_U such that the equalities $\omega_U - \omega_V = \theta d\xi_{VU}$ will hold on the intersections $U \cap V \neq \emptyset$.

Let $\omega_U^1, \ldots, \omega_U^k$ be the expansion coefficients of a forms ω_U on the canonical basis of algebra $\mathfrak{t}$ and $Exp : \mathbb{R}^k \to T^k = \mathbb{R}^k/\mathbb{Z}^k$ – the quotient map. Consider an arbitrary chart $\xi_U \in \mathcal{A}(\mathcal{U})$, a piecewise smooth path $x : I \to U$, an element $g_0^U \in T^k$ and a point $v = \xi_U(g_0^U, x(0))$. For every $t \in I$ we define path $x_t : I \to U$ as follows $x_t(s) = x(ts)$ and let

$$H_U(v, x)(t) = \xi_U(x(t), g_0^U - Exp(\text{int}_{x_t}\, \omega_U^1, \ldots, \text{int}_{x_t}\, \omega_U^k)).$$

Proposition 2. For a fibre bundle ξ there is a T^k-connection (path lifting operation) H such that $H|_U = H_U$ for each $U \in \mathcal{U}$.

The formula $R^U(\xi_U(a, t), \delta) = R_\delta^U(\xi_U(a, t)) = \xi_U(a \cdot \delta, t)$ defines Δ group action $R^U : E_U \times \Delta \to E_U$ on $E_U = p^{-1}(U)$.

Theorem 1. There is a T^k-connection H on $\xi = (E, p, B, T^k)$, which is invariant under the set of actions $\mathcal{R} = \{R^U | U \in \mathcal{U}\}$, if and only if ξ is an almost Δ-bundle.

Let's define a 2-form F on B as follows: $F|_U = d\omega_U$ for each $U \in \mathcal{U}$ and call it the base curvature form for the connection H. We denote the group of R-invariant $\mathfrak{t}$-valued smooth q-forms on B by $\Lambda_\Delta^q(B, \mathfrak{t})$, the cohomology group of the corresponding de Rham complex by $H_\Delta^q(B, \mathfrak{t})$, and its subgroup of cohomology classes of the forms of $\Lambda_\Delta^q(B, \mathfrak{t})$ with integer integrals over cycles by $H_\Delta^q(B, \mathfrak{t}|\mathbb{Z}^k)$.

Theorem 2. Let ξ be an almost Δ-bundle, F – a base curvature form of some T^k- and $\mathcal{R}$ - invariant connection H. Then $F \in \Lambda^2_\Delta(B, \mathfrak{t})$, cohomology class $[F]_\Delta \in H^2_\Delta(B, \mathfrak{t})$ lies in the subgroup $H^2_\Delta(B, \mathfrak{t}|\mathbb{Z}^k)$ and it is an invariant (a characteristic class) of the fiber bundle ξ in the category $\mathcal{KP}(B, T^k, \Delta)$. Formula $\eta([\xi]) = [F]_\Delta$, where $[\xi]$ is the equivalence class of the fiber bundle ξ, defines homomorphism $\eta : \mathcal{BP}(B, T^k, \Delta, R) \to H^2_\Delta(B, \mathfrak{t}|\mathbb{Z}^k)$.

Consider an almost Δ-bundle ξ such that $[\xi] \in Ker\eta$. With the help of an invariant connection H on ξ we define the holonomy homomorphism $\tau_H : H_1(B) \to T^k$. Let

$$\mathrm{Hom}^\Delta(H_n(B), \mathbb{R}^k) =$$

$$\{h \in \mathrm{Hom}(H_n(B), \mathbb{R}^k) | h([x \cdot \delta]) = h([x]) \ \forall \delta \in \Delta\}$$

and $\mathrm{Exp}^\Delta_* : \mathrm{Hom}^\Delta(H_1(B), R^k) \to \mathrm{Hom}(H_1(B), T^k)$ be a natural homomorphism induced by the map $\mathrm{Exp} : R^k \to T^k = R^k/Z^k$.

Theorem 3. The formula $\eta_0([\xi]) = \tau_H + \mathrm{im}\,\mathrm{Exp}^\Delta_*$ defines an isomorphism

$$\eta_0 : Ker\eta \to \mathrm{Hom}(H_1(B), T^k)/\,\mathrm{im}\,\mathrm{Exp}^\Delta_*.$$

Let $\mu = i \circ \eta_0^{-1}$, where $i : Ker\eta \to \mathcal{B}P(B, T^k, \Delta, R)$ is an inclusion.

Theorem 4. The sequence

$$0 \to \mathrm{Hom}(H_1(B), T^k)/\,\mathrm{im}\,\mathrm{Exp}^\Delta_* \xrightarrow{\mu} \mathcal{B}P(B, T^k, \Delta, R) \xrightarrow{\eta}$$

$$H^2_\Delta(B, R^k|Z^k) \to 0$$

is exact and splitting. Thus

$$\mathcal{B}P(B, T^k, \Delta, R) \cong$$

$$H^2_\Delta(B, R^k|Z^k) \oplus \mathrm{Hom}(H_1(B), T^k)/\,\mathrm{im}\,\mathrm{Exp}^\Delta_*.$$

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References

[1] E. I.Yakovlev "Bundles and geometric structures associated with gyroscopic systems," Journal of Mathematical Sciences, Vol. 153, No. 6, pp. 828 - 855, 2008.

[2] A. V. Ryzhkova, E. I.Yakovlev "Bundles with multivalued automorfism groups," Math. notes, v. 77, No. 3-4, pp. 553-567, 2005.

[3] D. Lemann "Homotopic theory of differential forms," Matematika. Novoe v zarubezhnoy nauke. Collected articles. M.: Mir, 1981, pp. 7 – 85.

Irina Zvina

Department of Mathematics, University of Latvia
Zellu 8, Riga, LV-1002, Latvia
e-mail: IrinaZvina@gmail.com

Frame embedding modulo compatible ideal: Application to generalized topological spaces and generalized spatial locales

Mathematics Subject Classification (MSC): 54A05, 06D22

Abstract. We consider the extension of Frame Embedding Theorem introduced in ICTA 2009 in Ankara. The extended theorem provides a better foundation for generalized topological spaces (gs-spaces). We discuss why the compatibility of the corresponding ideal of gt-space is the es-

sential property when dealing with interior operator and continuous mappings. Finally, we remind the notion of generalized spatial locale (gs-locale) also introduced in ICTA 2009 and show that the extended frame embedding theorem let us define the categories $\mathbf{GTop}_0$ (T_0 gt-spaces) and **GSLoc** (gs-locales) and consider the isomorphism of these categories.

Extended Frame Embedding Theorem. *Given a frame* $(T, \leq, \vee, \wedge)$ *and a complete Boolean lattice* $(F, \leq, \cup, \cap)$ *such that* $T \subseteq F$ *and* $\mathrm{id}\colon T \to F$ *is an order embedding preserving zero. Then there exists the least ideal* $I \subseteq F$ *such that:*

(i) for every $U \subseteq T$ *there is* $a \in I$ *such that* $\bigvee U = (\bigcup U) \cup a$*;*

(ii) for every $v, w \in T$ *there is* $b \in I$ *such that* $v \wedge w = (v \cap w) \setminus b$*;*

(iii) $I \cap T = \{0\}$*;*

(iv) for every $u, v \in T$*, it holds that* $u \setminus v \in I$ *iff* $u \leq v$*;*

(v) the ideal I *is compatible with* T*, t.i.* $a \in F$ *and* $U \subseteq T$ *with* $a \leq \bigvee U$ *and* $a \cap u \in I$*, for all* $u \in U$*, imply that* $a \in I$ *(compare with [1]).*

Given a nonempty set X and $\tau \subseteq 2^X$. The pair (X, τ) is called a *generalized topological space* provided that: $\varnothing, X \in \tau$, and $(\tau, \subseteq)$ is a frame. Given a frame T and a family $L \subseteq 2^T$. We call the pair (T, L) a *generalized spatial locale* provided that L *strongly separates elements of* T, t.i. every element $A \in L$ is a lower set and for every $u, v \in T$ with $v \nleq u$ there exists $A \in L$ such that $u \in A$ and $v \notin A$.

References

[1] Janković D., Hamlett T.R. *Compatible extensions of ideals.* Boll. Unione Mat. Ital., VII. Ser., B 6, No.3, 453-465 (1992)

O. V. Sipacheva
Moscow State University
e-mail: o-sipa@yandex.ru

Equations in Groups and Topologizability
Mathematics Subject Classification (MSC): 22A05, 54H11

Abstract. In 1944, Markov proved that any subset of a countable group which is unconditionally closed (that is, closed in any Hausdorff group topology on this group) must be algebraic (that is, representable as a finite union of solution sets of systems of equations) and asked whether this is true for any groups. This problem is closely related to other Markov's problem on the existence of a nondiscrete Hausdorff group topology on an arbitrary group, because a group is nontopologizable (does not admit such a topology) if and only if the complement of the identity element in this group is unconditionally closed.

In 1980, Ol'shanskii constructed an example of a countable nontopologizable group (in which the comlement to the identity is algebraic) and Shelah constructed a CH example of an uncountable nontopologizable group. In 2006, this author noticed that Shelah's group gives a consistent answer to Markov's question, namely, the complement to the identity element in this group is not algebraic. The author has found out since then that a ZFC example is contained in Hesse's 1979 unpublished dissertation.

The current state-of-the-art is delineated. It is proved

that any group can be embedded in a group with noncoinciding algebraic and unconditionally closed subsets.

A.B. Sossinsky
Independent University of Moscow
Poncelet Laboratory (CNRS-IUM)
e-mail: asossinsky@yandex.ru

Mechanical normal forms of knots
Mathematics Subject Classification (MSC):

Abstract. Vladimir Arnold once declared that "mathematics is that part of physics in which experiments are cheap." The talk will be an illustration of this thesis.

The first part will be a description of the "cheap experiments". In them, the behavior of a long, very thin, flexible, but very resilient cylindrical metallic tube will be demonstrated; this tube can be intertwined with itself, its extremities attached to each other, thus forming a knot, or more precisely, the tubular neighborhood of a knot. It will be shown that this device, under the action of its resilience (internal energy), almost instantly acquires a certain canonical position, which I call its *mechanical normal form*, or in more mathematical terms, the s-*normal form* of the knot, where s is the ratio of the length of the tube to the diameter of its cross section. For $s = 0.0005$, experiments show that the s-normal form is unique (up to isometry) for any knot diagram with 7 crossings or less.

These experiments are not models of any of the classical knot energies (studied by Moffat, Arnold, O'Hara, Freedman, Karpenkov, and others), because the energy of the tube (unlike, say, Möbius energy) has no self-repelling component, so that the tube usually has singularities: different parts of its boundary touch each other (but the the

s-normal form of the knot is nonsingular, i.e, is a smooth knot!). The device performs much better than computer simulations based on Möbius energy: for example, the figure eight knot has two different energy minima with respect to Möbius energy, but only one s-normal form.

The second part of the talk will be a mathematical discussion of the properties of s-normal forms. It will be shown that our device can perform Reidemeister moves (in certain situations), it can do both of the decreasing Markov moves for closed braids, it can carry out the Whitney trick (eliminate successive little loops provided that they are appropriately twisted). A mathematical description of the energy of our device will be presented and a few examples of computer animations (simulations) based on gradient descent of this energy in the space of tubular knots will be shown. The relationship between the non flat evolution of our device and the "jump number" (equal to the Whitney index of a given knot diagram minus the Whitney index of its mechanical normal form) will be specified.

It will be explained that the device does not always yield unique mechanical normal forms and a conjecture describing the cases when it doesn't will be formulated. We will also discuss *flat knots* which are, roughly speaking, modeled by our device placed between a plane horizontal table and a glass pane at the distance of $3s$ above the table. It turns out that the Kauffman bracket is an invariant of flat knots and some mechanical normal forms for flat knots will be presented.

T. E. Karakasidis and M. Peristeropoulou
University of Thessaly,
Pedion Areos, 38334 Volos, Greece

e-mail: thkarak@uth.gr

Application of phasespace reconstruction methods in system state identification
Mathematics Subject Classification (MSC):

Abstract. Phase space reconstruction methods are often employed in order to study system dynamics of systems in cases where the only information we have is time series of a system physical quantity. The methods recurrence plots (RP's) [1] and recurrence quantification analysis (RQA) [2] has attracted interest as a tool for time-series analysis through phase space reconstruction. In the present work we present an analysis of experimental results of laboratory experiments on a heated jet where temperature time series were recorded at various positions located on a cross section of the flow through RPs and RQA in conjunction with more conventional methods of analysis as average mutual information and correlation dimension [3]. The analysis shows that one can identify the region where the jet axis is located in accordance with the experimental results [4] and previous publications [5]. Close to the center of the jet, where we have fully developed turbulence, increased complexity is detected, while at the edges reduced complexity is observed.

References

[1] J.-P Eckmann, S.O. Kamphorst and D. Ruelle, Europhys. Lett. 4, 973 (1987).

[2] C.L. Jr, Webber, J.P. Zbilut, J. Appl. Physiol., 76, 965 (1994).

[3] H. Kantz and T. Schreiber, Nonlinear Time Series Analysis (Cambridge University Press, second edition, 2004).

[4] Michas SN, Papanicolaou PN, Horizontal round heated jets into calm uniform ambient, Desalination, 248, 803-815 (2009)

[5] Karakasidis TE, Liakopoulos A, Fragkou A, Papanicolaou P., Recurrence Quantification Analysis ofTemperature Fluctuations in a Horizontal Round Heated Jet, International Journal of Bifurcation and Chaos, 19, 2487-2498 (2009)

V. Farmaki and A. Koutsogiannis
Department of Mathematics,
Athens University, Athens 15784, Greece
e-mail: vfarmaki@math.uoa.gr,akoutsos@math.uoa.gr

Topological formulation of partition theorems in infinitary Ramsey theory
Mathematics Subject Classification (MSC):

Abstract. Fundamental results in infinitary Ramsey theory, such as the van der Waerden theorem and the Hindman theorem, have received equivalent topological formulations, in the work by Furstenberg and Weiss (J. d'analyse Math. 34, 1978), a formulation that proved useful, among others, in the deep result by Furstenberg and Katznelson on the density version of the Hales-Jewett theorem (J. d'analyse Math. 57, 1991). In a resent work (to appear) we have obtained strong combinatorial theorems, (unifying and extending the van der Waerden, Hindman, Carlson (Discrete Math. 68, 1988), and Farmaki-Negrepontis (Trans. Amar. Math. Soc. 358, 2006 and 360, 2008) theorems), involving the notion of w-Z*-located words over a countable (and not only finite) alphabet, and concerning Ramsey-type partition theorems for finite sequences of

such words over Schreier-type families. As every rational number can be represented as an w-Z*-located word, corresponding partition theorems may be obtained for the rational numbers. In the present work we obtain equivalent topological formulations of these combinatorial results, in the style of Furstenberg-Weiss.

Taras Banakh, Arkady Leiderman

National University of Lviv, Ukraine; Ben-Gurion University of the Negev, Israel
e-mail:arkady@math.bgu.ac.il

Uniform Eberlein compactifications of metrizable spaces

Mathematics Subject Classification (MSC):

Abstract. We prove that each metrizable space X (of size $|X| \leq \mathfrak{c}$) has a (first countable) uniform Eberlein compactification and each scattered metrizable space has a scattered hereditarily paracompact compactification. Each compact scattered hereditarily paracompact space is uniform Eberlein and belongs to the smallest class A of compact spaces, that contain the empty set, the singleton, and is closed under producing the Aleksandrov compactification of the topological sum of a family of compacta from the class A.

Alexander Sostak

University of Latvia, Department of Mathematics,
Riga, LV-1002 Latvia
e-mail:sostaks@lanet.lv

$\mathbb{M}$-approximative systems: a unified approach to Fuzzy Topology and Rough Sets

Mathematics Subject Classification (MSC): 54A40

Abstract. In 1968, that is only 3 years after L. Zadeh has published his famous work *"Fuzzy Sets"* [13], thus laying down the principles of what can be called *Mathematics of Fuzzy Sets*, his student C.L. Chang [1] introduced the concept of a fuzzy topological space thus marking the beginning of Fuzzy Topology, the conterpart of General Topology in the context of fuzzy sets. Later essentially different approaches to the concept of fuzzy topology see e.g. [2], [4], [7], [8], [6], [11] and now Fuzzy Topology is one of the most well developed fields of Mathematics of Fuzzy Sets.

In 1982 Z. Pawlak [5] has introduced the concept of a rough set which can be viewed as a certain alternative for the concept of a fuzzy set for the study of mathematical problems of applied nature. Pawlak's work was followed by many other publications where rough sets and mathematical structures on the basis of rough sets were introduced, studied, and applied.

Although at the first glance it may seem that the concepts of a fuzzy set, of a (fuzzy) topological space and of a rough set are of an essentially different nature and "have nothing in common", this is not the case. The works of different authors were devoted to the study of the relations between different concepts of this type, see e.g. [3], [11].

In [9] we introduced the category of $\mathbb{M}$-approximative systems generalizing all categories related to fuzzy sets, fuzzy topology and rough sets and presenting a unified approach to their study. This work was followed by [10]. The aim of this talk is to present introduction into the theory of $\mathbb{M}$-approximative systems and to discuss some recently obtained results.

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2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008.

References

[1] C.L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182–190.

[2] J.A. Goguen, *The fuzzy Tychonoff theorem*, J. Math. Anal. Appl., **43** (1973), 734–742.

[3] J. Kortelainen, On relationship between modified sets, topological spaces and rough sets, Fuzzy Sets and Syst. **61** (1994), 91-95.

[4] T. Kubiak, *On Fuzzy Topologies*, PhD Thesis, Adam Mickiewicz University, Poznań, Poland, 1985.

[5] Z. Pawlak, *Rough sets*, International J. of Computer and Information Sciences, **11** (1982), 341-356.

[6] S.E. Rodabaugh, A categorical accomodation of various notions of fuzzy topology, *Fuzzy Sets and Syst.*, **9** (1983), 241-265.

[7] A. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Palermo, Ser II **11** (1985), 125-186.

[8] A. Šostak, *Two decades of fuzzy topology: Basic ideas, notions and results*, Russian Math. Surveys, **44** (1989), 125–186

[9] A. Šostak *On approximative fuzzy operators.* In: 1st Czech – Latvian Seminar on Fuzzy Sets and Soft Computing, November 20–23, 2008, Trojanice, Czech Republik, Abstracts, pp.

[10] A. Šostak *On approximative operators and approximative systems*, In: Proceedings of Congress of the International Fuzzy System Association "IFSA2009", July 20–24, 2009, Lisbon, Portugale, pp.

[11] Y.Y. Yao, *A comparative study of fuzzy sets and rough sets*, Inf. Sci., **109** (1998), 227 - 242.

[12] Ying Mingsheng, *A new approach to fuzzy topology, Part* I, Fuzzy Sets and Syst., **39** (1991), 303-321.

[13] L.A. Zadeh, *Fuzzy sets*, Inform. and Control, **8** (1965), pp. 338–353.

Ingrida Uljane and Alexander Šostak
University of Latvia, Department of Mathematics,
Riga, LV-1002 Latvia
e-mail:uljane@mail.com

On bornological-type structures in the context of L-fuzzy sets
Mathematics Subject Classification (MSC): 54A40

Abstract. In [8], [9] S.T. Hu studied the problem of the possibility to define the concept of boundedness in a topological space. To do this he introduced a system of axioms which later gave rise to the concept of a bornology and a bornological space. In a certain sense a bornological space can be viewed as a counterpart of a topological space if one is mainly interested in the property of boundedness of mappings and not in their property of continuity. At the first stage of research bornological structures were mainly considered on Banach or, more general, on linear topological spaces, but later the reserach was extended to topological spaces without any linear structure, see e.g. [6], [2].

In the paper [1] the concept of a L-bornology (where L is a complete infinitely distributive lattice, or more generally a cl-monoid) on a set X was introduced. An L-bornology on a set X is a subset of the family L^X of its L-subsets $\mathcal{B} \subseteq \mathcal{L}^{\mathcal{X}}$ such that (1) $\mathcal{B}(\S) = \infty \ \forall \S \in \mathcal{X}$, (2) $U \leq V$ and $V \in \mathcal{B} \Longrightarrow \mathcal{V} \in \mathcal{B} \ \forall \mathcal{U}, \mathcal{V} \in \mathcal{L}^{\mathcal{X}}$, (3) $U, V \in \mathcal{B} \Longrightarrow \mathcal{U} \vee \mathcal{V} \in \mathcal{B} \ \forall \mathcal{U}, \mathcal{V} \in \mathcal{L}^{\mathcal{X}}$. A mapping $f : (X, \mathcal{B}_{\mathcal{X}}) \to (\mathcal{Y}, \mathcal{B}_{\mathcal{Y}})$ is called bounded if $f(U) \in \mathcal{B}_{\mathcal{Y}} \ \forall \mathcal{V} \in \mathcal{B}_{\mathcal{X}}$. L-bornological spaces and their bounded mappings form a category L-BORN whose basic properties were first considered in [1].

Further, in our talk at the conference "Mathematical Modelling and Analysis", the concept of a many-valued bornology on a set X was introduced [10]. Actually, a many-valued bornology on a set X is a mapping $\mathfrak{B} : 2^X \to L$ satisfying the following properties (1) $\mathfrak{B}(x) = 1 \ \forall x \in X$, (2) $U \subseteq V \Longrightarrow \mathfrak{B}(U) \geq \mathfrak{B}(V) \ \forall U, V \in 2^X$, (3) $\mathfrak{B}(U \cup V) \geq \mathfrak{B}(U) * \mathfrak{B}(V) \ \forall U, V \in 2^X$, where $*$ is an arbitrary t-norm on L, in particular, $* = \wedge$. A mapping $f : (X, \mathfrak{B}_X) \to (Y, \mathfrak{B}_Y)$ is called bounded if $\mathfrak{B}(U) \leq \mathfrak{B}_Y(f(U)) \ \forall U \subseteq X$. L-valued bornologies and bounded mappings between them form a category $BORN(L)$.

While category L-BORN is a certain bornological counterpart of the category of L-topological spaces in the sense of Chang-Goguen [3], [4], [5] when introducing the category BORN(L) we had in mind the category of fuzzifying topological spaces in the sense of Hóhle-Ying [7], [11] as its topological analogue.

The aim of this talk to go further in the study of L-bornological and L-valued bornological spaces and the corresponding categories. In particular we show that the families of L-bornologies and L-valued bornologies on a set X ordered in a natural way is a complete infinitely distributive lattice and prove that the categories L-BORN and BORN(L) are topological over the category of sets **SET**.

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References

[1] M. Abel, A. Šostak, Towards the theory of L-bornological spaces, *Iranian Journal of Fuzzy Systems*, (2010), to appear.

[2] G Beer, Metric bornologies and Kuratowska-Painleve convergence to the empty set, *Journal of Convex Analysis* **8** (2001), 273–289.

[3] C.L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, **24** (1968), 182–190.

[4] J.A. Goguen, L-fuzzy sets, *J. Math. Anal. Appl.*, **18** (1967) 145–174.

[5] J.A. Goguen, The fuzzy Tychonoff theorem, *J. Math. Anal. Appl.*, **18** (1973), 734–742.

[6] H. Hogbe-Nlend Bornology and Functional Analysis, *Math. Studies* **26**, North-Holland, Amsterdam, 1977.

[7] U. Höhle, Upper semicontinuous fuzzy sets and applications, *J. Math. Anal. Appl.*, **78** (1980), 659–673.

[8] S.-T. Hu Boundedness in a topological space, *J. Math. Pures Appl.*, **28** (1949), 287–320.

[9] S.-T. Hu Introduction to General Topology , Holden-Day, San-Francisko, 1966.

[10] I. Uljane, A. Šostak, On many-valued bornological structures 15^{th} *International Conference "Mathematical Modelling and Analysis"*, Druskeninkai, Lithuania, May 26–29, 2010, Abstracts, p. 106.

[11] Ying Mingsheng, A new approach to fuzzy topology, Part I, *Fuzzy Sets and Syst.*, **39** (1991), 303–321.

Sergey Matveev

Chelyabinsk State University, Chelyabinsk,
454001, Russia
e-mail:matveev@csu.ru

Prime Decompositions and the Diamond Lemma
Mathematics Subject Classification (MSC): 57M

Abstract. We develop a new version of the famous Diamond Lemma [1] and describe several results on prime decompositions of different geometric objects. All results are obtained by using that version and the standard technics for removing intersections of surfaces.

(i) The Kneser-Milnor prime decomposition theorem of 3-manifolds into connected sums of prime factors (new proof).

(ii) The similar theorem of Swarup for decompositions into boundary connected sums (new proof).

(iii) A prime decomposition theorem for knotted graphs in 3-manifolds containing no non-separating 2-spheres.

(iv) Counterexamples to prime decomposition theorems for knots in 3-manifolds and for 3-orbifolds.

(v) A new theorem on annular splittings of 3-manifolds, which is independent of the JSJ-splitting theorem.

(vi) An existence and uniqueness theorem for prime decompositions of homologically trivial knots in direct products of surfaces and intervals.

(vii) A theorem on the exact structure of the semigroup of theta-curves in 3-manifolds.

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References [1] M. H. A. Newman, On theories with a combinatorial definition of "equivalence", Ann. Math. 43 (1942), 223-243.

A. Bartoszewicz, M. Filipczak, and T. Natkaniec
Institute of Mathematics,
University of Gdańsk, Poland
e-mail: mattn@mat.ug.edu.pl

Weak and strong forms of Smital properties
Mathematics Subject Classification (MSC): Primary 22A30, 28C10; Secondary 28A05, 22A10.

Abstract. Let $\langle X, +\rangle$ be a group, τ be a topology on X, $\mathcal{A} \subset \mathcal{P}(X)$ be an algebra and $\mathcal{J}$ be an ideal in $\mathcal{P}(X)$. We will say that the set $X_0 \subset X$ is

- $\langle\mathcal{A}, \mathcal{J}\rangle$-large if $Y \in \mathcal{A}\backslash\mathcal{J}$;
- $\mathcal{J}$-semilarge if $Y \notin \mathcal{J}$;
- $\mathcal{J}$-residual if the complement Y^c of Y belongs to $\mathcal{J}$;

- $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual if Y^c does not contain any $\langle \mathcal{A}, \mathcal{J} \rangle$-large set.

A triple $\langle \mathcal{A}, \mathcal{J}, \tau \rangle$ has

(i) the Steinhaus Property (SP) if for any $\langle \mathcal{A}, \mathcal{J} \rangle$-large sets A, B the set $-B + A$ has an interior point;

(ii) the Extended Steinhaus Property (ESP) if, for any $\langle \mathcal{A}, \mathcal{J} \rangle$-large set A and $\mathcal{J}$-semilarge set B, the set $-B + A$ has an interior point;

(iii) the Smital Property (SmP) if, for any $\langle \mathcal{A}, \mathcal{J} \rangle$-large set A and any dense set D, the set $A + D$ is $\mathcal{J}$-residual;

(iv) the Extended Smital Property (ESmP) if, for any $\mathcal{J}$-semilarge set B and any dense set D, $B + D$ is $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual.

It is known that under some natural assumptions on τ, $\mathcal{A}$ and $\mathcal{J}$ all above properties are equivalent. We will generalize the Smital properties in two directions.

Weak forms. Assume that τ is invariant. The triple $\langle \mathcal{A}, \mathcal{J}, \tau \rangle$ has:

- the Weaker Smital Property (WSmP), if there exists a dense set $D \subset X$ such that $|D| = d(X)$ and $(A + D)^c \in \mathcal{J}$ for each $A \in \mathcal{A} \setminus \mathcal{J}$;

- the Weaker Extended Smital Property (WESmP), if there exists a dense set $D \subset X$ such that $|D| = d(X)$ and $Y + D$ is $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual for each $Y \notin \mathcal{J}$;

- the Weak Smital Property (W^{2}SmP), if for each $A \in \mathcal{A} \setminus \mathcal{J}$ there exists a dense set $D \subset X$ such that $|D| = d(X)$ and $(A + D)^c \in \mathcal{J}$;

- the Weak Extended Smital property (W^{2}ESmP), if for each $Y \subset X$, $Y \notin \mathcal{J}$ there exists a dense set $D \subset X$ such that $|D| = d(X)$ and $Y + D$ is $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual.

Strong forms. For $A, B \subset X$ define

$$A +_{\mathcal{J}} B = \{z \in X : A \cap (z - B) \notin \mathcal{J}\}$$

We say that the triple $\langle \mathcal{A}, \mathcal{J}, \tau \rangle$ has:

- the Strong Smital Property (SSmP) if for every $A \in \mathcal{A} \setminus \mathcal{J}$ and each dense set $D \subset X$, if $D \notin \mathcal{J}$ then the set $A +_{\mathcal{J}} D$ is $\mathcal{J}$-residual;

- the Strong Extended Smital Property (SESmP) if for every $A \notin \mathcal{J}$ and dense set $D \subset X$, if $D \notin \mathcal{J}$ then the set $A +_{\mathcal{J}} D$ is $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual

- the S^{2}SmP (S^{2}ESmP, respectively) iff for each $A \in \mathcal{A} \setminus \mathcal{J}$ ($Z \notin \mathcal{J}$) and for every $\mathcal{J}$-dense set D, the set $A +_{\mathcal{J}} D$ is $\mathcal{J}$-residual ($Z +_{\mathcal{J}} D$ is $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual);

- the S^{3}SmP (S^{3}ESmP) iff for each $A \in \mathcal{A} \setminus \mathcal{J}$ ($Z \notin \mathcal{J}$) and for every $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual set D, the set $A +_{\mathcal{J}} D$ is $\mathcal{J}$-residual ($Z +_{\mathcal{J}} D$ is $\langle \mathcal{A}, \mathcal{J} \rangle$-semiresidual).

We consider relationships between those properties. Some examples are constructed, some open question will be posed.

References [1] A. Bartoszewicz, M. Filipczak, T. Natkaniec, *On Smital properties*, a manuscript.

Dmitri Shakhmatov and Dikran Dikranjan
Division of Mathematics,
Physics and Earth Sciences,
Graduate School of Science and Engineering,
Ehime University,
Matsuyama 790-8577, Japan
e-mail: dmitri@dpc.ehime-u.ac.jp

Topological properties of the Markov-Zariski topology of an abelian group
Mathematics Subject Classification (MSC): 20A99 (Primary), 20A45, 22A05, 54H11 (Secondary)

Abstract. In 1944, Markov [Mar] introduced four special families of subsets of a group G. A subset X of a group G is called:

(a) *elementary algebraic* if there exist an integer $n > 0$, elements $a_1, \dots, a_n \in G$ and $\varepsilon_1, \dots, \varepsilon_n \in \{-1, 1\}$, such that

$$X = \{x \in G : x^{\varepsilon_1} a_1 x^{\varepsilon_2} a_2 \dots a_{n-1} x^{\varepsilon_n} a_n = 1\},$$

(b) *algebraic* if X is an intersection of finite unions of elementary algebraic subsets of G,

(c) *unconditionally closed* if X is closed in *every* Hausdorff group topology on G,

(d) *potentially dense* if G admits *some* Hausdorff group topology $\mathcal{T}$ such that X is dense in $(G, \mathcal{T})$.

The family of all unconditionally closed subsets of G coincides with the family of closed sets of a T_1 topology $\mathfrak{M}_G$ on G, namely the infimum (taken in the lattice of all topologies on G) of all Hausdorff group topologies on G.

This topology has been introduced in [DS], [OPIT] [DS] [JGT] as the *Markov topology* of G.

Recall that a Hausdorff group topology $\mathcal{T}$ on a group G is called *precompact* (or *totally bounded*) provided that $(G, \mathcal{T})$ is (isomorphic to) a subgroup of some compact Hausdorff group or, equivalently, if the completion of $(G, \mathcal{T})$ with respect to the two-sided uniformity is compact. Let $\mathfrak{P}_G$ be the infimum of all precompact Hausdorff group topologies on G. Clearly, $\mathfrak{P}_G$ is a T_1 topology on G, which we call the *precompact Markov topology* of G [DS] [OPIT].

One can easily see that the family of all algebraic subsets of G is closed under finite unions and arbitrary intersections, and contains G and all finite subsets of G; thus, it can be taken as the family of closed sets of a unique T_1 topology $\mathfrak{Z}_G$ on G. Markov [Mar1], [Mar] defined the *algebraic closure* of a subset X of a group G as the intersection of all algebraic subsets of G containing X, i.e., the smallest algebraic set that contains X. This definition satisfies the conditions necessary for introducing a topological closure operator on G. Since a topology on a set is uniquely determined by its closure operator, it is fair to say that Markov was the first to (implicitly) define the topology $\mathfrak{Z}_G$, though he did not name it. To the best of our knowledge, the first name for this topology appeared explicitly in print in a 1977 paper by Bryant [Bryant], who called it a *verbal topology* of G. In a more recent series of papers beginning with [BMR], Baumslag, Myasnikov and Remeslennikov have developed algebraic geometry over an abstract group G. In an analogy with the celebrated Zariski topology from algebraic geometry, they introduced the *Zariski topology* on the finite powers G^n of a group G. In the particular case when $n = 1$, this topology coincides with the verbal topology of Bryant. For this reason, the topology $\mathfrak{Z}_G$ is also called the

Zariski topology of G in [DS] [OPIT], [DS] [JGT].

Note that $(G, \mathfrak{Z}_G)$, $(G, \mathfrak{M}_G)$ and $(G, \mathfrak{P}_G)$ are quasi-topological groups, i.e., their inversion and shifts are continuous. The inclusion $\mathfrak{Z}_G \subseteq \mathfrak{M}_G \subseteq \mathfrak{P}_G$ holds for every group G.

In 1944, Markov [Mar1] (see also [Mar]) posed his celebrated problem: *is every unconditionally closed subset of a group algebraic*? Using the language of Markov and Zariski topologies, this question can be naturally reformulated as the problem of coincidence of these topologies: *does the equality $\mathfrak{Z}_G = \mathfrak{M}_G$ hold for every group G*? Markov himself obtained a positive answer in the case when G is countable [Mar]. The authors proved that $\mathfrak{Z}_G = \mathfrak{M}_G = \mathfrak{P}_G$ for every abelian group G; see [JA]. (The proof of the first equality has appeared already in [DS] [JGT].)

In this talk, we overview topological properties of the Markov-Zariski topology of an abelian group. This topology is known to be Noetherian [Bryant], so hereditarily compact (=every subset is compact), and thus, notoriously non-Hausdorff. Among other things, we show that the Markov-Zariski topology of an abelian group is always hereditarily separable and Fréchet-Urysohn.

References

[BMR] G. Baumslag, A. Myasnikov, V. Remeslennikov, *Algebraic geometry over groups. I. Algebraic sets and ideal theory*, J. Algebra 219 (1999) 16–79.

[Bryant] R. Bryant, *The verbal topology of a group*, J. Algebra 48 (1977) 340–346.

[DS] [OPIT] D. Dikranjan, D. Shakhmatov, *Selected topics from the structure theory of topological groups*, pp. 389-406 in: Open Problems in Topology II (E. Pearl, ed.), Elsevier 2007.

[DS] [JGT] D. Dikranjan, D. Shakhmatov, *Reflection principle characterizing groups in which unconditionally closed sets are algebraic*, J. Group Theory 11 (2008) 421-442.

[JA] D. Dikranjan, D. Shakhmatov, *The Markov-Zariski topology of an abelian group*,
J. Algebra (2010), doi:10.1016/j.jalgebra.2010.04.025 (available also as arXiv:1005.1149 preprint).

[Mar1] A. A. Markov, *On unconditionally closed sets*, Comptes Rendus Dokl. AN SSSR (N.S.) 44 (1944) 180–181 (in Russian).

[Mar] A. A. Markov, *On unconditionally closed sets*, Mat. Sbornik 18 (1946), 3–28 (in Russian). English translation in: Markov, A. A. Three papers on topological groups: I. On the existence of periodic connected topological groups, II. On free topological groups, III. On unconditionally closed sets. Amer. Math. Soc. Translation 1950, (1950). no. 30, 120 pp.; yet another English translation in: Topology and Topological Algebra, Translations Series 1, vol. 8, pp. 273–304. American Math. Society, 1962.

Alexander Dranishnikov
University of Florida,
Department of Mathematics,
P.O. Box 118105, 358 Little Hall,
Gainesville, FL 32611-8105
e-mail: dranish@math.ufl.edu

The Lusternik-Schnirelmann-category and the fundamental group

Mathematics Subject Classification (MSC): 55M30

Abstract. Whitehead's theorem states that the Lusternik-Schnirelmann category $\operatorname{cat} X$ of a simply connected com-

plex X does not exceed $\dim X/2$. We show that basically the same holds true for complexes whose fundamental group has small cohomological dimension. Precisely, we prove that

$$\operatorname{cat} X \leq cd(\pi_1(X)) + \left\lceil \frac{\dim X - 1}{2} \right\rceil$$

where $cd(\pi_1(X))$ denotes the cohomological dimension of the fundamental group of X. The proof uses a trick from dimension theory which goes back to Kolmogorov.

References

[CLOT] Cornea, O.; Lupton, G.; Oprea, J.; Tanré, D.: Lusternik-Schnirelmann category. *Mathematical Surveys and Monographs*, **103**. American Mathematical Society, Providence, RI, 2003. MR1990857

[Br] Brown, K.: Cohomology of groups, Springer, 1982. MR0672956

[DKR] Dranishnikov, A.; Katz, M.; Rudyak, Y.: Small values of Lusternik-Schnirelmann category and systolic categories for manifolds. Geometry and Topology, 12 (2008) no. 3, 1711-1727. MR2421138

[Dr] Dranishnikov, A.: On the Lusternik-Schnirelmann category of spaces with 2-dimensional fundamental group. Proc. Amer. Math. Soc. 137 (2009), no. 4, 1489–1497. MR2465675

[EG] Eilenberg, Samuel; Ganea, Tudor: On the Lusternik-Schnirelmann category of abstract groups. *Ann. of Math.* (2) **65** (1957), 517–518. MR0085510

[Gro] Grossman, D.P.: On the estimate for the category of Lusternik-Schnirelmann, *Dokl. Akad. Nauk SSSR* (1946), 109-112. MR0018820

[Hu] Hurewicz, Witold. On the concept of fiber space. *Proc. Nat. Acad. Sci. U. S. A.* 41 (1955), 956•961. MR0073987

[Os] Ostrand, Ph. : Dimension of metric spaces and Hilbert's problem 13, Bull. Amer. Math. Soc. 71 (1965), 619-622. MR0177391

[Stal] Stallings, J.: Groups of dimension 1 are locally free. *Bull. Amer. Math. Soc.* **74** (1968), 361–364. MR0223439

[St] Strom, J.: Lusternik-Schnirelmann category of spaces with free fundamental group. *AGT*, **7** 2007, 1805-1808. MR2366179

[Sv] Švarc, A.: The genus of a fibered space. *Trudy Moskov. Mat. Obšč* **10, 11** (1961 and 1962), 217–272, 99–126, (in *Amer. Math. Soc. Transl.* Series 2, vol **55** (1966)). MR0154284

[Swan] Swan, R.: Groups of cohomological dimension one. *J. Algebra* **12** (1969), 585–610. MR0154284

[Wh] Whitehead, G.W.: The homotopy suspension *Colloque de Topologie algebrique tenu á Louvain* (1956) 89-95. MR0094794

Wuria Muhammad Ameen Hussein
Department of Mathematics,
College of Science,
Salahaddin University-Hawler,
Erbil-Iraq.
e-mail: wuria_khoshnaw@yahoo.com

Strongly Semi Open Sets, $\delta\delta$ -Sets and More about δ-Sets in Topological Spaces

Mathematics Subject Classification (MSC): 54A05, 54C10

Abstract. The aim of this paper is to define two types of generalized open sets in topological spaces with our acknowledgment, one of them called strongly semi open set and the other called $\delta\delta$-set. By using these sets, we may add some implications and finding some relations between the sets which given in it, also we investigated some properties of strongly pre open sets and $\delta\delta$-sets, some more properties of -continuous functions have been investigated.

REFERENCES

[1] N.Levin, semi open sets and semicontinuity in topological space, Amer. Math. Monthly, 70 (1963), 36-41.

[2] O.Njasted, on some classes of nearly open sets, Pacific J.Math., 15 (1965), 961-970.

[3] Mashour A.S, Abd El-Monsef M.E and El-Deeb S.N., on pretopological spaces, Bull. Math. De la Soc. R.S. de Roumanie 28(76), (1984), 39-45.

[4] M.E. Abd-El-Monsef, S.N. Al-Deeb and R.A.Mahmoud, -open sets and -continuous mappings, Bull.Fac.Sci.Assiut Univ., (1983), 1-18.

[5] D.E.Cameron, properties of S-closed spaces, PAMS 72(3), (1978), 581-586.

[6] S.F. Tadros and A.B.Khalaf, on regular semi-opens and S*-closed spaces, The Journal of the college of Education, Salahaddin Univ.,2,4,(1990).

[7] Chattopadhyay C. and Bandyopadhyay C., on structure of -sets, Bull. Cal. Math. Soc. 83, 281-290, (1991).

[8] Chattopadhyay C., on strongly pre-open sets and a decomposition of continuity, 57, 121-125, (2005).

[9] Sundaram P. and Nagaveni N., on weakly generalized closed sets, weakly generalized continuous maps and Twg-spaces in topological spaces. , Bull. Fukuoka Univ. Ed. Part III, 48, 33-40, (1997).

A. M. Gurin
Kharkov, Ukraina
e-mail: alexgu101@ya.ru

On topology of the arrangement of balls on a sphere and Stoker's problem
Mathematics Subject Classification (MSC):

Abstract. Arbitrary topology of the arrangement of equal balls on the central sphere was considered. New examples of topology complyting with the Coxeter's definition of any conditionally steady topology are found.

Under the requirement of stability of topology some conditionally steady topologies become steady as a whole.

Examples of such topologies are given. The existence of topology steady as a whole results in the statement of validity of Stoker's hypothesis for polyhedra of corresponding topology.

B. A. Pasynkov
Moscow State University, Russia
e-mail: bpasynkov@gmail.com

What is a non-metrizable analog of metrizable compacta?
Mathematics Subject Classification (MSC):

Abstract. The metrizable compacta is a model class of spaces in general topology. It would be very interesting

to find a class $\mathcal{C}$ of non-metrizable compacta, properties of which are closed to ones of metrizable compacta. In particular, 1) every metrizable space X must have a compactification $cX \in \mathcal{C}$ with $w(cX) = w(X)$ and 2) basic dimensional properties of $\mathcal{C}$ must be similar to ones of the class of metrizable compacta.

Below a space is a topological space. The Alexandroff sequence A_τ of weight τ is the one-point compactification of the discrete space D_τ of weight τ.

Definition 1. A family λ of subsets of a space X will be called *jointly functionally open* if there exists a continuous function $f : X \to [0,1]$ such that

$(*)$ $\cup\lambda = f^{-1}(0,1]$ and for any $O \in \lambda$, the function f_O with $f_O|_O = f|_O$ and $f_O(x) = 0$, $x \in X \setminus O$, is continuous.

Definition 2. A family λ of open subsets of a space X will be called *weakly discrete in entourage* O_λ if $O_\lambda \in \lambda$, $\cup\lambda = O_\lambda$, all $O \in \lambda^- = \lambda \setminus \{O_\lambda\}$ are closed in O_λ and λ^- is disjoint. If, additionally, λ^- is dense in O_λ (i.e. $O_\lambda \subset cl \cup \lambda^-$), then λ is called *dense weakly discrete in entourage* O_λ.

Definition 3. We shall say that a compactum X *has property* $(\mathfrak{M})$ if

$(**)$ there exists an open family λ in X that T_0-separates X and is the union of jointly functionally open and dense weakly discrete families $\lambda(i)$ in entourages $O_{\lambda(i)}$, $i \in \mathbb{N}$,

and X *has weak property* $(\mathfrak{M})$ if in $(**)$ families $\lambda(i)$ are not necessary dense in entourages $O_{\lambda(i)}$.

Below, $Q \prod\{I_i = [0,1] : i \in \mathbb{N}\}$, pr_i is the projection of the product Q onto the factor I_i, $B = \{Q_i = pr_i^{-1}(0,1] : i \in \mathbb{N}\}$; $A_{\tau i}$ is a copy of A_τ and $D_{\tau i}$ consists of all isolated points of $A_{\tau i}$.

Theorem 1. *For a compactum X the following conditions are equivalent:*

1. X has weak property (has property) $(\mathfrak{M})$;

2. there exists a topological embedding e of X in the partial topological product $\Psi^\tau = P(Q, B, \{A_{\tau i} : i \in \mathbb{N}\}$ (and the intersection of eX with the subproduct $\Phi^\tau = P(Q, B, \{D_{\tau i} : i \in \mathbb{N}\})$ of the partial product Ψ^τ is dense in eX).

Note that Φ^τ is metrizable.

Addition to Theorem 1. *If X has property $(\mathfrak{M})$, then we have $(**)$ so that* (see Definition 2) *for $R_{\lambda(i)} = O_{\lambda(i)} \setminus (\cup\lambda(i))^-$ and $M_\lambda = X \setminus \cup\{R_{\lambda(i)} : i \in \mathbb{N}\}$, $\lambda(i\cap) = M_\lambda \wedge (\lambda(i))^-$ is discrete in M_λ, $\cup\{\lambda(i\cap) : i \in \mathbb{N}\}$ is a base for M_λ and M_λ is a dense G_δ-set in X (and so M_λ is completely metrizable and d-posed in X and dim $M_\lambda \leq$ dim X).*

Corollary 2. Ψ^τ *has property $(\mathfrak{M})$ and it is a universal element in the class of all compacta of weight $\leq \tau$ having week property $(\mathfrak{M})$.*

Theorem 2. *Every metrizable space X of weight τ has an $(\mathfrak{M})$-compactification cX (i.e., cX has property $(\mathfrak{M})$) of weight τ. Besides, if dim $X = n$, then we can suppose that dim $cX = n$, $n = 0, 1, \ldots$*

Every metrizable compactum has $(\mathfrak{M})$, every compactum X having weak $\mathfrak{M}$ is an Eberlein compactum and so it is a Fréchet-Urysohn space; $w(X) = d(X)$; if X has weak $(\mathfrak{M})$, then all subcompacta of X also have weak $(\mathfrak{M})$; the class of compacta having (weak) $(\mathfrak{M})$ is countably productive. Every compactum X having weak $(\mathfrak{M})$ has a 0-dimensional map to a metrizable compactum. It follows from this that the following properties are equivalent: *dim* $X \leq n$, *ind* $X \leq n$, *Ind* $X \leq n$, there exists a compactum X^0 with *dim* $X^0 \leq 0$ and onto map $f : X^0 \to X$

such that $|f^{-1}x| \leq n+1$ for any $x \in X$, $n \in \mathbb{N}$.

Taras Banakh and Igor Zarichnyi

Instytut Matematyki,
Uniwersytet Humanistyczno-Przyrodniczy
Jana Kochanowskiego w Kielcach, Poland,
and Department of Mathematics,
Ivan Franko National University of Lviv, Ukraine
e-mail: tbanakh@yahoo.com; ihor.zarichnyj@gmail.com

Characterizing the Cantor bi-cube in various categories
Mathematics Subject Classification (MSC): 54E35, 54E40

Abstract. In the talk we shall present characterizations of the extended Cantor set

$$\mathsf{EC} = \Big\{ \sum_{i=-n}^{\infty} \frac{2x_i}{3^i} : n \in \mathbb{N},\ (x_i)_{i\in\mathbb{Z}} \in \{0,1\}^{\mathbb{Z}} \Big\} \subset \mathbb{R}$$

in categories having topological, uniform or asymptotic nature.

All these categories have metric spaces as their objects. Morphisms of these categories are multi-maps. A *multi-map* from a set X to a set Y is a subset $\Phi \subset X \times Y$ which can be thought as the map $\Phi : X \Rightarrow Y$ assigning to each point $x \in X$ the subset $\Phi(x) = \{y \in Y : (x, y) \in \Phi\}$ of Y and to each subset $A \subset X$ the subset $\Phi(A) = \bigcup_{a\in A} \Phi(a) \subset Y$. The inverse to $\Phi \subset X \times Y$ is the multi-map $\Phi^{-1} = \{(y, x) : (x, y) \in \Phi\} \subset Y \times X$. It assigns to each point $y \in X$ the subset $\Phi^{-1}(y) = \{x \in X : y \in \Phi(x)\}$.

By the *oscillation* of a multi-map $\Phi : X \Rightarrow Y$ between two metric spaces X, Y we understand the function ω_Φ :

$[0,\infty) \to [0,\infty]$ defined by

$$\omega_\Phi(\delta) = \sup\{\operatorname{diam}\Phi(A) : A \subset X,\ \operatorname{diam} A \le \delta\}.$$

A multi-map $\Phi : X \Rightarrow Y$ between metric spaces is called

- *micro-uniform* (= uniformly continuous) if for each $\varepsilon > 0$ there is $\delta > 0$ with $\omega_\Phi(\delta) < \varepsilon$;
- *macro-uniform* (= large scale uniform) if for each $\delta < \infty$ there is $\varepsilon < \infty$ with $\omega_\Phi(\delta) < \varepsilon$;
- *bi-uniform* if Φ is both micro- and macro-uniform.

A multi-map $\Phi : X \Rightarrow Y$ between metric spaces X, Y is called *micro-uniform* (resp. *macro-uniform*, *bi-uniform*) *equivalence* if $\Phi(X) = Y$, $\Phi^{-1}(Y) = X$ and both Φ and Φ^{-1} are micro-uniform (resp. macro-uniform, bi-uniform). In this case metric spaces X, Y are called *micro-uniformly* (resp. *macro-uniformly*, *bi-uniformly*) *equivalent*.

Our aim is to present characterization of metric spaces that are micro-, macro, or bi-uniformly equivalent to the extended Cantor set EC. The space EC is bi-uniformly equivalent to the space

$$2^{<\mathbb{Z}} = \{(x_i)_{i\in\mathbb{Z}} \in \{0,1\}^{\mathbb{Z}} : \exists n \in \mathbb{N}\ \forall i \ge n\ \ x_i = 0\}$$

endowed with the metric

$$d\big((x_i),(y_i)\big) = \max_{i\in\mathbb{Z}} 2^i |x_i - y_i|$$

and called the *Cantor bi-cube.*

It is easy to see that the map

$$f : 2^{<\mathbb{Z}} \to \mathsf{EC}, \quad f : (x_i)_{i\in\mathbb{Z}} \mapsto \sum_{i\in\mathbb{Z}} 2x_i 3^i,$$

is a bi-uniform equivalence between the Cantor bi-cube $2^{<\mathbb{Z}}$ and the extended Cantor set EC.

The Cantor bi-cube can be written as the product $2^{<\mathbb{Z}} = 2^\omega \times 2^{<\mathbb{N}}$ of

- the *Cantor micro-cube* $2^\omega = \{(x_i)_{i\in\mathbb{Z}} \in 2^{<\mathbb{Z}} : \forall i > 0\ \ x_i = 0\} \subset 2^{<\mathbb{Z}}$ and
- the *Cantor macro-cube* $2^{<\mathbb{N}} = \{(x_i)_{i\in\mathbb{Z}} \in 2^{<\mathbb{Z}} : \forall i \le 0\ \ x_i = 0\} \subset 2^{<\mathbb{Z}}$.

It is easy to see that the projection $\mathrm{pr} : 2^{<\mathbb{Z}} = 2^\omega \times 2^{<\mathbb{Z}} \to 2^{<\mathbb{N}}$ of the Cantor bi-cube onto the Cantor macro-cube is a macro-uniform equivalence. So, $2^{<\mathbb{Z}}$ is macro-uniformly equivalent to $2^{<\mathbb{N}}$. It is also clear that the Cantor micro-cube 2^ω is homeomorphic to the standard Cantor set $\mathsf{EC} \cap [0,1]$.

The space 2^ω, $2^{<\mathbb{N}}$ and $2^{<\mathbb{Z}}$ are universal in the respective classes of zero-dimensional spaces, defined as follows.

For a real number $\varepsilon > 0$ by the *ε-connected component* of a point x of a metric space X we understand the set $C_\varepsilon(x)$ of all points x' that can be linked with x by an ε-chain $x = x_0, x_1, \dots, x_n = x'$ with $\mathrm{dist}(x_{i-1}, x_i) \le \varepsilon$ for all $i \le n$. Let $\mathcal{C}_\varepsilon(X) = \{C_\varepsilon(x) : x \in X\}$ and

$$\mathrm{mesh}\,\mathcal{C}_\varepsilon(X) = \sup_{x\in X} \mathrm{diam}\, C_\varepsilon(x).$$

For two real numbers $\delta \le \varepsilon$ let

$$\theta^\varepsilon_\delta(X) = \min_{x\in X} |C_\varepsilon(x)/\mathcal{C}_\delta(X)| \quad \text{and} \quad \Theta^\varepsilon_\delta(X) = \sup_{x\in X} |C_\varepsilon(x)/\mathcal{C}_\delta(X)|$$

where $\mathcal{C}_\varepsilon(x)/\mathcal{C}_\delta(X) = \{C_\delta(y) : y \in C_\varepsilon(x)\}$.

A metric space X has

- *topological dimension zero* if each open cover of X has a disjoint open refinement;

- *micro-uniform dimension zero* if for each $\varepsilon > 0$ there is $\delta > 0$ with $\operatorname{mesh} \mathcal{C}_\delta(X) < \varepsilon$;
- *macro-uniform dimension zero* if for each $\delta < \infty$ there is $\varepsilon < \infty$ with $\operatorname{mesh} \mathcal{C}_\delta(X) < \varepsilon$;
- *bi-uniform dimension zero* if X has both micro- and macro-uniform dimensions zero.

Theorem 1. *A metric space X has topological (resp. micro-uniform, macro-uniform, bi-uniform) dimension zero if and only if X is topologically (resp. micro-uniformly, macro-uniformly, bi-uniformly) equivalent to an ultrametric space.*

We recall that a metric space X is caled an *ultrametric space* if its metric d satisfies the strong triangle inequality $d(x,z) \leq \max\{d(x,y), d(y,z)\}$ for all $x, y, z \in X$.

The following theorems present universal properties of the Cantor micro-, macro, and bi-cubes.

Theorem 2 (Classics). *A metric space X is topologically equivalent to a subspace of the Cantor bi-cube $2^{<\mathbb{Z}}$ if and only if X is separable and has topological dimension zero.*

Theorem 3. *A metric space X is micro-equivalent to a subspace of the Cantor bi-cube $2^{<\mathbb{Z}}$ if and only if X is separable, has micro-uniform dimension zero and there is $\varepsilon > 0$ such that $\Theta^\varepsilon_\delta(X)$ is finite for all $0 < \delta \leq \varepsilon$.*

Theorem 4 (Dranishnikov-Zarichnyi). *A metric space X is macro-equivalent to a subspace of the Cantor bi-cube $2^{<\mathbb{Z}}$ if and only if X has macro-uniform dimension zero and there is $\delta > 0$ such that $\Theta^\varepsilon_\delta(X)$ is finite for all $\delta \leq \varepsilon < \infty$.*

Theorem 5. *A metric space X is bi-equivalent to a subspace of the Cantor bi-cube $2^{<\mathbb{Z}}$ if and only if X has bi-uniform dimension zero and $\Theta^\varepsilon_\delta(X)$ is finite for all $0 < \delta \leq \varepsilon < \infty$.*

Next, we present characterizations of the Cantor bi-cube $2^{<\mathbb{Z}}$ in various categories. We start with the following classical result of Brouwer:

Theorem 6 (Brouwer). *For a metric space X the following conditions are equivalent:*

(i) *X is topologically equivalent to the Cantor micro-cube 2^ω;*

(ii) *X is micro-uniformly equivalent to 2^ω;*

(iii) *X is bi-uniformly equivalent to 2^ω;*

(iv) *X is a zero-dimensional metric compact space without isolated points.*

This theorem of Brouwer implies the following (well-known) topological characterization of the Cantor bi-cube:

Theorem 7 (Topological Characterization of $2^{<\mathbb{Z}}$). *A metric space X is topologically equivalent to the Cantor bi-cube $2^{<\mathbb{Z}}$ if and only if*

(i) *X has topological dimension zero;*

(ii) *X is separable, locally compact and non-compact;*

(iii) *X has no isolated points.*

In the next three theorems we present characterizations of the extended Cantor set in the micro-, macro-, and bi-uniform categories.

Theorem 8 (Micro-Uniform Characterization of $2^{<\mathbb{Z}}$). *A metric space X is micro-uniformly equivalent to the Cantor bi-cube $2^{<\mathbb{Z}}$ if and only if* *(i)* *X is a non-compact complete metric space of micro-uniform dimension zero;*

(ii) *there is* $\varepsilon > 0$ *such that* $\Theta_\delta^\varepsilon(X)$ *is finite for all positive* $\delta \le \varepsilon$*;*

(iii) $\lim_{\delta \to +0} \theta_\delta^\varepsilon(X) = \infty$.

Theorem 9 (Macro-Uniform Characterization of $2^{<\mathbb{Z}}$). *A metric space X is macro-uniformly equivalent to the Cantor bi-cube* $2^{<\mathbb{Z}}$ *if and only if*

(i) *X has macro-uniform dimension zero;*

(ii) *there is* $\delta > 0$ *such that* $\Theta_\delta^\varepsilon(X)$ *is finite for all positive* $\varepsilon \ge \delta$*;*

(iii) $\lim_{\varepsilon \to \infty} \theta_\delta^\varepsilon(X) = \infty$.

Theorem 10 (Bi-Uniform Characterization of $2^{<\mathbb{Z}}$). *A metric space X is bi-uniformly equivalent to the Cantor bi-cube* $2^{<\mathbb{Z}}$ *if and only if*

(i) *X is a complete metric space of bi-uniform dimension zero;*

(ii) $\Theta_\delta^\varepsilon(X)$ *is finite for all* $0 < \delta \le \varepsilon < \infty$*;*

(iii) $\lim_{\varepsilon \to \infty} \theta_\delta^\varepsilon(X) = \infty$ *for all* $\delta < \infty$*;*

(iv) $\lim_{\delta \to +0} \theta_\delta^\varepsilon(X) = \infty$ *for all* $\varepsilon > 0$.

We shall apply these characterization theorem to classification of isometrically homeogeneous metric spaces. A metric space X is called

- *isometrically homogeneous* if for any points $x, y \in X$ there is a bijective isometry $f : X \to X$ such that $f(x) = y$;
- *proper* if X is unbounded and each closed ball in X is compact.

Corollary 1. *A proper isometrically homogeneous space X is*

(i) topologically equivalent to $2^{<\mathbb{Z}}$ iff X is uncountable and has topological dimension zero;

(ii) micro-uniformly equivalent to $2^{<\mathbb{Z}}$ iff X is uncountable and has micro-uniform dimension zero;

(iii) bi-uniformly equivalent to $2^{<\mathbb{Z}}$ iff X is uncountable and has bi-uniform dimension zero;

(iv) macro-uniformly equivalent to $2^{<\mathbb{Z}}$ iff X has macro-uniform dimension zero.

References

[1] T. Banakh, I. Zarichnyi, *Characterizing the Cantor bi-cibe set in asymptotic categories*, ArXiv:0908.3687.

[2] A. Dranishnikov, M. Zarichnyi, *Universal spaces for asymptotic dimension,* Topology Appl. **140** (2004), 203–225.